

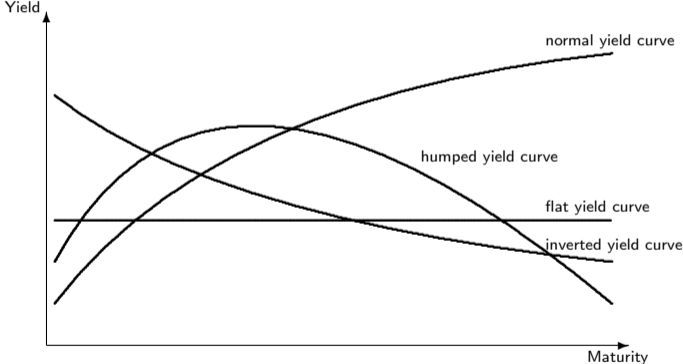
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Yield curves

Term structures and yield curves

- ▶ The yield of risk-free bonds changes with its time to maturity, this is referred to as the term structure of interest rates
- ▶ The graphical representation of the term structure is known as the yield curve
- ▶ The yield curve also changes over time and such changes to future yield curves can be predicted from the current yield curve

Typical yield curves



Investing into long-term and short-term bonds

- ▶ Investors can invest into a single bond with a long maturity, yielding a total return of $(1 + r_{0,T})^T$
- ▶ Investors can invest into a bond with a shorter maturity first, yielding a return of $(1 + r_{0,T_1})^{T_1}$
- ▶ Investors can after this bond has matured, invest into another bond with a short maturity
- ▶ The yield he receives is not known, but expectations can be formed, yielding a return of $(1 + \mathbb{E}[r_{T_1,T-T_1}])^{T-T_1}$
- ▶ The total return of the investor is then $(1 + r_{0,T_1})^{T_1} (1 + \mathbb{E}[r_{T_1,T-T_1}])^{T-T_1}$

Expected future yields

- ▶ Such that long-term and short-term bonds are demanded, the total return of both investment strategies must be the same

$$\Rightarrow (1 + r_{0,T})^T = (1 + r_{0,T_1})^{T_1} (1 + \mathbf{E}[r_{T_1,T-T_1}])^{T-T_1}$$

$$\Rightarrow 1 + \mathbf{E}[r_{T_1,T-T_1}] = \sqrt[T-T_1]{\frac{(1+r_{0,T})^T}{(1+r_{0,T_1})^{T_1}}}$$

- ▶ We can now interpret the yield curve as showing expectations about future interest rates

Yield curves as predictors

- ▶ Short-term interest rates are heavily influenced by monetary policy
- ▶ Expectations about short-term interest rates will reflect expectations about monetary policy
- ▶ Monetary policy is influenced by inflation and the growth of the economy
- ▶ The yield curve can be used to predict macroeconomic performance of an economy

