Andreas Krause

Limits to diversification

- Diversification is seen as beneficial to investors as it reduces risks and hence they are advised to spread their investments across many assets.
- We will show why this assertion is correct and also link the effect of diversification back to the Capital Asset Pricing Model.

Benefits of diversification

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  - When conducting out portfolio analysis, increasing the number of assets to be considered by investors increases the opportunity set , thus for a given return, the risk of the efficient portfolio reduces.
  - With risk averse investors, diversification therefore increases their utility level.
- However, as we will see below, unless all correlations between assets are zero, risks will never disappear completely, some level of risk remains.
- We will also see that as the number of assets increases, the benefits of diversification are becoming ever smaller.

→ We can now determine the risk of a portfolio as we increase the number of assets included.

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Portfolio risk with increasing number of assets

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→ We will now investigate the variance of a portfolio with a given number of assets included.

- For simplicity we assume that the N assets included all have the same weight in the portfolio. This assumption is not critical as the same results would hold if we merely assume that as the number of assets increases, the weight of each asset reduces and will eventually become zero.
- The variance is determined by the weighted sum of all the combinations of assets, giving their covariances; the weight of each asset is $\frac{1}{N}$.
- [] We can separate the variances out, as they are the covariance of the asset with itself, σii.
- [] We can now define the average variance as $\overline{\sigma}_i^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sigma_i^2$ and average covariance as $\overline{\sigma}_{ij} = \frac{1}{N(N-1)} \sum_{j=1, j \neq i}^{N} \sigma_{ij}$. We insert this into the previous line, noting that there are $N(N-1)$ covariances.
- [] We can re-arrange this expression.
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- The risk of a well-diversified portfolio is the average covariance of the assets with each other. This is sometimes also referred to as the covariance risk.
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    - Risks other than the covariances can be eliminated through diversification as these risks are not included in the above result.
    - These other risks are called unsystematic risk,
- The covariance risk, the risk that remains, is called the systematic risk.

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Determining the systematic risk

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→ We can now determine the systematic risk as identified by the CAPM and compare it to the systematic risk we have defined here.

- We look at the covariance of an asset with the market, the numerator in the β_i of the CAPM. We here simply use the definition of the covariance.
- [] The market portfolio, which will be well-diversified, consists of the individual assets, where we again assume the weights of the assets are all equal.
- [] Due to the linearity of covariances, we can take the $\frac{1}{N}$ out of the covariance and do the same with the summation.
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- The systematic defined above is the covariance of the asset with the market.
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→ The CAPM therefore determines the expected excess return of an asset relative to the risk of the well-diversified market portfolio, σ_M^2; if the covariance of an asset is higher, it has a higher systematic risk than the market portfolio and attracts a higher excess return. Similarly, if the covariance of an asset is lower, it has a lower systematic risk than the market portfolio and attracts a lower excess return. This relationship we found to be linear.

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- $$\begin{aligned}\sigma_{iM} &= \text{Cov}\left[R_i, R_M\right] \\ &= \text{Cov}\left[R_i, \sum_{j=1}^{N} \frac{1}{N} R_j\right] \\ &= \frac{1}{N} \sum_{j=1}^{N} \text{Cov}\left[R_i, R_j\right] \\ &= \frac{1}{N} \sum_{j=1}^{N} \sigma_{ij}\end{aligned}$$

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Determining the systematic risk

- $$\begin{aligned}\sigma_{iM} &= \text{Cov}\left[R_i, R_M\right] \\ &= \text{Cov}\left[R_i, \sum_{j=1}^{N} \frac{1}{N} R_j\right] \\ &= \frac{1}{N}\sum_{j=1}^{N} \text{Cov}\left[R_i, R_j\right] \\ &= \frac{1}{N}\sum_{j=1}^{N} \sigma_{ij} \\ &= \overline{\sigma}_{ij}\end{aligned}$$

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Costs and benefits of diversification

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→ Diversification reduces unsystematic risk and because of this it increases the utility of investors; this is achieved by investing into more assets. However, investing into more assets might increase out costs.

- Due to the reduction in unsystematic risk, the portfolio risk overall reduces as the number of assets goes up.
- Looking at the formula from above for the total risk of the portfolio, we now know that the second terms denotes the systematic risk and hence the first term denotes the unsystematic risk. As we increase the number of assets, the reduction in unsystematic risk will reduce the larger N becomes as every additional asset reduces $\frac{1}{N}$ by less and less.
- *Formula*
- If we assume that investing into an asset is costly, there will be a point where these costs outweigh the benefits. Costs to investors might not only be trading costs, which are usually very small, but also collecting and assessing information about each asset.
- In these cases it can be more cost effective to invest into fewer assets. Those assets chosen ideally have a low correlation to reduce the average covariance and obtain most of the diversification benefits, despite being exposed to unsystematic risk.

→ Diversification reduces risks, but cannot eliminate systematic risk. The costs of investing into a large number of assets might limit the benefits of diversification and it can be optimal to retain some unsystematic (idiosyncratic) risk.

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