

Limits to diversification



Benefits of diversification

- ▶ Diversification reduces the risks of a portfolio, therefore investing into more assets is beneficial
- ▶ Unless a correlation is zero, risks will not be eliminated
- ▶ The positive effect of diversification will diminish the more assets are already invested in

Portfolio risk with increasing number of assets

- ▶ Assume we have a portfolio of equally weighted assets

$$\begin{aligned}\sigma_P^2 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} \\ &= \frac{1}{N^2} \sum_{i=1}^N \sigma_i^2 + \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sigma_{ij} \\ &= \frac{1}{N} \bar{\sigma}_i^2 + \frac{N(N-1)}{N^2} \bar{\sigma}_{ij} \\ &= \frac{1}{N} (\bar{\sigma}_i^2 - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij} \\ &\rightarrow_{N \rightarrow +\infty} \bar{\sigma}_{ij}\end{aligned}$$

Systematic risk

- ▶ If we diversify, the risk of a portfolio converges towards the average covariance
- ▶ Any other risk can be eliminated through diversification and is called unsystematic risk or idiosyncratic risk
- ▶ The remaining risk is called the systematic risk

Determining the systematic risk

- ▶ $\sigma_{iM} = \text{Cov}[R_i, R_M]$
$$= \text{Cov}\left[R_i, \sum_{j=1}^N \frac{1}{N} R_j\right]$$
$$= \frac{1}{N} \sum_{j=1}^N \text{Cov}[R_i, R_j]$$
$$= \frac{1}{N} \sum_{j=1}^N \sigma_{ij}$$
$$= \bar{\sigma}_{ij}$$
- ▶ The systematic risk is the covariance of the asset with the market
- ▶ As systematic risk cannot be eliminated through diversification, this risk is compensated for through higher returns in the CAPM

Costs and benefits of diversification

- ▶ Diversification reduces portfolio risk as we increase the number of assets
- ▶ As the number of assets increases, the reduction in risk becomes smaller for each added asset
- ▶ $\sigma_P^2 = \frac{1}{N} (\bar{\sigma}_i - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij}$
- ▶ If investing into assets is costly, a widespread diversification might not be cost-effective
- ▶ Investing into fewer assets with low correlations (covariances) will be more effective



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