

Yield curves



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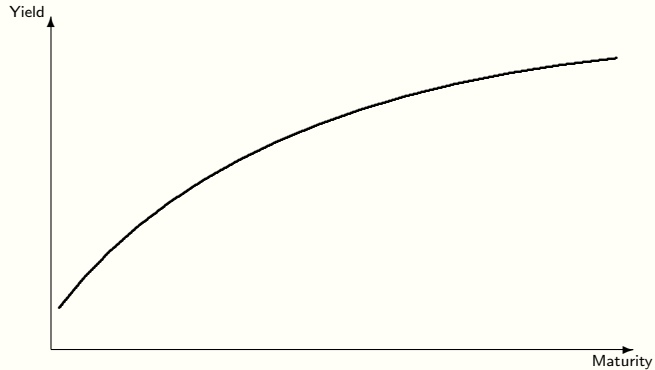


# Typical yield curves

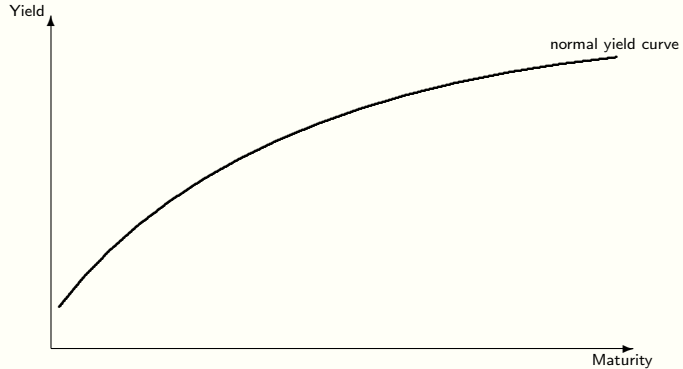
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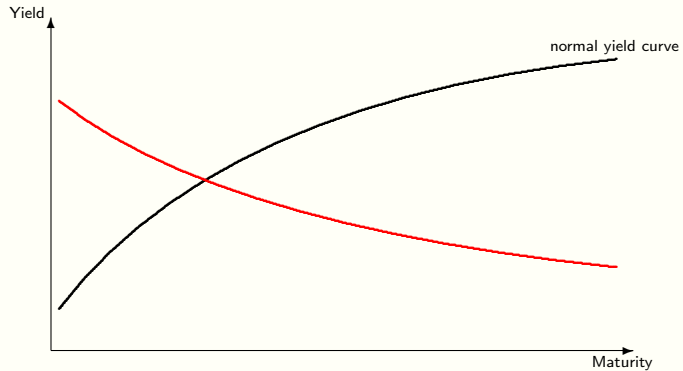
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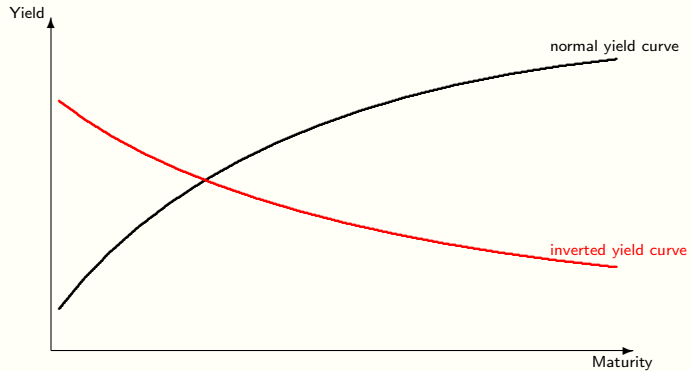
