Andreas Krause

Yield curves

- Interest rates on bonds vary depending on the time to maturity of the bond.
- As the time to maturity is increasing, the length of time an investor foregoes access to his money would be increasing, implying that interest rates should increase with time to maturity, although bonds can be sold at any time making this argument less viable. Some element can be rescued by the fact that when selling bonds, there is a risk on the price obtained and this risk increases the longer the time to maturity of the bond. However, interest rates are not always increasing in the time to maturity of the bond, but might well be decreasing.
- We will here explore an alternative explanation for the observed interest rates.

Term structures and yield curves

Term structures and yield curves

→ The interest rate of bonds is often referred to as the yield, the interest rate often refers to the coupon payment the bond makes, that is the interest the bond holder pays, The yield will also be affected by the value of the bond, which is the present value of all coupon payments and the repayment of the bond itself.

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  - We observe that the yield on risk-free government bonds are not identical, but vary with the time to maturity, at time significantly, at other time there is not much difference.
  - These yields for the different times to maturity is called the 'term structure'.
- ▶ When the term structure is shown graphically, this is called the 'yield curve'.
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  - Yield curves are not fixed but vary over time considerably.
  - These variations are not random but they can very well be predicted using only the current yield curve.

→ We will seek to explain the structure of the yield curve and how they will evolve over time.

Term structures and yield curves

- The yield of risk-free bonds changes with its time to maturity

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Typical yield curves

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- We will look at how the yield changes as the time to maturity increases.
- The usual assumption for a yield curve is that it is upwards sloping, with the argument being that longer times to maturity need a premium as the investor gives up its liquidity for longer.
- This is often referred to as the normal yield curve.
- But we also find yield curves that are downward sloping,
- called an inverted yield curve.
- In some instances the yields for all maturities might be identical,
- which is knows as a flat yield curve.
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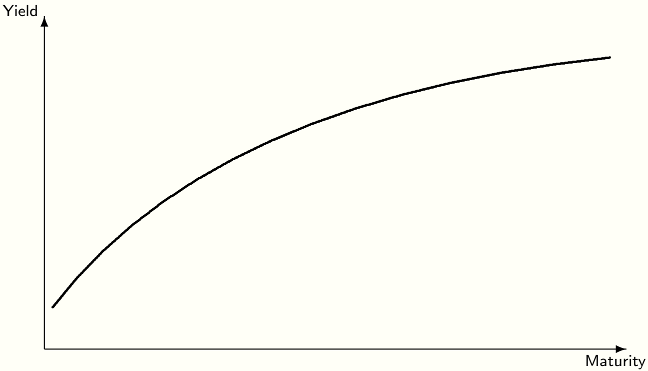

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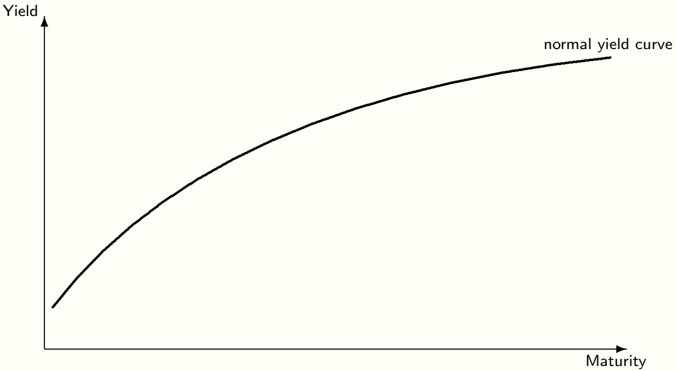

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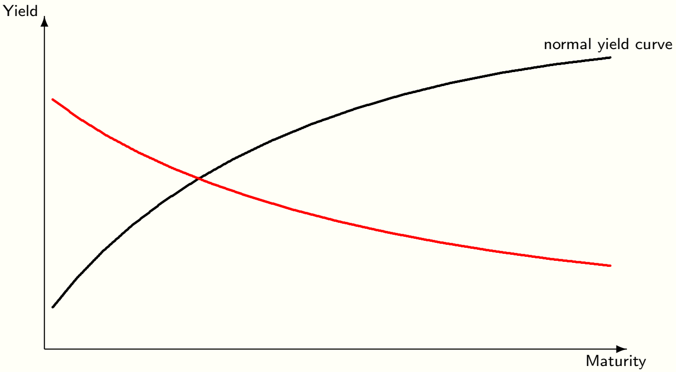

