

The background is a complex, abstract composition of glowing lines and shapes. It features a prominent candlestick chart on the right side, with green and red bars. Overlaid on this are various other lines, including a thick yellow curve that peaks and then declines, and several thinner, multi-colored lines in shades of blue, green, and red. The overall aesthetic is high-tech and financial, with a cool color palette dominated by blues and greens, accented with warm yellows and oranges from the glowing lines.

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Discounted Cash Flows

- We will now see how the value of stocks are determined using the projected dividends and results from asset pricing models.

Stock returns

- ▶ Returns on investments consist of capital gains and dividend payments

- ▶
$$R_{t+1} = \frac{P_{t+1} - P_t}{P_t} + \frac{D_{t+1}}{P_t}$$

- ⇒
$$P_t = \frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}}$$

- ▶ The current price should then be the present value of the future price and any dividends obtained

- ▶ As the future stock prices and dividends are not known, we need to form expectations

- ⇒
$$P_t = \frac{E[P_{t+1}] + E[D_{t+1}]}{1 + E[R_{t+1}]}$$

- ▶ In efficient markets expected returns are not changing: $E[R_{t+1}] = E[R_t] = \mu$

- ⇒
$$P_t = \sum_{\tau=1}^T \frac{E[D_{t+\tau}]}{(1+\mu)^\tau} + \frac{E[P_{t+T}]}{(1+\mu)^T}$$

- ▶ The stock price should be the present value of future dividends and the final value

- We will initially start by looking at the returns of stocks before then inverting the results to solve for the stock value.
 - ▶
 - When investing into stocks gains (or losses) can be made from an increase in value, here expressed as a percentage gain.
 - In addition, stocks pay dividends that are due to investors. These two components make up the returns an investor achieves.
 - ▶ *Formula*
- ⇒ We can solve this equation for the current value of the stock.
 - ▶ This expression can be interpreted as the present value (dividing by $1 + R_{t+1}$) of the future price and the dividend that are paid until then.
 - ▶ The future stock value and dividends are not known in advance, therefore the value can only be obtained by forming expectations about these future values.
- ⇒ *Formula*
 - ▶
 - We assume that markets are efficient and anticipate any future news, therefore the expected future returns are not changing. If investors anticipate that returns are increasing in the future, they would increase their demand now as the value should be increasing more in the future, increasing the value now and reducing future returns.
 - *Formula*
- ⇒ Inserting this result and now iteratively inserting for $P_{t+\tau}$, we get this *formula*.
 - ▶
 - We see that the stock value is given by the present value of the expected future dividends.
 - In addition with a certain time horizon T the investor will obtain the present value of this stock value.
- The choice of the investment horizon T is arbitrary and stocks are not having a fixed time to maturity, unlike bonds. We will now seek to lift this assumption.

Stock value with an infinite time horizon

- ▶ We assume that stock prices increase less than their expected returns
- ▶ This is the case if the dividends are paid out regularly, reducing the stock price after every payment

$$\Rightarrow \lim_{T \rightarrow \infty} \frac{E[P_{t+T}]}{(1+\mu)^T} = 0$$

$$\Rightarrow P_t = \sum_{\tau=1}^{\infty} \frac{E[D_{t+\tau}]}{(1+\mu)^\tau}$$

- ▶ If the time horizon is infinite, the stock value is the present value of future dividends
- ▶ The discount rate is the expected stock return

Stock value with an infinite time horizon

- We will now extend the time horizon of investors to infinity.
- ▶ If we want the time horizon to increase, we must ensure the value does not go beyond bounds. To this effect we assume that the stock value increases less than the expected returns.
- ▶ Essentially it means, that the increase in the stock price is reduced by dividends to achieve this requirement.
- ⇒ The formal requirement is that the time horizon goes to infinity, the present value of the future price goes to zero.
- ⇒ Using this result, we can now let T go to infinity and obtain the stock value as this *formula*.
- ▶ The stock value will simply be given as the present value of all future dividends.
- ▶ The discount rate applied is the expected return of the stock. This expected return of the stock can be determined using asset pricing models, such as the Capital Asset Pricing model or Arbitrage Pricing Theory.
- We can now turn our attention to forming expectations about dividends, the aspect that has not yet been addressed in the stock valuation formula.

Constant dividends

- ▶ We need to form expectations about future dividends
- ▶ Assume for now that dividends are constant and at the current level:
 $E[D_{t+\tau}] = D_t$
- ⇒ $P_t = \frac{D_t}{\mu}$
- ▶ If we assume that earnings are paid out fully, then we can identify dividends with earnings
- ▶ The price-earnings ratio is given by $\frac{P_t}{D_t} = \frac{1}{\mu}$
- ▶ Using the price-earnings ratio from the market we can determine which discount rate the market applies

- We will first look at a stock that pays the same dividend every time period. We will see what this implies for the stock value.
- ▶ It is obvious from the stock valuation formula, that we need to form expectations about future dividends.
- ▶ The simplest possible assumption is that dividends are constant and at the current level of dividends.
- ⇒ If we insert this and solve the resulting equation as a geometric series, the stock value is given by this *formula*. The stock value is the capitalisation of the current dividend.
- ▶ If earnings are paid out fully, then when determining the stock value, we can use earnings instead; using the dividend irrelevance from corporate finance, we can replace dividends by earnings even if some or all earnings are retained.
- ▶ Financial analysts like to express stock valuations using the price-earnings ratio, which in this case is given by the *formula*.
- ▶ Using the price earnings ration from the market is another way to determine the expected return of a stock, by taking the inverse of this relationship. This method is an alternative to using asset pricing models and instead relies in market data, the stock price.
- ▶ We can solve the equation for the discount rate and if we know the price-earning-ratio can determine the expected return of the stock.
- Assuming constant dividends seems to be a strong assumption, so we will now consider the case of growing dividends.