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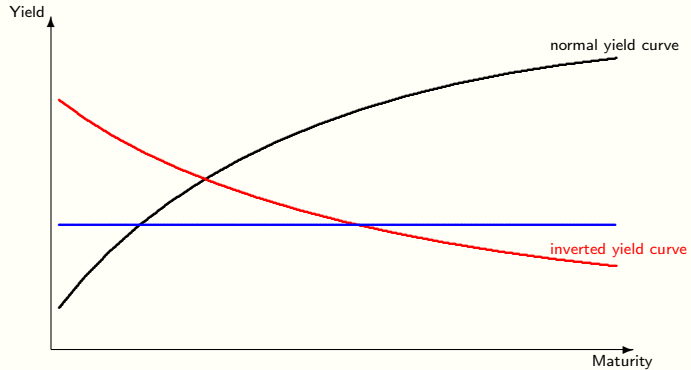
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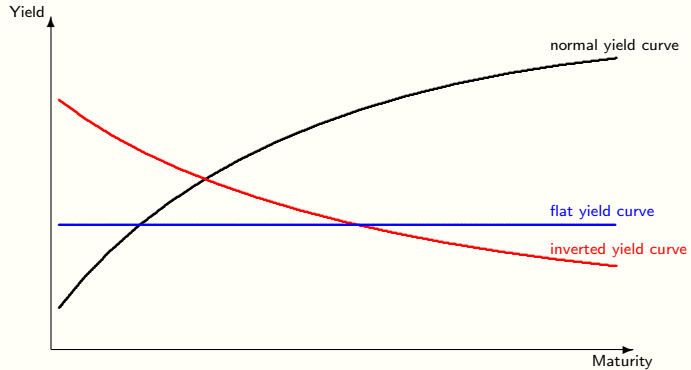
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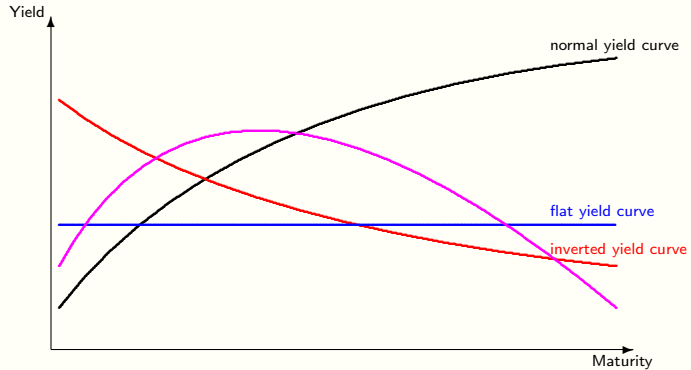