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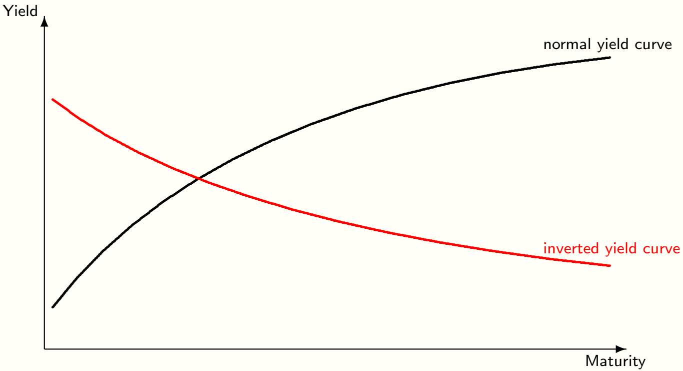

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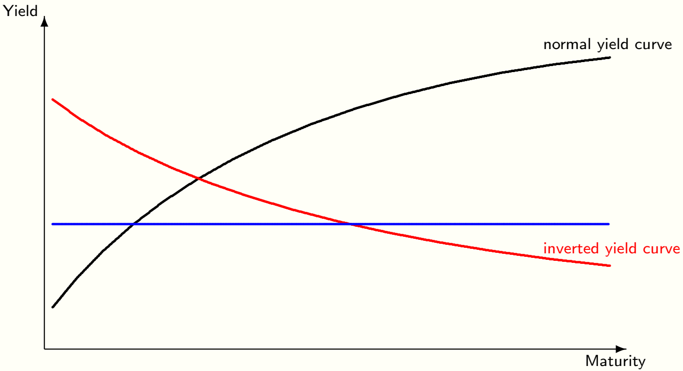

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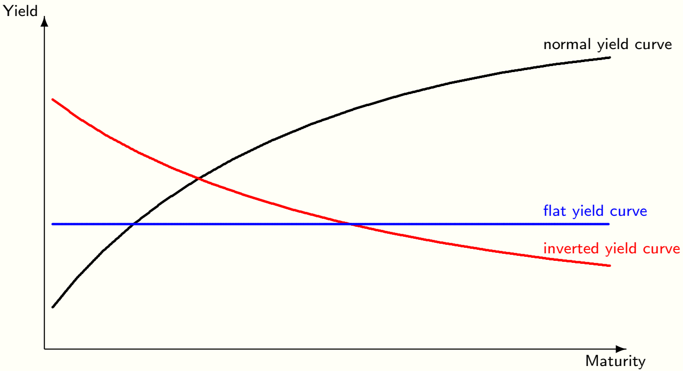

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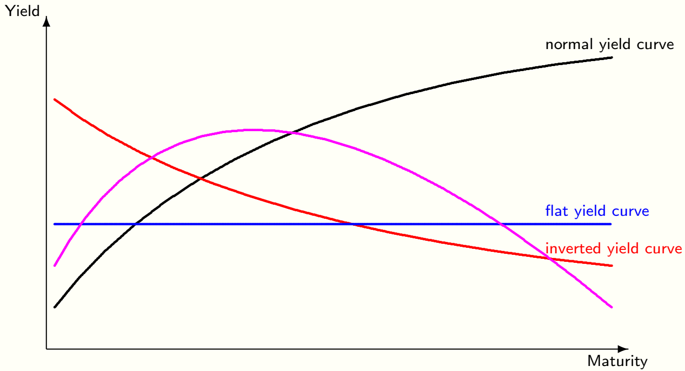

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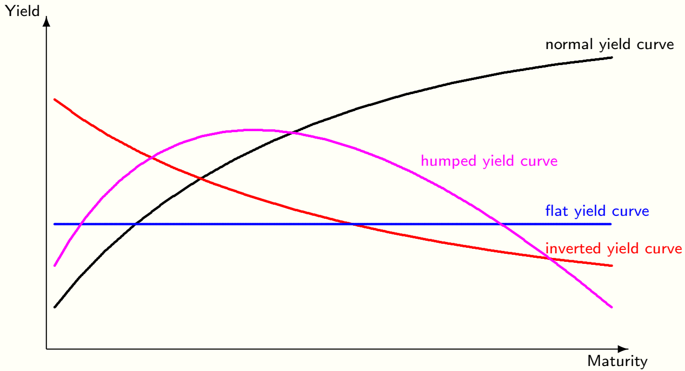

