

Limits to diversification



Benefits of diversification

Benefits of diversification

- ▶ Diversification **reduces the risks** of a portfolio

Benefits of diversification

- ▶ Diversification reduces the risks of a portfolio, therefore investing into **more assets** is beneficial

Benefits of diversification

- ▶ Diversification reduces the risks of a portfolio, therefore investing into more assets is beneficial
- ▶ Unless a correlation is zero, risks will **not** be eliminated

Benefits of diversification

- ▶ Diversification reduces the risks of a portfolio, therefore investing into more assets is beneficial
- ▶ Unless a correlation is zero, risks will not be eliminated
- ▶ The positive effect of diversification will **diminish** the more assets are already invested in

Benefits of diversification

- ▶ Diversification reduces the risks of a portfolio, therefore investing into more assets is beneficial
- ▶ Unless a correlation is zero, risks will not be eliminated
- ▶ The positive effect of diversification will diminish the more assets are already invested in

Portfolio risk with increasing number of assets

Portfolio risk with increasing number of assets

- ▶ Assume we have a portfolio of equally weighted assets

Portfolio risk with increasing number of assets

- ▶ Assume we have a portfolio of **equally weighted assets**
- ▶ $\sigma_P^2 = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij}$

Portfolio risk with increasing number of assets

- ▶ Assume we have a portfolio of equally weighted assets

- ▶
$$\begin{aligned}\sigma_P^2 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} \\ &= \frac{1}{N^2} \sum_{i=1}^N \sigma_i^2 + \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sigma_{ij}\end{aligned}$$

Portfolio risk with increasing number of assets

- ▶ Assume we have a portfolio of equally weighted assets

- ▶
$$\begin{aligned}\sigma_P^2 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} \\ &= \frac{1}{N^2} \sum_{i=1}^N \sigma_i^2 + \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sigma_{ij} \\ &= \frac{1}{N} \bar{\sigma}_i^2 + \frac{N(N-1)}{N^2} \bar{\sigma}_{ij}\end{aligned}$$

Portfolio risk with increasing number of assets

- ▶ Assume we have a portfolio of equally weighted assets

- ▶
$$\begin{aligned}\sigma_P^2 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} \\ &= \frac{1}{N^2} \sum_{i=1}^N \sigma_i^2 + \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sigma_{ij} \\ &= \frac{1}{N} \bar{\sigma}_i^2 + \frac{N(N-1)}{N^2} \bar{\sigma}_{ij} \\ &= \frac{1}{N} (\bar{\sigma}_i^2 - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij}\end{aligned}$$

Portfolio risk with increasing number of assets

- ▶ Assume we have a portfolio of equally weighted assets

$$\begin{aligned}\sigma_P^2 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} \\ &= \frac{1}{N^2} \sum_{i=1}^N \sigma_i^2 + \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sigma_{ij} \\ &= \frac{1}{N} \bar{\sigma}_i^2 + \frac{N(N-1)}{N^2} \bar{\sigma}_{ij} \\ &= \frac{1}{N} (\bar{\sigma}_i^2 - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij} \\ &\rightarrow_{N \rightarrow +\infty} \bar{\sigma}_{ij}\end{aligned}$$

Portfolio risk with increasing number of assets

- ▶ Assume we have a portfolio of equally weighted assets

- ▶
$$\begin{aligned}\sigma_P^2 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \sigma_{ij} \\ &= \frac{1}{N^2} \sum_{i=1}^N \sigma_i^2 + \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sigma_{ij} \\ &= \frac{1}{N} \bar{\sigma}_i^2 + \frac{N(N-1)}{N^2} \bar{\sigma}_{ij} \\ &= \frac{1}{N} (\bar{\sigma}_i^2 - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij} \\ &\rightarrow_{N \rightarrow +\infty} \bar{\sigma}_{ij}\end{aligned}$$

Systematic risk

Systematic risk

- ▶ If we diversify, the risk of a portfolio converges towards the **average covariance**

Systematic risk

- ▶ If we diversify, the risk of a portfolio converges towards the average covariance
- ▶ **Any other risk** can be eliminated through diversification

Systematic risk

- ▶ If we diversify, the risk of a portfolio converges towards the average covariance
- ▶ Any other risk can be eliminated through diversification and is called **unsystematic risk**

Systematic risk

- ▶ If we diversify, the risk of a portfolio converges towards the average covariance
- ▶ Any other risk can be eliminated through diversification and is called unsystematic risk or **idiosyncratic risk**

Systematic risk

- ▶ If we diversify, the risk of a portfolio converges towards the average covariance
- ▶ Any other risk can be eliminated through diversification and is called unsystematic risk or idiosyncratic risk
- ▶ The remaining risk is called the **systematic risk**

