



Andreas Krause

Yield curves

Term structures and yield curves

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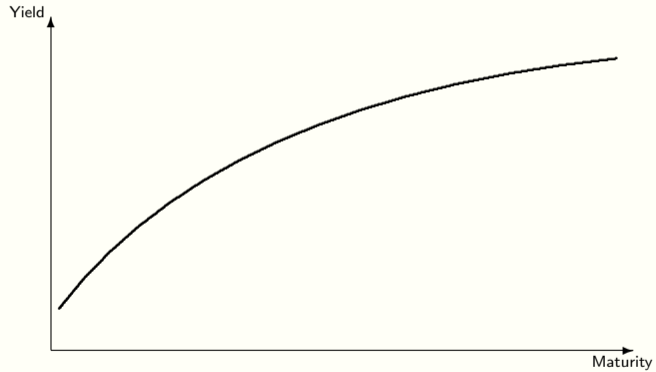
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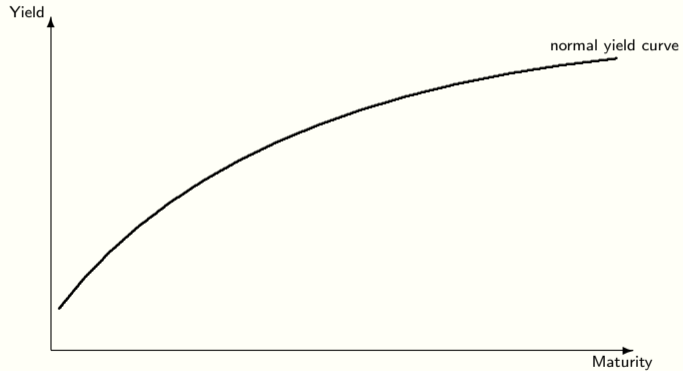
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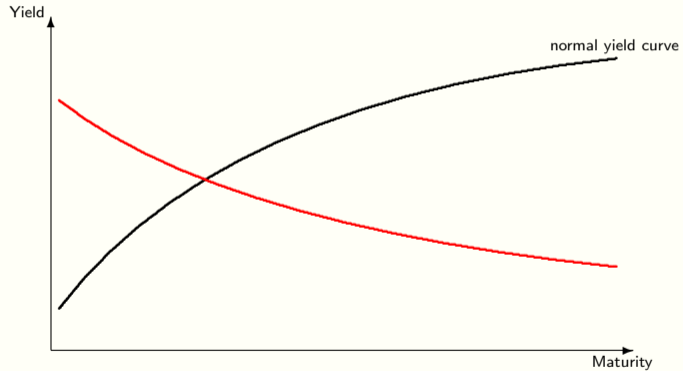
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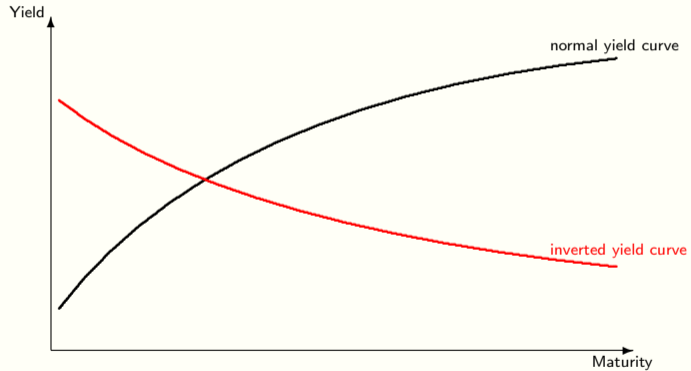
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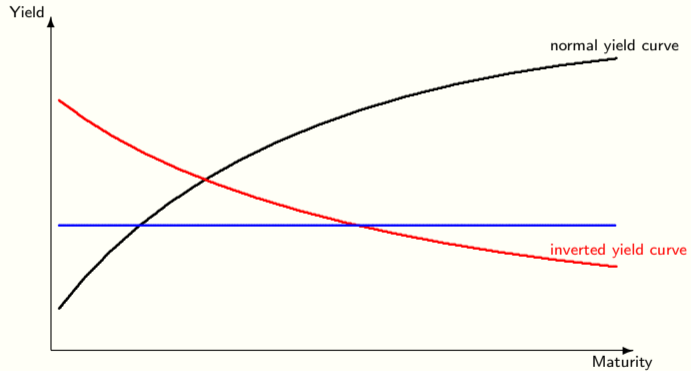
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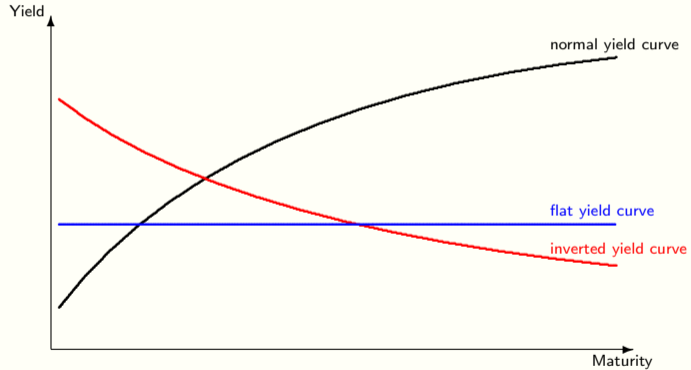