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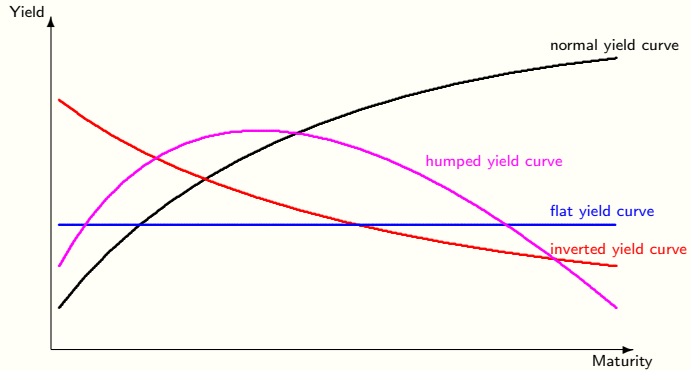




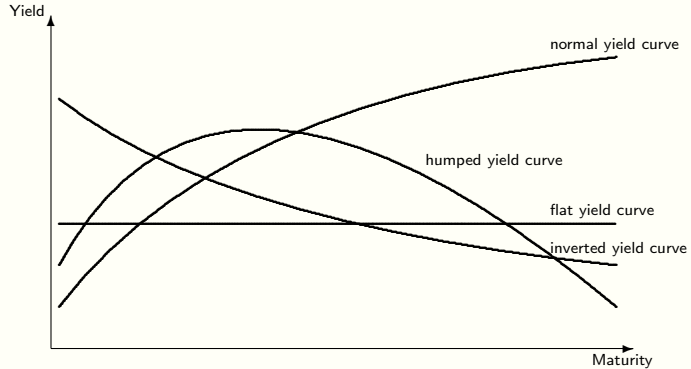
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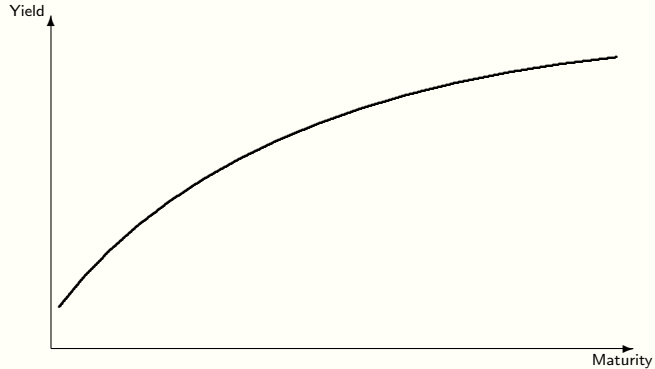
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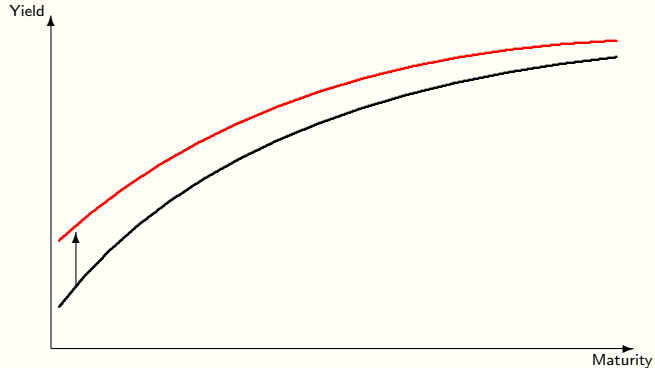


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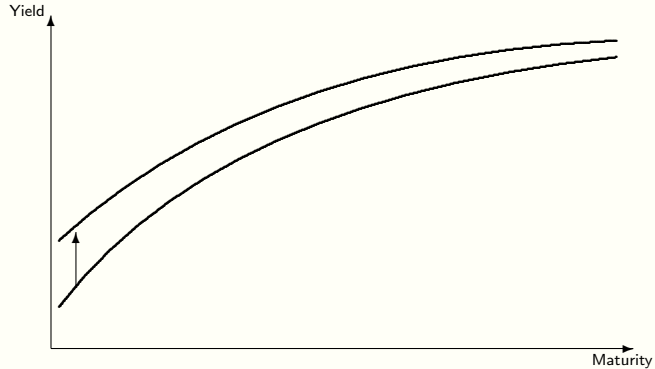




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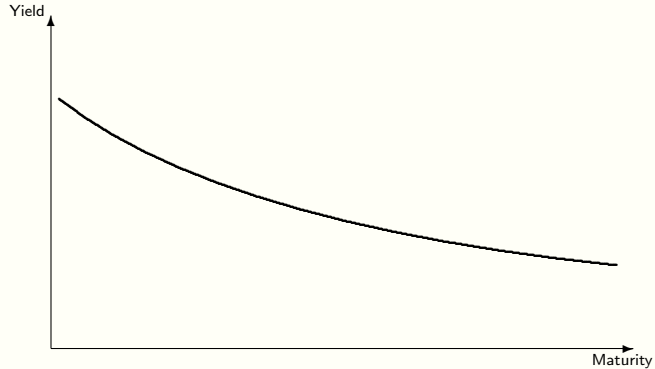


