



Andreas Krause

Market efficiency

- Market efficiency is a key concept in finance and presuming market efficiency at the heart of many financial theories.
- We will look at what market efficiency means and what the implications for asset prices are.

# Definition of market efficiency

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- We first define market efficiency and its different forms.
- ▶ In its simplest definition, we state that a market is efficient if all information about the asset is reflected in its price. In even simpler terms, the market price should be equal to the value of the asset, taking into account all information. We now have different version of market efficiency, depending on the information that is available.
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    - We might include only information on past prices; this implies that in a weak form efficient market, traders using technical analysis (those looking for trends in charts, using moving averages, and similar) cannot make profits. This is because the price will reflect this information and hence there would be no profitable trading opportunity.
    - We might increase the definition of information to any information that is publicly available; such information can include disclosures by the company, including annual or quarterly reports, ad hoc disclosures of significant information, but also reports in the financial press. In this case a fundamental analysis of the asset, for example stocks, would not generate any profitable trading strategy; all the information an investor would use and analyse will already be included into the price.
    - We might include more than just publicly available information, but any information anyone might have. This would make even insider trading, for example by company managers, not profitable as all they information they hold is already included into the price. As insider trading is in many cases illegal, prices cannot reflect thi information, so this efficiency form has to be restricted to information that can be used legally.
- Given the nature of private information not being easily observable, the focus of efficiency is mainly on semi-strong and weak form efficiency.

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A market is **efficient** if prices include all relevant information

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**Weak form efficiency** Prices reflect information from past prices

**Semi-strong form efficiency** Prices reflect all **publicly available information**

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**Weak form efficiency** Prices reflect information from past prices

**Semi-strong form efficiency** Prices reflect all publicly available information

**Strong form efficiency** Prices reflect **all available information**, including private information

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# Asset returns

- We can now start to investigate the implications of market efficiency for asset returns. We focus on stock markets, but the procedure can easily be adapted to other assets.
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    - We know that the stock value is given by future dividends, including the dividend that is about to be paid; thus we consider a stock just before its ex-dividend day.
    - This will be discounted to obtain the present value of these future dividends. In general the discount rate will be given as the expected returns of the stock as determined by asset pricing models. The discount factor used here would then be  $\rho = 1 + \mu$ .
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    - We will determine the expected dividend of the stock using all available information  $\Omega_t$ .
    - We can rewrite this also as the current dividend, which is about to be paid and hence not discounted, and the present value of the future expected price.
  - ▶ If we look at short term returns, the dividend paid in this short time period will most likely be zero as dividends are paid quarterly at most.
  - ▶ [⇒]
    - We can now solve the above equation for the discount factor.
    - We can repeat the same, just for the next time period by shifting all time periods by 1.
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    - As we are at time  $t$ , we do not know the future information available at  $t + 1$ , but can only use the current information; therefore we will take expectations, given the information we have at time  $t$ .
    - These expectations can now be taken into the fraction, at least approximately.
    - The denominator can be replaced by solving the previous equation for  $E [P_{t+1} | \Omega_t]$ .
  - ▶ [⇒] The final term can now be transformed into this *formula*.
- We have now established the value for the ratio between the expected price in the next time period and the current price,  $\rho$ , as well as the ratio of the expected price in two time period and the current price,  $\rho^2$ . We can now use this result to obtain an implication of market being efficient.



# Asset returns

- ▶ Asset values are derived from the **future income** they generate

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$$P_t = \sum_{\tau=0}^{+\infty} \frac{E[D_{t+\tau}]}{r^\tau}$$

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- ▶ Asset values are derived from the **future income** they generate, **discounted** to the present value

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  - ▶ For short-term returns, we can neglect the future income and set  $E[d_{t+\tau}|\Omega_t] = 0$
- ⇒  $\rho = \frac{E[P_{t+1}|\Omega_t]}{P_t} = \frac{E[P_{t+2}|\Omega_{t+1}]}{P_{t+1}}$
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- We can now start to investigate the implications of market efficiency for asset returns. We focus on stock markets, but the procedure can easily be adapted to other assets.
- ▶
    - We know that the stock value is given by future dividends, including the dividend that is about to be paid; thus we consider a stock just before its ex-dividend day.
    - This will be discounted to obtain the present value of these future dividends. In general the discount rate will be given as the expected returns of the stock as determined by asset pricing models. The discount factor used here would then be  $\rho = 1 + \mu$ .
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    - We will determine the expected dividend of the stock using all available information  $\Omega_t$ .
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    - We can now solve the above equation for the discount factor.
    - We can repeat the same, just for the next time period by shifting all time periods by 1.
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    - As we are at time  $t$ , we do not know the future information available at  $t + 1$ , but can only use the current information; therefore we will take expectations, given the information we have at time  $t$ .
    - **These expectations can now be taken into the fraction, at least approximately.**
    - The denominator can be replaced by solving the previous equation for  $E [P_{t+1} | \Omega_t]$ .
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- We have now established the value for the ratio between the expected price in the next time period and the current price,  $\rho$ , as well as the ratio of the expected price in two time period and the current price,  $\rho^2$ . We can now use this result to obtain an implication of market being efficient.

# Asset returns

- ▶ Asset values are derived from the future income they generate, discounted to the present value
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# Serial correlation of returns

- We will look at the autocorrelation of returns that emerge in such efficient markets.
- ▶ We can determine the covariance of the returns between returns in the next time period and the time period after that. We then apply the displacement theorem to rewrite the covariance as the expected value of the product of returns less the product of expected returns.
- ▶ □ We simplify the first term by eliminating the numerator in the first expression and the denominator in the second expression.
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  - We then use the result from above and see that the first term is the ration of the price in two time periods and the current price, which we found to be  $\rho^2$  and the second term each fraction is  $r h_0$  as they represent the ration of the price in two subsequent periods.
  - Of course, these expressions subtracted from each other gives zero.
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- If the information is included in the price, the price in the market will be given by the value which we have determined using stock valuation models. Hence, while the result here formally states that the returns on the value of stocks are serially uncorrelated, the definition of market efficiency requires the prices to be equal to the value. Hence, a test for market efficiency would be whether prices are serially correlated.