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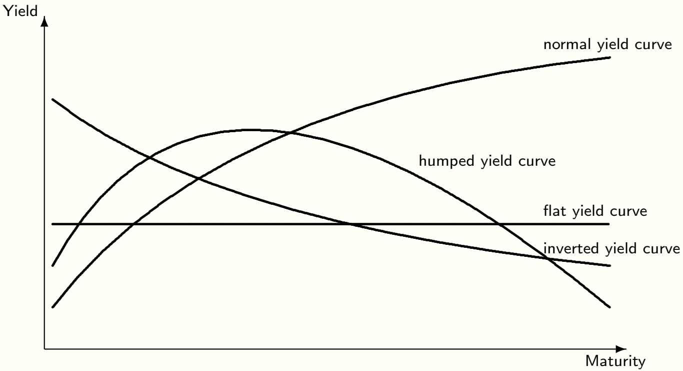
Yield
normal yield curve
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Maturity

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Investing into long-term and short-term bonds

Investing into long-term and short-term bonds

→ We will now look at investing into bonds and how we can combine long-term and short-term bonds.

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  - Assume an investor chooses a single bond with a long time to maturity.
  - If we assume that interest is accumulated in the bond rather than paid out, then the final payment from the bond is as given in the *formula*. Assuming that coupons are accumulated like in zero-bonds (bonds that pay no coupon) as a convenience and only simplifies the analysis.
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  - Alternatively the investor could choose two bonds successively. He will first invest into a bond with a shorter time to maturity.
  - The final repayment of this bond at its maturity is as given in the *formula*.
- After this bond has matured, the investor can use the total amount he received and invest into another bond with a shorter maturity that is repaid at the same time as the long-term bond.
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  - As the bond is only bought in the future, its yield is not known, but we have to form expectations about the yield it will provide.
  - This will then give an expected total return as given in the *formula*.
- By investing to these two short-term bonds, the total return is given as in the *formula*. The amount returned from the first bond is then fully reinvested into the second bond.

→ We can now see how this approach can result in explaining the yield curve.

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- Investors can invest into a single bond with a long maturity

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- Investors can invest into a single bond with a long maturity, yielding a total return of $(1+r_{0,T})^T$

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- The yield he receives is not known, but expectations can be formed

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- Investors can invest into a single bond with a long maturity, yielding a total return of $(1 + r_{0,T})^T$
- Investors can invest into a bond with a shorter maturity first, yielding a return of $(1 + r_{0,T_1})^{T_1}$
- Investors can after this bond has matured, invest into another bond with a short maturity
- The yield he receives is not known, but expectations can be formed, yielding a return of $(1 + \mathsf{E}\left[r_{T_1,T-T_1}\right])^{T-T_1}$

Investing into long-term and short-term bonds

→ We will now look at investing into bonds and how we can combine long-term and short-term bonds.

- ▶
    - Assume an investor chooses a single bond with a long time to maturity.
    - If we assume that interest is accumulated in the bond rather than paid out, then the final payment from the bond is as given in the *formula*. Assuming that coupons are accumulated like in zero-bonds (bonds that pay no coupon) as a convenience and only simplifies the analysis.
- ▶
    - Alternatively the investor could choose two bonds successively. He will first invest into a bond with a shorter time to maturity.
    - The final repayment of this bond at its maturity is as given in the *formula*.
- ▶ After this bond has matured, the investor can use the total amount he received and invest into another bond with a shorter maturity that is repaid at the same time as the long-term bond.
- ▶
    - As the bond is only bought in the future, its yield is not known, but we have to form expectations about the yield it will provide.
    - This will then give an expected total return as given in the *formula*.
- ▶ By investing to these two short-term bonds, the total return is given as in the *formula*. The amount returned from the first bond is then fully reinvested into the second bond.

