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Limits to diversification

Benefits of diversification

- Diversification reduces the risks of a portfolio, therefore investing into more assets is beneficial
- Unless a correlation is zero, risks will not be eliminated
- The positive effect of diversification will diminish the more assets are already invested in

▶ Assume we have a portfolio of equally weighted assets

▶ $\sigma_P^2 = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij}$

$ = \frac{1}{N^2} \sum_{i=1}^N \sigma_i^2 + \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1, j\neq i}^N \sigma_{ij}$

$ = \frac{1}{N} \overline{\sigma}_i^2 + \frac{N(N-1)}{N^2} \overline{\sigma}_{ij}$

$ = \frac{1}{N} \left(\overline{\sigma}_i^2 - \overline{\sigma}_{ij} \right) + \overline{\sigma}_{ij}$

$ \rightarrow_{N \to +\infty} \overline{\sigma}_{ij}$

Systematic risk

- If we diversify, the risk of a portfolio converges towards the average covariance
- Any other risk can be eliminated through diversification and is called unsystematic risk or idiosyncratic risk
- The remaining risk is called the systematic risk

- $\sigma_{iM} = \text{Cov}\left[R_i, R_M\right]$

  $= \text{Cov}\left[R_i, \sum_{j=1}^{N} \frac{1}{N} R_j\right]$

  $= \frac{1}{N} \sum_{j=1}^{N} \text{Cov}\left[R_i, R_j\right]$

  $= \frac{1}{N} \sum_{j=1}^{N} \sigma_{ij}$

  $= \overline{\sigma}_{ij}$

- The systematic risk is the covariance of the asset with the market

- As systematic risk cannot be eliminated through diversification, this risk is compensated for through higher returns in the CAPM

Costs and benefits of diversification

▶ Diversification reduces portfolio risk as we increase the number of assets

▶ As the number of assets increases, the reduction in risk becomes smaller for each added asset

▶ $\sigma_P^2 = \frac{1}{N} \left(\overline{\sigma}_i - \overline{\sigma}_{ij} \right) + \overline{\sigma}_{ij}$

▶ If investing into assets is costly, a widespread diversification might not be cost-effective

▶ Investing into fewer assets with low correlations (covariances) will be more effective

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