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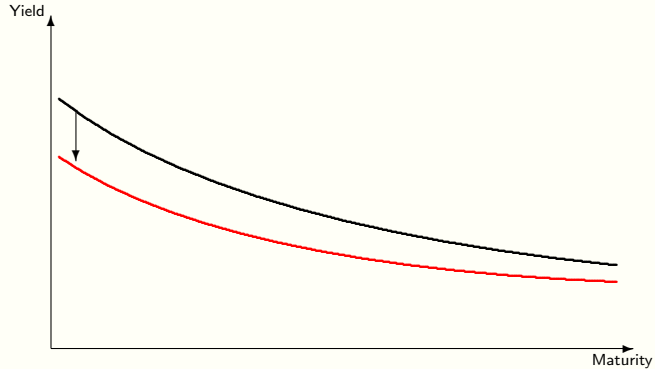
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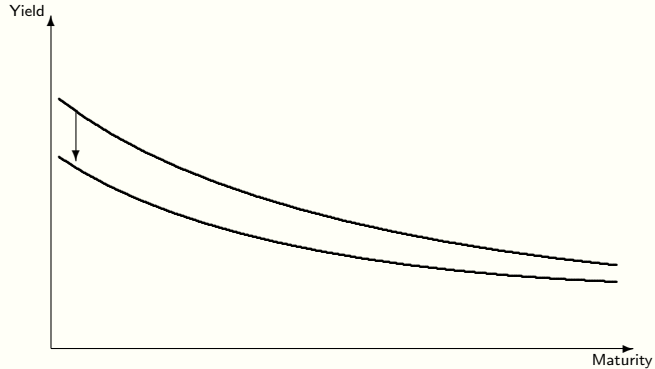
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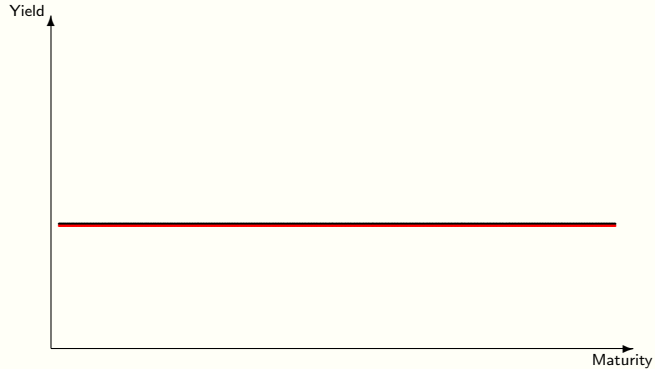
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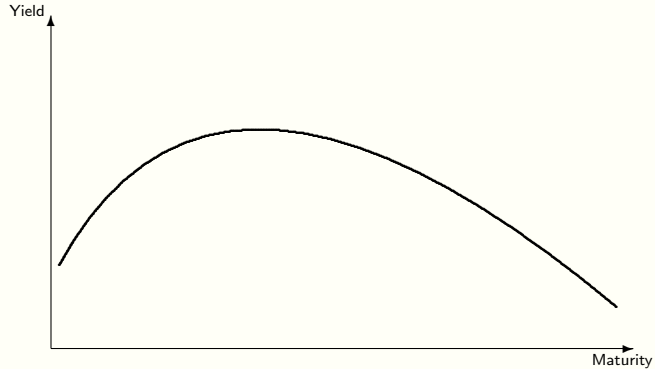


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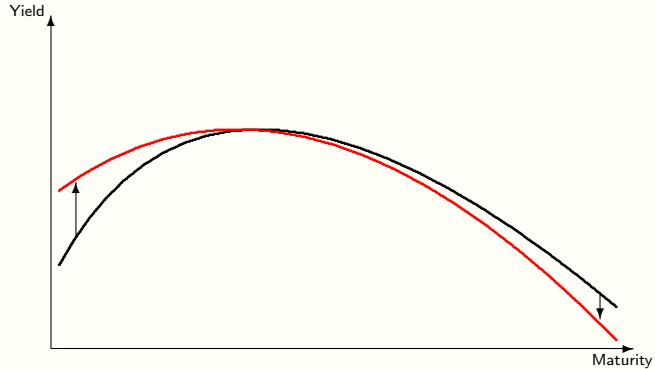
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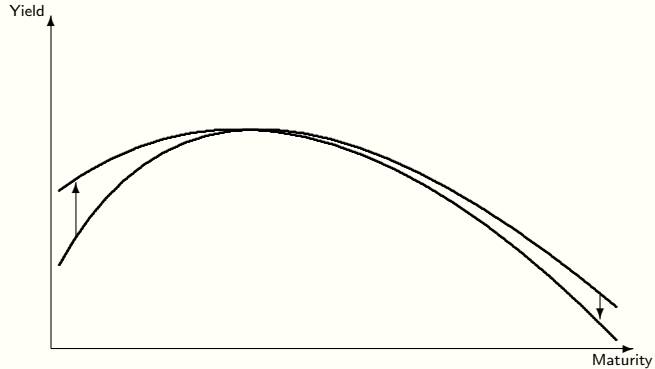


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- ⇒ The market expects a recession



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