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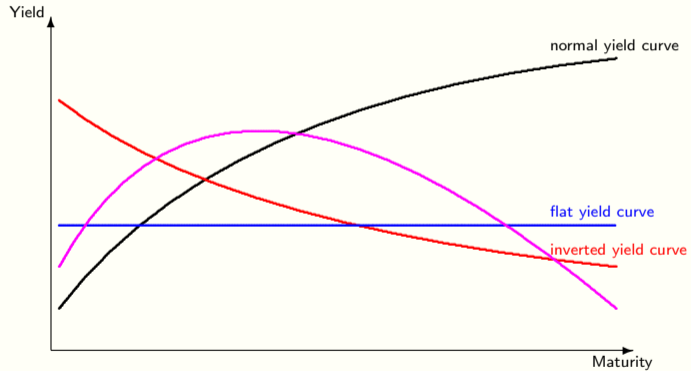
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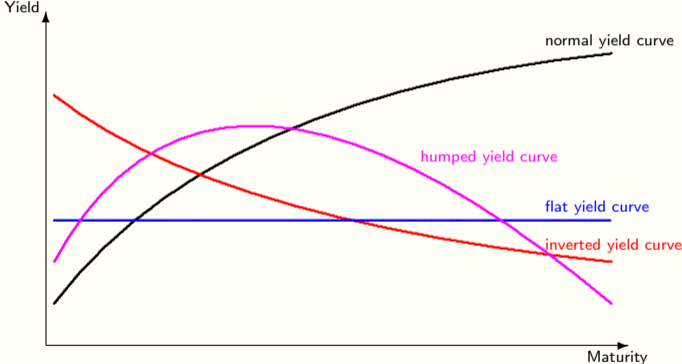
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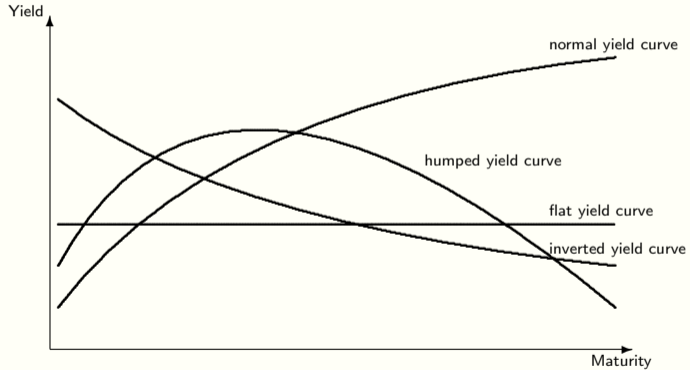
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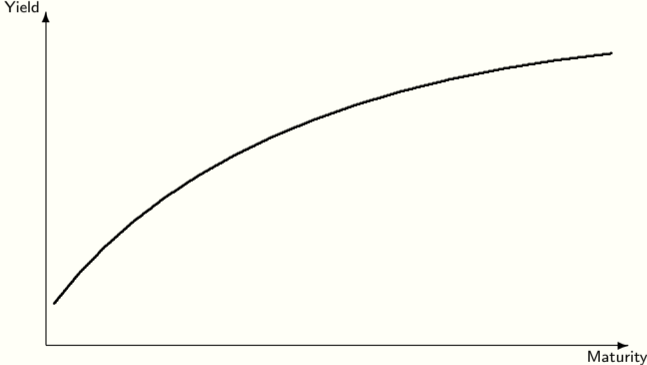
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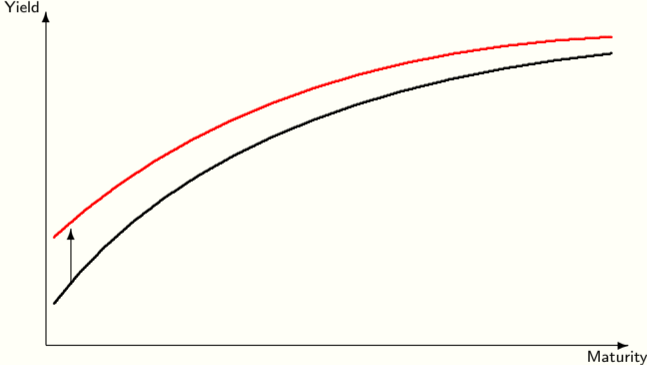
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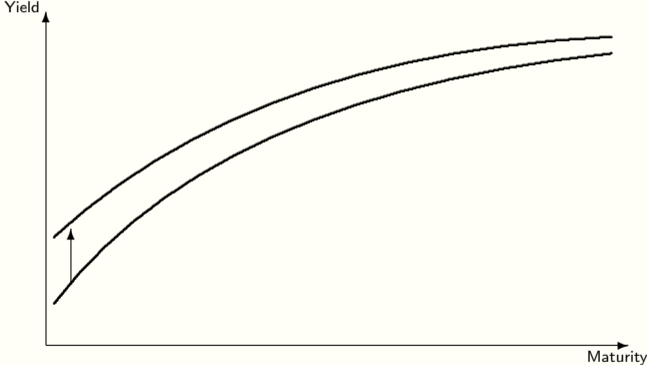
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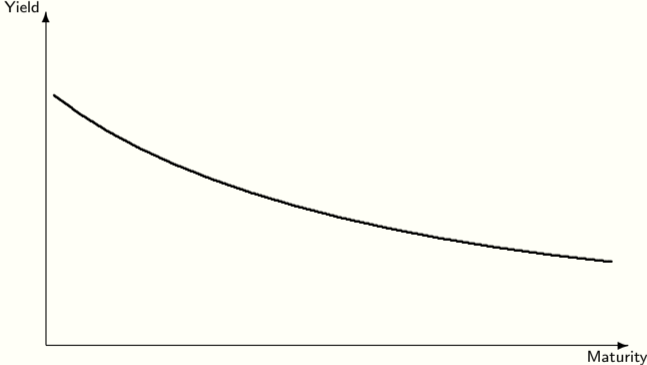