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- The total return of the investor is then $(1 + r_{0,T_1})^{T_1}(1 + \mathsf{E}[r_{T_1,T-T_1}])^{T-T_1}$

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Expected future yields

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- → We can now obtain the expected yield of a bond in the future from these two investment strategies.
- ▶ If investing into the long-term bond were giving a higher outcome, then there would be no demand for the short-term bonds; if investing into the short-term bond were giving a higher outcome, then there would be no demand for the long-term bonds. Thus the two outcomes must be identical.
- ▶ [⇒] *Formula*
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- ▶ Thus the yield curve includes information about the expected yield of bonds in the future. Here it is the yield of a bond with time to maturity $T - T_1$ in T_1 time periods.
- → With the yield curve giving us the expectation about future yields, we can now provide a more general assessment of the information contained in yield curves.

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Yield curves as predictors

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→ We can now use yield curves as predictors for future bond yields.

- We know that short-term bond yields are directly affected by monetary policy as the decisions of central banks on interest rates and money supply affect the lending costs of banks which get transmitted onto the yields of short-term bonds.
- If we have formed expectations about the short-term yields, this would mainly reflect any expectations about the monetary policy that will be enacted.
- Monetary policy decisions are driven by concerns about economic growth and inflation, mainly, thus will be driven by macroeconomic concerns.
- We can thus use the yield curve not only to predict future monetary policy decisions, but it will also give an indication about the expected state of the economy in general.

→ The yield curve provides a tool to determine the market perception of future monetary policy decisions and the macroeconomic conditions in general.

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Upward sloping yield curve

Upward sloping yield curve

- → We can now assess the properties of features of the yield curve, starting with a yield curve that is rising.
- ▶
    - We again consider two short term bonds
    - and a long term bond, where the successive investment into the two short-term bonds last as long as the long-term bond.
- ▶ We now compare the outcome of investing into the two short-term bonds but using the current yield of the bonds, rather than the expected yields of the bond to be invested into in the future (left hand side), with a long-term investment (right hand side). Assume a hypothetical long-term bond that has a yield of either of these short-term bonds, one of these yields will be lower than the other and hence investing with the higher of these yield for the full length would give a higher outcome.
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    - If the yield curves is upwards sloping, the yields with longer times to maturity are higher.
    - In this case the yield to maturity of both short-term bonds will be lower than that of the long-term bond.
- ▶ [⇒] We can thus obtain the relationship of the final payoff with the short-term bond and the long-term bond. Using the relationship of the yields we see that the outcome from investing into the short-term bonds would yield a lower outcome than the long-term bond.
- ▶ [⇒] As the two investment strategies of using the short-term bonds and long-term bond, respectively, must be the same and we have to use expected yields as above, we have to increase the left-hand side and this is only possible if we increase the yield of the future short-term bond (the initial short-term bond is given by current market data). Thus the expected yield of this future short-term bond will be higher than the current yield.
- ▶ [⇒] We can conclude that we expect short-term bond yields to increase in the future.
- → We thus see that an upward sloping yield curve implies that short-term interest rates are expected to rise.

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- $\left(1 + r_{0,T_1}\right)^{T_1} \left(1 + r_{0,T-T_1}\right)^{T-T_1} < \left(1 + \max\left\{r_{0,T_1}, r_{0,T-T_1}\right\}\right)^{T}$
- Assume the yield curve is upward slowing, longer times to maturity have a higher yield: $\max\left\{r_{0,T_1}, r_{0,T-T_1}\right\} < r_{0,T}$

$\Rightarrow$ $\left(1 + r_{0,T_1}\right)^{T_1} \left(1 + r_{0,T-T_1}\right)^{T-T_1} < \left(1 + \max\left\{r_{0,T_1}, r_{0,T-T_1}\right\}\right)^{T} < \left(1 + r_{0,T}\right)^{T}$

Upward sloping yield curve

→ We can now assess the properties of features of the yield curve, starting with a yield curve that is rising.

