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Arbitrage Pricing Theory

- The Capital Asset Pricing Model (CAPM) is not the only model of determining expected returns.
- The CAPM is often criticised for only considering systematic risk with respect to the market as the driver of expected asset returns.
- We will therefore here look at a model in which a number of factors influencing asset returns are considered.

Determinants of asset returns

- ▶ Asset returns are affected by a wide range of factors
- ▶ Possible factors are inflation, economic growth, interest rates, commodity prices, exchange rates, market sentiment, ...
- ▶ The CAPM aggregates all these influences into a the market portfolio
- ▶ To understand asset returns, it is important to differentiate the importance of individual factors

- We will now look what factors might drive the returns of assets.
- ▶ When listening to market professionals, it is obvious that other factors are also considered to be driving asset returns. Models in macroeconomic theory, for example, also suggest other influences on asset returns.
 - ▶
 - The possible factor affecting assets returns are numerous, but they include inflation. Higher inflation might increase the (nominal) return of assets; for example stocks might increase their returns if the company is able to maintain real profits through price increases. On the other hand, bonds might reduce in value as higher inflation usually is followed by higher interest rates (the risk-free rate), which are the basis of the discounting and future value.
 - Economic growth will affect stock returns as higher growth is usually associated with higher profits for companies, but also higher future interest rates, giving the same argument as above for the bond market.
 - The effect of interest rates (the risk-free rate) has been mentioned in the previous cases.
 - Commodity prices might affect stock directly if they are bought or sold by the company, but there might also be indirect effects on the wider economic growth through investments, for example.
 - Exchange rates can increase or decrease profits, depending on whether companies have to pay the foreign currency or receive it. Again, there is also a wider impact on economic growth.
 - We might also consider aspects like market sentiment, that is how investors view the assets overall. This might be driven by aspects from behavioural finance, where psychological factors are considered.
 - Many other factors can be identified. A famous example is the Fama-Franck Three Factor model, the Carhart Four Factor Model, and other variations of these models.
 - ▶ What the Capital Asset Pricing model does is assume that all these factors are affecting all stocks, it is mostly considered only for stocks, and thus the average influence on the market can be used.
 - ▶ For a better understanding of asset returns, it would be useful to identify the different factors individually. It might especially the case that the influence of these factors varies by company. Some companies might have more exposure to some of the risk factors than other companies; or, like in the case of commodities and foreign exchange, the influences might even have opposite signs.
- We can now use the idea of having multiple factors to investigate how they will affect asset returns.

Linear dependence on factors

- ▶ We now assume that each factor has an influence on the expected asset return and that this influence is linear
 - ▶ $\mu_i = \beta_{i0} + \sum_{k=1}^K \beta_{ik} F_k$
 - ▶ Consider an asset that is not affected by any of the factors, it will be risk-free
- ⇒ $\mu_i = \beta_{i0} = r$

- How various factors influence asset returns can take many different forms. We will focus here on the most simple possible dependence, a linear relationship.
- ▶
 - We assume now we have identified a number of factors that can affect asset returns
 - and we combine the influence of these factors in a linear way
 - ▶ *Formula*
 - ▶
 - Firstly, we can consider the risk-free asset, which is not affected by any of the possible factors considered, thus $\beta_{ik} = 0$.
 - The return of this asset is then the risk-free rate.
 - ▶ [\Rightarrow] *Formula*
- We can now continue considering special cases and make use of the result that $\beta_i 0 = r$.

Deriving the APT equation

- ▶ Consider now an asset that for factor k has $\beta_{ik} = 1$ and for all other factors has $\beta_{ij} = 0$
- ▶ Such an asset would be affected by factor k alone
- ⇒ $\mu_k = r + F_k$
- ▶ This 'asset' k mimics factor k and is often referred to as a factor portfolio
- ⇒ $\mu_i = r + \sum_{k=1}^K \beta_{ik} (\mu_k - r)$
- ▶ This equation represents the Arbitrage Pricing Theory (APT)

Deriving the APT equation

- Using more special cases, we can now build the complete APT pricing equation.
 - ▶
 - We now consider a special kind of asset, one that has as the coefficient with one specific factor as $\beta_{ik} = 1$.
 - This asset is not influenced by any other factors.
 - ▶ Hence the asset is affected by this single factor only.
 - ▶ [⇒] Inserting this into the asset returns from above, we get the asset return as in this *formula*.
 - ▶
 - This asset is as close to the factor itself as it can be, it essentially is the factor.
 - We call such an asset a factor portfolio.
 - ▶ [⇒] We can now solve this relationship for the factor and insert into the asset return to obtain this *formula*.
 - ▶ This equation represents what is known as Arbitrage Pricing Theory (APT).
- When estimating the coefficients β_{ik} this can be interpreted as a multivariate linear regression and the coefficients will be the ration of the covariance of the asset with the factor portfolio, divided by the variance of the factor portfolio. This is the same structure as the β_i in the CAPM, it represents the covariance risk with respect to the factor portfolio.

Factor portfolios

- ▶ A factor portfolio is a combination of assets that perfectly track the movements of one of the factors
- ▶ In practice finding such portfolios is difficult as it needs to be free of unsystematic risk and be independent of all other factors
- ▶ Identifying factors themselves can also be difficult, but statistical methods for both problems exist

- A problem in practical applications of Arbitrage Pricing Theory is the determination of the expected return of the asset that mimics the factor, μ_k . The factor itself cannot be used for this purpose as it has no asset return.
- ▶ We have to create an asset that has the desired property, only being affected by the factor. This would be a portfolio of assets that are specifically put together to have these properties and we can then determine the return on this portfolio.
- ▶
 - In reality obtaining such a pure portfolio is difficult; the portfolio needs to be well diversified to have no systematic risk.
 - In addition, it must be independent of all the other factors considered. The resulting coefficient, β_{ik} would then be the systematic risk of the asset with respect to this specific factor.
- ▶
 - In addition to determining the appropriate portfolio, identifying the factors itself can also be difficult.
 - For both problems, statistical methods exist that can help in overcoming these complications in using the Arbitrage Pricing Theory.
- While there are some practical problems when implementing APT, it is possible to use statistical methods to obtain a good approximation of the factor portfolios and then use these to determine asset returns.

- ▶ The Capital Asset Pricing model can be interpreted as a special case of Arbitrage Pricing Theory
- ▶ Assume that we only have a single factor, the market
- ⇒ $\mu_i = r + \beta_{iM} (\mu_M - r)$
- ▶ The factor portfolio will be the market portfolio

- The Capital Asset Pricing Model and Arbitrage Pricing Theory are the most widely used models to determine the returns of individual stocks, where the CAPM is by far the more dominant model that is used. We will now compare these two models to see what relationship they have with each other.
 - ▶ We will argue that the CAPM is nothing more than a special case of the APT.
 - ▶ We will consider a case where we only identify a single factor.
 - This factor is the market.
 - ▶ [⇒] Using the expected return of the APT in this case gives this *formula*.
 - ▶ In this case the factor portfolio will be the market portfolio and can be more readily identified than most other factor portfolios.
- We thus see that the CAPM is a special case of a one-factor APT. Given the difficulties in identifying factors and then determining the factor portfolios to determine the expected returns, APT is not a widely used approach in determining asset returns. It is often, however, that the factors are directly applied to the asset return; this is not consistent with the APT as no expected returns of this factor are determined.



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