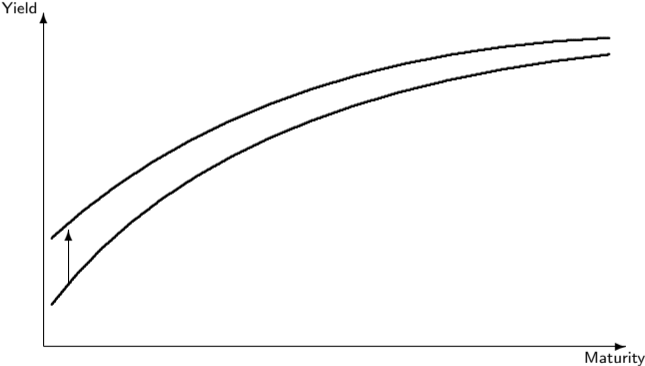
Upward sloping yield curve

- ▶ Consider two bonds with short maturities T_1 and $T - T_1$ and a bond with long maturity T
- ▶ $(1 + r_{0,T_1})^{T_1} (1 + r_{0,T-T_1})^{T-T_1} < (1 + \max\{r_{0,T_1}, r_{0,T-T_1}\})^T$
- ▶ Assume the yield curve is upward sloping, longer times to maturity have a higher yield: $\max\{r_{0,T_1}, r_{0,T-T_1}\} < r_{0,T}$
- ⇒ $(1 + r_{0,T_1})^{T_1} (1 + r_{0,T-T_1})^{T-T_1} < (1 + \max\{r_{0,T_1}, r_{0,T-T_1}\})^T < (1 + r_{0,T})^T$
- ⇒ $E[r_{T_1,T-T_1}] > r_{0,T-T_1}$
- ⇒ Short-term interest rates are expected to rise

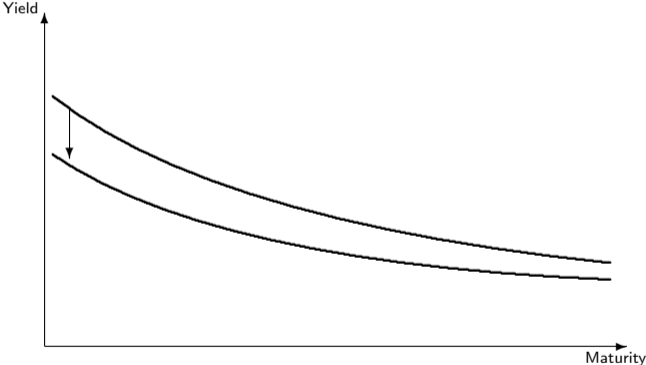
Downward sloping yield curve

- ▶ Consider two bonds with short maturities T_1 and $T - T_1$ and a bond with long maturity T
- ▶ $(1 + r_{0,T_1})^{T_1} (1 + r_{0,T-T_1})^{T-T_1} > (1 + \min \{r_{0,T_1}, r_{0,T-T_1}\})^T$
- ▶ Assume the yield curve is downward sloping, longer times to maturity have a lower yield: $\min \{r_{0,T_1}, r_{0,T-T_1}\} > r_{0,T}$
- ⇒ $(1 + r_{0,T_1})^{T_1} (1 + r_{0,T-T_1})^{T-T_1} > (1 + \max \{r_{0,T_1}, r_{0,T-T_1}\})^T > (1 + r_{0,T})^T$
- ⇒ $E[r_{T_1,T-T_1}] < r_{0,T-T_1}$
- ⇒ Short-term interest rates are expected to fall

Expected interest rate changes for normal yield curves



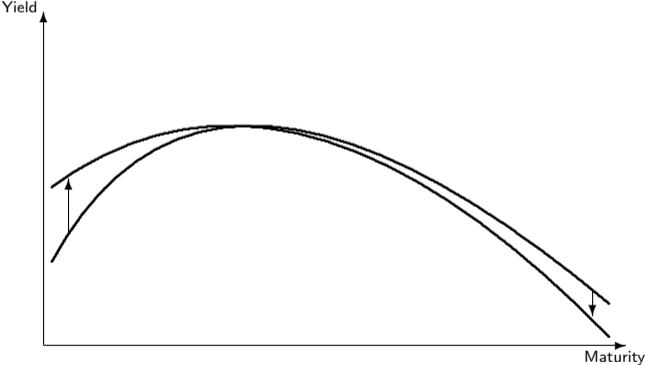
Expected interest rate changes for inverted yield curves



Expected interest rate changes for flat yield curves



Expected interest rate changes for humped yield curves



Prediction macroeconomic performance

- ▶ The expectations theory of the yield curve asserts that the term structure reflects expectations about future short-term interest rates
- ▶ The steepness of the slope indicates the magnitude of the change in the short-term interest rate
- ▶ Upward sloping yield curves indicate future short-term interest rates to rise
- ⇒ The market expects the economy to perform well
- ▶ Downward sloping yield curves indicate future short-term interest rates to fall
- ⇒ The market expects a recession



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