-
  - We again consider two short term bonds
  - and a long term bond, where the successive investment into the two short-term bonds last as long as the long-term bond.
- We now compare the outcome of investing into the two short-term bonds but using the current yield of the bonds, rather than the expected yields of the bond to be invested into in the future (left hand side), with a long-term investment (right hand side). Assume a hypothetical long-term bond that has a yield of either of these short-term bonds, one of these yields will be lower than the other and hence investing with the higher of these yield for the full length would give a higher outcome.
-
  - If the yield curves is upwards sloping, the yields with longer times to maturity are higher.
  - In this case the yield to maturity of both short-term bonds will be lower than that of the long-term bond.
- [⇒] We can thus obtain the relationship of the final payoff with the short-term bond and the long-term bond. Using the relationship of the yields we see that the outcome from investing into the short-term bonds would yield a lower outcome than the long-term bond.
- [⇒] As the two investment strategies of using the short-term bonds and long-term bond, respectively, must be the same and we have to use expected yields as above, we have to increase the left-hand side and this is only possible if we increase the yield of the future short-term bond (the initial short-term bond is given by current market data). Thus the expected yield of this future short-term bond will be higher than the current yield.
- [⇒] We can conclude that we expect short-term bond yields to increase in the future.

→ We thus see that an upward sloping yield curve implies that short-term interest rates are expected to rise.

Upward sloping yield curve

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$\Rightarrow$ $\mathsf{E}\left[r_{T_1,T-T_1}\right] > r_{0,T-T_1}$

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$\Rightarrow$ $\mathsf{E}\left[r_{T_1,T-T_1}\right] > r_{0,T-T_1}$

$\Rightarrow$ Short-term interest rates are expected to rise

Upward sloping yield curve

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$\Rightarrow$ $\mathsf{E}\left[r_{T_1,T-T_1}\right]>r_{0,T-T_1}$

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Upward sloping yield curve

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Downward sloping yield curve

Downward sloping yield curve

→ Rather than a rising yield curve, we now consider a falling yield curve.

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    - We again consider two short term bonds
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    - If the yield curves is downwards sloping, the yields with longer times to maturity are lower.
    - In this case the yield to maturity of both short-term bonds will be higher than that of the long-term bond.
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- [⇒] We can conclude that we expect short-term bond yields to decrease in the future.

→ We thus see that a downward sloping yield curve implies that short-term interest rates are expected to fall.

Downward sloping yield curve

- Consider two bonds with short maturities T_1 and $T - T_1$

Downward sloping yield curve

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Downward sloping yield curve

- Consider two bonds with short maturities T_1 and $T - T_1$ and a bond with long maturity T

Downward sloping yield curve

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Downward sloping yield curve

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Downward sloping yield curve

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Downward sloping yield curve

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⇒ $\mathsf{E}\left[r_{T_1,T-T_1}\right] < r_{0,T-T_1}$

⇒ Short-term interest rates are expected to fall

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- [⇒] We can thus obtain the relationship of the final payoff with the short-term bond and the long-term bond. Using the relationship of the yields we see that the outcome from investing into the short-term bonds would yield a higher outcome than the long-term bond.
- [⇒] As the two investment strategies of using the short-term bonds and long-term bond, respectively, must be the same and we have to use expected yields as above, we have to decrease the left-hand side and this is only possible if we decrease the yield of the future short-term bond (the initial short-term bond is given by current market data). Thus the expected yield of this future short-term bond will be lower than the current yield.
- [⇒] We can conclude that we expect short-term bond yields to decrease in the future.

→ We thus see that a downward sloping yield curve implies that short-term interest rates are expected to fall.

Downward sloping yield curve

- Consider two bonds with short maturities T_1 and $T - T_1$ and a bond with long maturity T
- $(1 + r_{0,T_1})^{T_1} (1 + r_{0,T-T_1})^{T-T_1} > (1 + \min\{r_{0,T_1}, r_{0,T-T_1}\})^{T}$
- Assume the yield curve is downward slowing, longer times to maturity have a lower yield: $\min\{r_{0,T_1}, r_{0,T-T_1}\} > r_{0,T}$

$\Rightarrow$ $(1 + r_{0,T_1})^{T_1} (1 + r_{0,T-T_1})^{T-T_1} > (1 + \max\{r_{0,T_1}, r_{0,T-T_1}\})^{T} > (1 + r_{0,T})^{T}$

$\Rightarrow$ $\mathsf{E}\left[r_{T_1,T-T_1}\right] < r_{0,T-T_1}$

$\Rightarrow$ Short-term interest rates are expected to fall

Downward sloping yield curve

→ Rather than a rising yield curve, we now consider a falling yield curve.