## Serial correlation of returns

$$\blacktriangleright \text{Cov} \left[ \frac{E[P_{t+1}|\Omega_t]}{P_t}, \frac{E[P_{t+2}|\Omega_t]}{E[P_{t+1}|\Omega_t]} \right] = E \left[ \frac{E[P_{t+1}|\Omega_t]}{P_t} \frac{E[P_{t+2}|\Omega_t]}{E[P_{t+1}|\Omega_t]} \right] - \frac{E[P_{t+1}|\Omega_t]}{P_t} \frac{E[P_{t+2}|\Omega_t]}{E[P_{t+1}|\Omega_t]}$$

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⇒ Returns are **serially uncorrelated**

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# Random returns

- We have considered expected values and hence expected prices in our derivation of efficient markets being serially uncorrelated, but not the actual prices. We will consider this difference now.
- ▶ If returns are serially uncorrelated, they may still fluctuate randomly as the expected value averaged out all these random fluctuations. Such fluctuations might arise if the information investors hold are imperfect and therefore the realised prices might be different from those investors expect.
  - ▶ We can say that the actually observed return is the discount factor plus a random disturbance,  $\varepsilon_t$ , reflecting any missing information.
  - ▶ This is often called an error term and it we propose that it would be unbiased, having a mean of zero (any bias with a positive or negative value would be known and could be anticipated by investors). It will also have a variance that reflects the amount of information that is missing, or the precision of information. Less precise information, more information missing, will give a larger variance of this error term.
  - ▶ [⇒] Taking expectations of this return, we see that the expected return equals the discount factor as we had derived previously.
  - ▶ [] The returns will now have some variance due to the imprecise information, which is exactly the variance of the error term. We obtain this by taking the expectations of the return as defined above, noting that  $\rho$  is fixed and have no variance.
- Thus in an efficient market, stock returns have a mean equal to that obtained by asset pricing models, they will have a variance that depends on the precision of information available to investors, and be serially uncorrelated. The variance of stock returns would be zero if information by investors would be complete and perfect, variances of returns arise because of the imperfect information investors hold. If information were perfect, any future changes would be fully anticipated and the prices would immediately adjust such that there are no changes to returns and these returns would be risk-free. In reality, returns cannot be risk-free, but stocks on which less information is available will be more risky, not because of the nature of their business, but because of the lack of information available.



# Random returns

- ▶ If returns are uncorrelated, they will fluctuate randomly around the expected return
- ▶  $\frac{P_{t+1}}{P_t} = \rho + \varepsilon_t$

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- ▶ The error term will have a mean of 0 and a variance of  $\sigma_\varepsilon^2$

- We have considered expected values and hence expected prices in our derivation of efficient markets being serially uncorrelated, but not the actual prices. We will consider this difference now.
  - ▶ If returns are serially uncorrelated, they may still fluctuate randomly as the expected value averaged out all these random fluctuations. Such fluctuations might arise if the information investors hold are imperfect and therefore the realised prices might be different from those investors expect.
  - ▶ We can say that the actually observed return is the discount factor plus a random disturbance,  $\varepsilon_t$ , reflecting any missing information.
  - ▶ This is often called an error term and it we propose that it would be unbiased, having a mean of zero (any bias with a positive or negative value would be known and could be anticipated by investors). It will also have a variance that reflects the amount of information that is missing, or the precision of information. Less precise information, more information missing, will give a larger variance of this error term.
  - ▶ [⇒] Taking expectations of this return, we see that the expected return equals the discount factor as we had derived previously.
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- Thus in an efficient market, stock returns have a mean equal to that obtained by asset pricing models, they will have a variance that depends on the precision of information available to investors, and be serially uncorrelated. The variance of stock returns would be zero if information by investors would be complete and perfect, variances of returns arise because of the imperfect information investors hold. If information were perfect, any future changes would be fully anticipated and the prices would immediately adjust such that there are no changes to returns and these returns would be risk-free. In reality, returns cannot be risk-free, but stocks on which less information is available will be more risky, not because of the nature of their business, but because of the lack of information available.

# Random returns

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# Profitability of trading strategies

- We can now finally review briefly the profitability of trading strategies in efficient markets based on the results we have obtained.
- ▶ If returns are unpredictable, as they are if they are serially uncorrelated, traders cannot make any profits. This is because they do not know in which direction stock prices will move; they will only know the trend given by the expected returns, and they can earn this return, but this does not represent an economic profit as the trader only obtains a return for the risk he takes.
- ▶ If the market is strong form efficient, then no investor could make economic profits from trading, that is generate returns in excess of the expected return from asset pricing models. This is because given the information investors have, prices fluctuate randomly.
- ▶ If the market is semi-strong efficient, anyone relying on publicly available information could not make profits with the same argument as before; therefore a fundamental analysis of stocks is not profitable. However, those investors having information that is not publicly available (private information), could make economic profits as this information will not be included into the price.
- ▶ For weak form efficient market, technical traders relying on charts, moving averages and similar would not be able to make economic profits. However, anyone using public or private information would be able to make profits.
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Andreas Krause  
Department of Economics  
University of Bath  
Claverton Down  
Bath BA2 7AY  
United Kingdom

E-mail: [mnsak@bath.ac.uk](mailto:mnsak@bath.ac.uk)