Expected interest rate changes for inverted yield curves

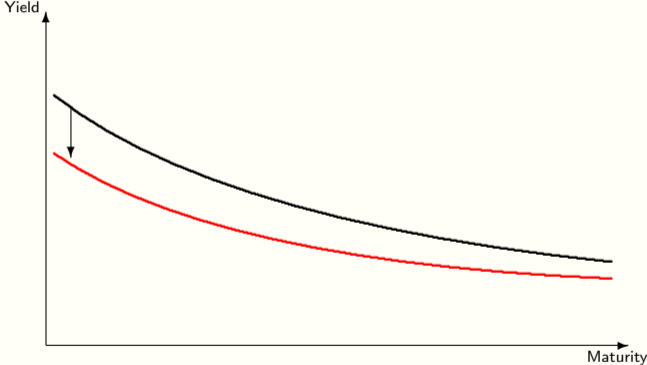
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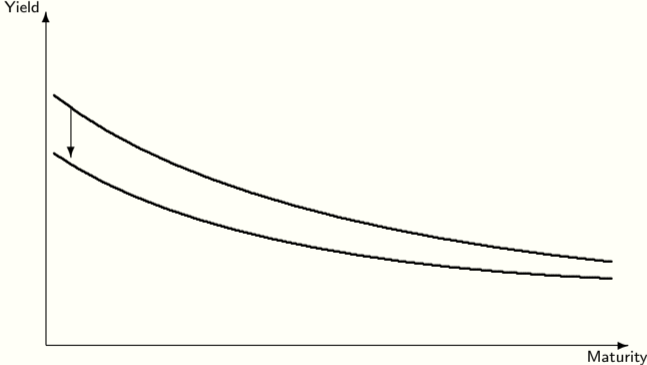
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Expected interest rate changes for flat yield curves