-
  - We again consider two short term bonds
  - and a long term bond, where the successive investment into the two short-term bonds last as long as the long-term bond.
- We now compare the outcome of investing into the two short-term bonds but using the current yield of the bonds, rather than the expected yields of the bond to be invested into in the future (left hand side), with a long-term investment (right hand side). Assume a hypothetical long-term bond that has a yield of either of these short-term bonds, one of these yields will be lower than the other and hence investing with the lower of these yield for the full length would give a lower outcome.
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  - If the yield curves is downwards sloping, the yields with longer times to maturity are lower.
  - In this case the yield to maturity of both short-term bonds will be higher than that of the long-term bond.
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Expected interest rate changes for normal yield curves

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→ We can illustrate graphically how we expect the yield curve to change if the current yield curve is upward sloping.

- We again look at the yield for a variety to times to maturity of the bond.
- The current yield curve is upward sloping.
- Then the interest rates are expected to increase throughout. The larger the slope of the yield curve, the larger the increase.

→ Thus, if the yield curve is upward sloping, it is expected to shift upwards in the future.

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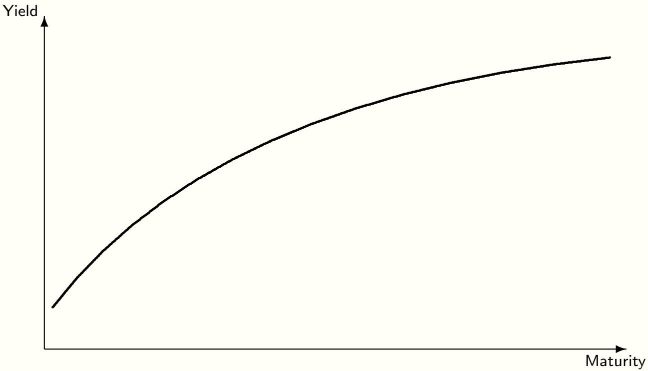

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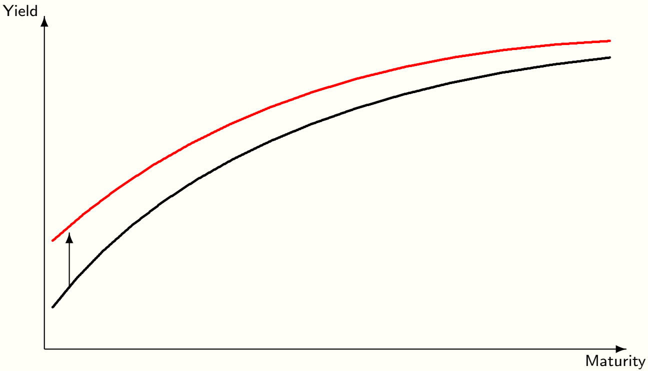

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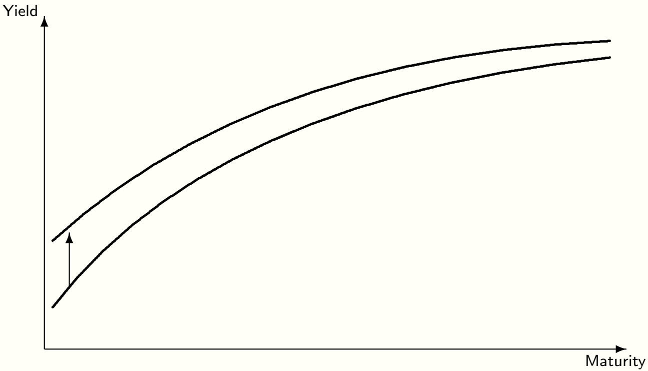
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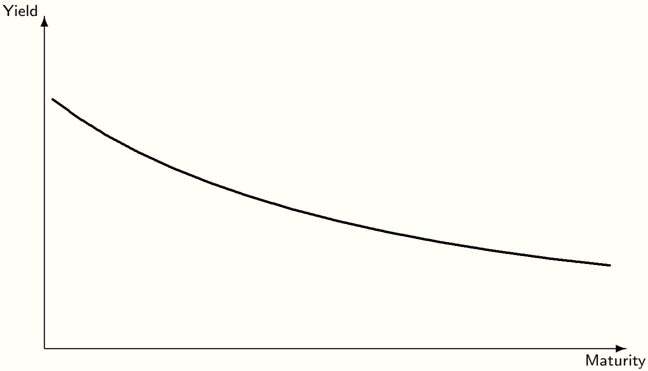