Gordon growth model

- ▶ We can assume that dividends are growing at a constant rate every time period
- ▶ $E[D_{t+\tau+1}] = (1+g)E[D_{t+\tau}] = (1+g)^{\tau+1}D_t$
- ⇒ $P_t = \frac{1+g}{\mu-g}D_t$
- ▶ For a feasible solution we require that $g < \mu$
- ▶ The dividends cannot grow too fast, otherwise the present value of future dividends grows ever larger and the stock value is infinite
- ▶ The price-earnings ratio is given by $\frac{P_t}{D_t} = \frac{1+g}{\mu-g}$

- We can now assume that dividends are growing continuously rather than being constant. This is commonly known as the Gordon growth model.
- ▶ We assume that dividends are growing at a given rate in each time period; this growth continues ad infinitum.
- ▶ The dividend in the next time period is supposed to increase by a factor g . This implies that we can use the current dividend as a starting point and for each time period ahead increase its value.
- ⇒ If we insert this and solve the resulting equation as a geometric series, the stock value is given by this *formula*.
- ▶ We see that if $g > \mu$, the stock value would be negative.
- ▶
 - If the dividends grow faster than the expected return of the stock, the return an investor would get from obtaining the dividends would already be higher than the total expected return they are requiring.
 - This case would imply that the present value of the future price does not go to zero, violating our above assumption. We therefore require that $g < \mu$ to obtain a feasible result.
- ▶ We can again obtain the price-earnings ratio. Compared to constant dividends, this expression is now more complex and does not allow directly to use the price-earnings ratio of the market to obtain the expected return of the stock, but this is only possible if we know the growth rate of dividends.
- The Gordon growth model takes into account that dividends tend to grow over time. The stock value will, however, be sensitive to the assumptions on the growth rate.

Sensitivity to assumptions on the growth rate

- ▶ The stock price will depend on the assumptions on the growth rate and discount rate
- ▶ $\frac{\partial P_t}{\partial g} = \frac{1+\mu}{(\mu-g)^2} D_t$
- ▶ The price is very sensitive to the growth rate, especially if it is close to the discount rate
- ▶ Small changes in growth expectations or the applied discount rate can affect the stock value significantly

Sensitivity to assumptions on the growth rate

- If we change the growth rate of dividends, which is an assumption an investor makes when valuing stocks, the stock value will change; we will now explore how important this assumption on the growth rate is for the stock value the investor will obtain.
- ▶ We can look at the sensitivity of the stock value to changing our assumption on the growth rate.
- ▶ We can differentiate the stock value with respect to the growth rate and obtain this *formula*.
- ▶
 - Due to the squared item in the denominator and growth rates being small, we see that this expression will be quite large, suggesting a high sensitivity of the stock value to changing the growth rate.
 - This effect is even more pronounced once the growth rate of dividends gets close to the expected stock return, making the denominator close to zero and hence the expression overall very large.
- ▶ Even if we change the growth rate only marginally (or the expected return if it is close to the growth rate), the stock value can change substantially. The stock value we obtain can be very sensitive to the assumption on the growth rate.
- This sensitivity can be used by investment banks advising companies in mergers and acquisitions to make small changes in the assumptions on the growth rate, or sometimes on the expected returns, and obtain a significantly different stock value, which they can use to justify their recommendations; usually this is a recommendation to go ahead with the transaction.

Determination of the discount rate

- ▶ The discount rate was identified as the expected stock return
- ▶ This can be determined by asset pricing models
- ▶ Most common is to use the Capital Asset Pricing Model

Determination of the discount rate

- The focus of our discussion has been on the assumption regarding the future dividends. However, the second important component is the expected returns of the stock.
 - ▶ The expected stock returns acted as the discount rate to obtain the present value of future dividends.
 - ▶ The common way to determine the expected return is to use asset pricing models.
 - ▶ In most cases, the Capital Asset Pricing Model is used to this effect. The CAPM is used for its ability to focus solely on the stock market by using a stock index as a proxy for the market and it avoids the empirical complications of identifying and then using other factors as in the Arbitrage Pricing Theory.
- We can now focus briefly on the use of dividends in stock valuation models.

Dividends and earnings

- ▶ Rather than dividends, it is common to use earnings in determining stock values
- ▶ If we assume that retained earnings are re-invested and generate the same return as existing investments, dividend payments are irrelevant
- ▶ The value of retained earnings is accumulated in the stock price, giving the same return to investors

- Stock valuation models suggest that the relevant variable are dividends as these represent payments to the investor. However, dividends are discretionary payments by the company and maybe affected by more than the performance of the company itself as theories from corporate finance suggest.
 - ▶ In many applications of stock valuation models earning are used rather than dividends. In other cases the cash-flow of a company is chosen (simplified, the cash flow is the amount the company brings in less than what it pays out, it excludes depreciations and other aspects of earnings recognition) or the free cash-flow (which is the cash-flow less necessary replacement investment).
 - ▶ We know from corporate finance that dividends are irrelevant for the value of companies as long as the company can make investments that generate the same return as their current investments. In this case there would be no difference to the value when using dividends or earnings for the valuation.
 - ▶ If we use earnings, then those retained earnings are accumulated in the company affect the ability to pay dividends in the future. Using the present value of these accumulated retained earnings and the fraction paid out as dividends, will give us the same value as using the full earnings in the stock valuation model.
- We thus have seen that the stock value will be the present value of future dividends, where the discount rate is determined using asset pricing models and dividends can be replaced by earnings. The resulting stock value will be every sensitive to assumptions on the growth rate of dividends or earnings



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