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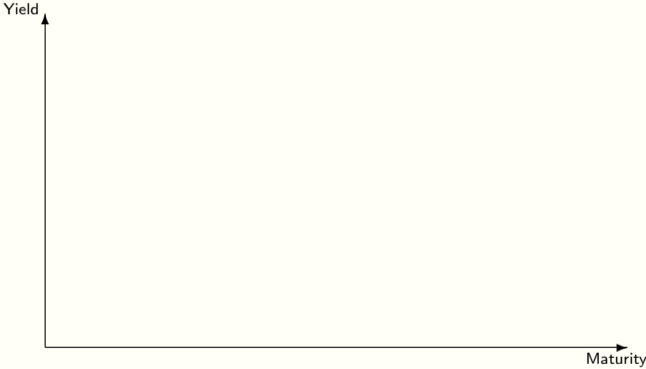


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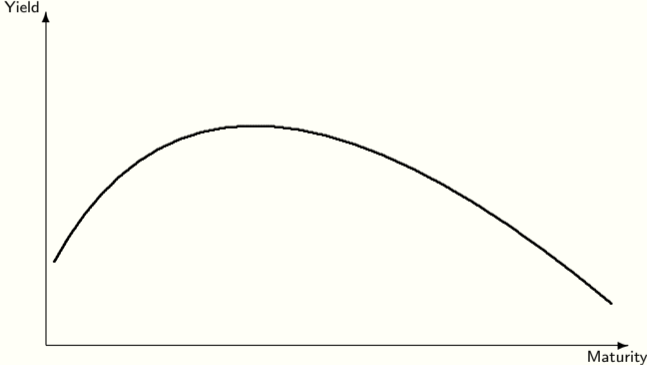


Expected interest rate changes for humped yield curves

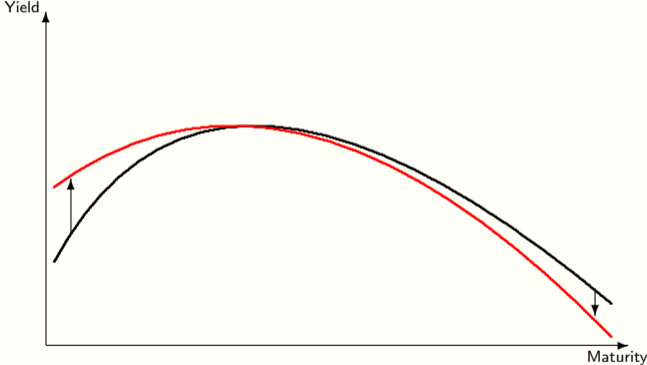
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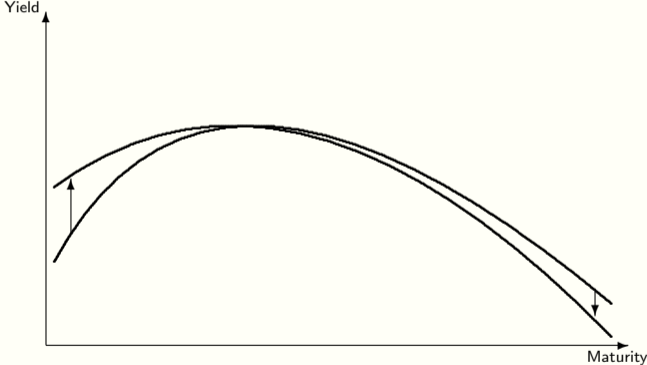
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