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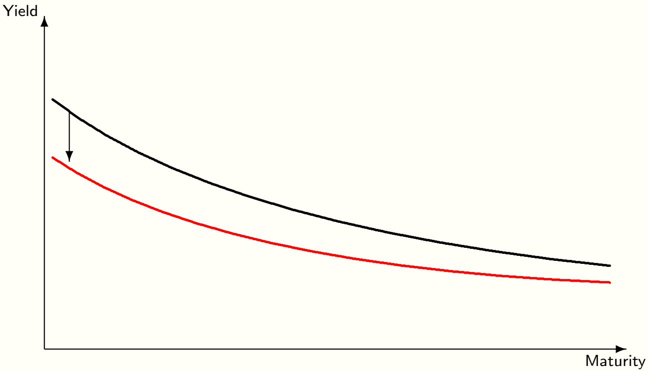

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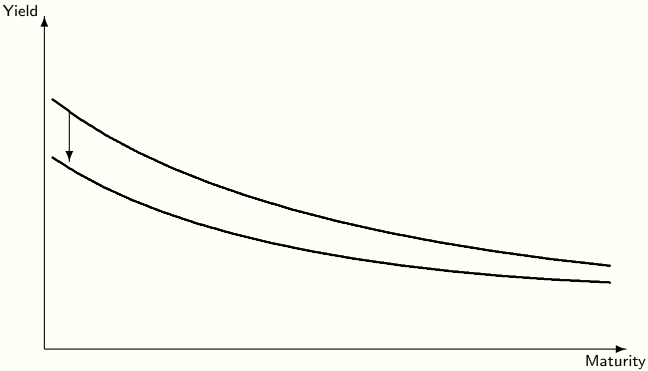
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Expected interest rate changes for flat yield curves

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→ We can illustrate graphically how we expect the yield curve to change if the current yield curve is flat.

- We again look at the yield for a variety to times to maturity of the bond.
- The current yield curve is flat.
- Then the interest rates are expected to remain unchanged. Using the same arguments as above we can show that expectations are identical to the current yield.

→ Thus, if the yield curve is flat, the yield is expected to remain unchanged.

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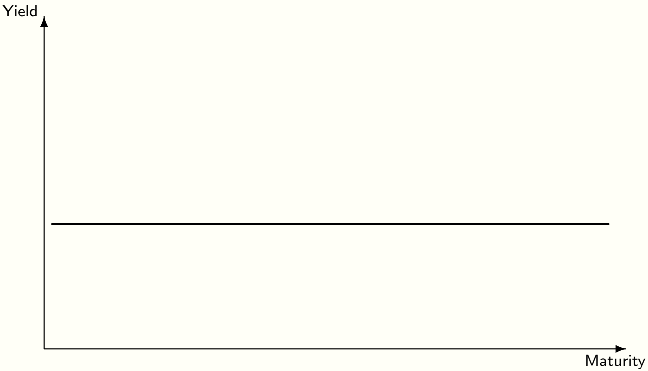

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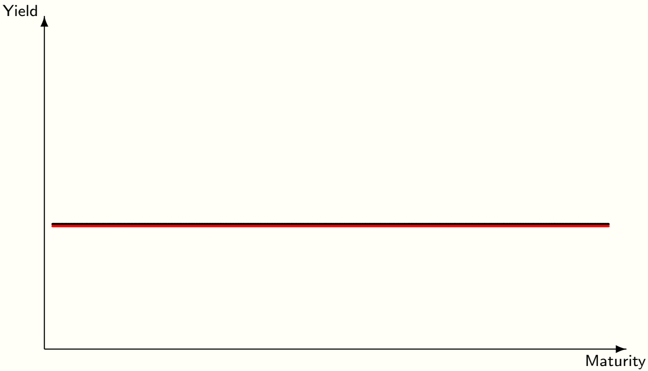

