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Arbitrage Pricing Theory

Determinants of asset returns

- ▶ Asset returns are affected by a wide range of factors
- ▶ Possible factors are inflation, economic growth, interest rates, commodity prices, exchange rates, market sentiment, ...
- ▶ The CAPM aggregates all these influences into a the market portfolio
- ▶ To understand asset returns, it is important to differentiate the importance of individual factors

Linear dependence on factors

- ▶ We now assume that each factor has an influence on the expected asset return and that this influence is linear
 - ▶ $\mu_i = \beta_{i0} + \sum_{k=1}^K \beta_{ik} F_k$
 - ▶ Consider an asset that is not affected by any of the factors, it will be risk-free
- $\Rightarrow \mu_i = \beta_{i0} = r$

Deriving the APT equation

- ▶ Consider now an asset that for factor k has $\beta_{ik} = 1$ and for all other factors has $\beta_{ij} = 0$
- ▶ Such an asset would be affected by factor k alone
- ⇒ $\mu_k = r + F_k$
- ▶ This 'asset' k mimics factor k and is often referred to as a factor portfolio
- ⇒ $\mu_i = r + \sum_{k=1}^K \beta_{ik} (\mu_k - r)$
- ▶ This equation represents the Arbitrage Pricing Theory (APT)

Factor portfolios

- ▶ A factor portfolio is a combination of assets that perfectly track the movements of one of the factors
- ▶ In practice finding such portfolios is difficult as it needs to be free of unsystematic risk and be independent of all other factors
- ▶ Identifying factors themselves can also be difficult, but statistical methods for both problems exist

CAPM vs. APT

- ▶ The Capital Asset Pricing model can be interpreted as a special case of Arbitrage Pricing Theory
- ▶ Assume that we only have a single factor, the market
- ⇒ $\mu_i = r + \beta_{iM} (\mu_M - r)$
- ▶ The factor portfolio will be the market portfolio



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