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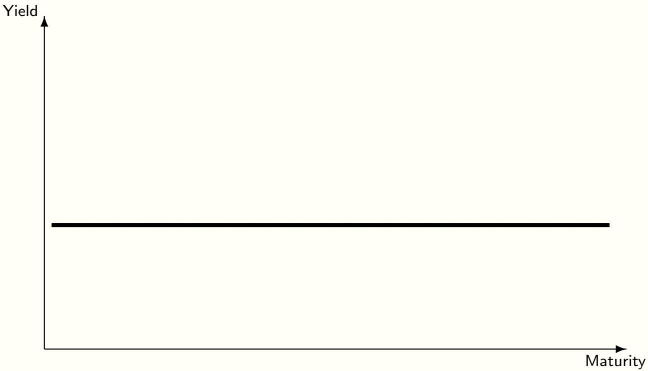

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→ We can illustrate graphically how we expect the yield curve to change if we currently have a humped yield curve.

- We again look at the yield for a variety to times to maturity of the bond.
- The current yield curve is humped.
- In this case the yields for the upward sloping part are expected be increasing, while the yields for the downwards sloping part are expected to fall. As long as the slope between two points increasing (decreasing) the yield is expected to increase (decrease) for bonds with a maturity equalling the difference between these two points. The time into the future for this expectation is the starting point of the line.

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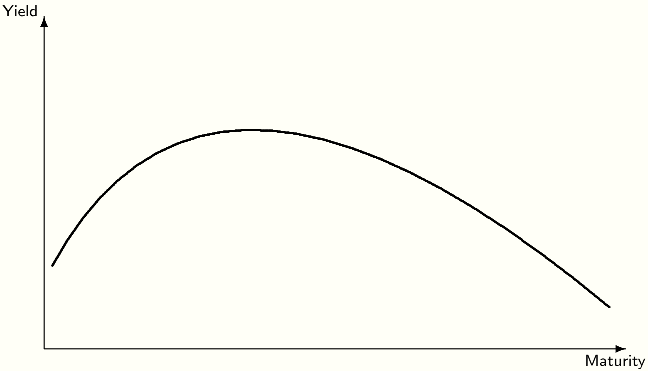

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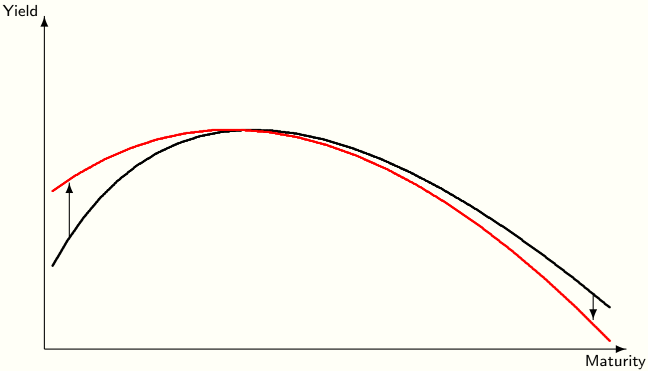

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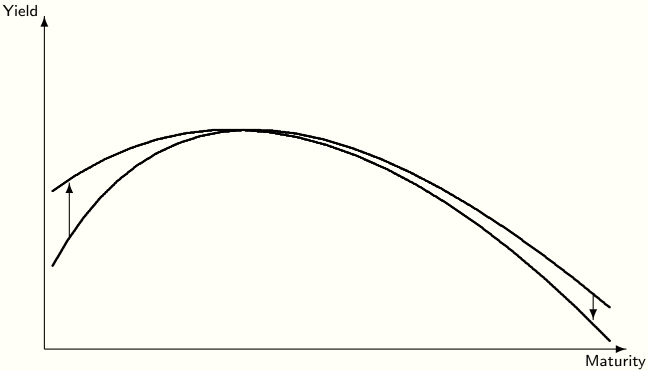
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Prediction macroeconomic performance

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→ We can now use the predictions of the future yield to make an assessment of the expected macroeconomic conditions in the future.

- We have seen that this so called 'expectations hypothesis' for yield curves allows us to infer market expectations of the yields on bonds in the future.
- How steep the yield curve is indicates how big the expected change is from the current state,
- If the yield curve is upwards sloping, the short-term yields are expected to rise.
- [⇒] As higher interest rates are commonly associated with higher economic growth, this would indicate an expectation that the economy will be performing well.
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→ Thus the yield curve does not only serve as an indicator for expected future yields, but indirectly makes an assessment of the expected performance of the economy.

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Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: `mnsak@bath.ac.uk`