Systematic risk

- ▶ If we diversify, the risk of a portfolio converges towards the average covariance
- ▶ Any other risk can be eliminated through diversification and is called unsystematic risk or idiosyncratic risk
- ▶ The remaining risk is called the systematic risk

Determining the systematic risk

Determining the systematic risk

▶ $\sigma_{iM} = \text{Cov}[R_i, R_M]$

Determining the systematic risk

$$\begin{aligned}\blacktriangleright \sigma_{iM} &= \text{Cov}[R_i, R_M] \\ &= \text{Cov}\left[R_i, \sum_{j=1}^N \frac{1}{N} R_j\right]\end{aligned}$$

Determining the systematic risk

$$\begin{aligned}\blacktriangleright \sigma_{iM} &= \text{Cov}[R_i, R_M] \\ &= \text{Cov}\left[R_i, \sum_{j=1}^N \frac{1}{N} R_j\right] \\ &= \frac{1}{N} \sum_{j=1}^N \text{Cov}[R_i, R_j]\end{aligned}$$

Determining the systematic risk

$$\begin{aligned}\blacktriangleright \sigma_{iM} &= \text{Cov}[R_i, R_M] \\ &= \text{Cov}\left[R_i, \sum_{j=1}^N \frac{1}{N} R_j\right] \\ &= \frac{1}{N} \sum_{j=1}^N \text{Cov}[R_i, R_j] \\ &= \frac{1}{N} \sum_{j=1}^N \sigma_{ij}\end{aligned}$$

Determining the systematic risk

$$\begin{aligned}\blacktriangleright \sigma_{iM} &= \text{Cov}[R_i, R_M] \\ &= \text{Cov}\left[R_i, \sum_{j=1}^N \frac{1}{N} R_j\right] \\ &= \frac{1}{N} \sum_{j=1}^N \text{Cov}[R_i, R_j] \\ &= \frac{1}{N} \sum_{j=1}^N \sigma_{ij} \\ &= \bar{\sigma}_{ij}\end{aligned}$$

Determining the systematic risk

- ▶ $\sigma_{iM} = \text{Cov} [R_i, R_M]$
$$= \text{Cov} \left[R_i, \sum_{j=1}^N \frac{1}{N} R_j \right]$$
$$= \frac{1}{N} \sum_{j=1}^N \text{Cov} [R_i, R_j]$$
$$= \frac{1}{N} \sum_{j=1}^N \sigma_{ij}$$
$$= \bar{\sigma}_{ij}$$
- ▶ The systematic risk is the covariance of the asset with the **market**

Determining the systematic risk

- ▶ $\sigma_{iM} = \text{Cov}[R_i, R_M]$
$$= \text{Cov}\left[R_i, \sum_{j=1}^N \frac{1}{N} R_j\right]$$
$$= \frac{1}{N} \sum_{j=1}^N \text{Cov}[R_i, R_j]$$
$$= \frac{1}{N} \sum_{j=1}^N \sigma_{ij}$$
$$= \bar{\sigma}_{ij}$$
- ▶ The systematic risk is the covariance of the asset with the market
- ▶ As systematic risk cannot be eliminated through diversification, this risk is compensated for through higher returns in the **CAPM**

Determining the systematic risk

- ▶ $\sigma_{iM} = \text{Cov} [R_i, R_M]$
$$= \text{Cov} \left[R_i, \sum_{j=1}^N \frac{1}{N} R_j \right]$$
$$= \frac{1}{N} \sum_{j=1}^N \text{Cov} [R_i, R_j]$$
$$= \frac{1}{N} \sum_{j=1}^N \sigma_{ij}$$
$$= \bar{\sigma}_{ij}$$
- ▶ The systematic risk is the covariance of the asset with the market
- ▶ As systematic risk cannot be eliminated through diversification, this risk is compensated for through higher returns in the CAPM

Costs and benefits of diversification

Costs and benefits of diversification

- ▶ Diversification **reduces portfolio risk** as we increase the number of assets

Costs and benefits of diversification

- ▶ Diversification reduces portfolio risk as we increase the number of assets
- ▶ As the number of assets increases, the **reduction** in risk becomes **smaller** for each added asset
- ▶ $\sigma_P^2 = \frac{1}{N} (\bar{\sigma}_i - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij}$

Costs and benefits of diversification

- ▶ Diversification reduces portfolio risk as we increase the number of assets
- ▶ As the number of assets increases, the reduction in risk becomes smaller for each added asset
- ▶ $\sigma_P^2 = \frac{1}{N} (\bar{\sigma}_i - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij}$
- ▶ If investing into assets is costly, a widespread diversification might **not** be **cost-effective**

Costs and benefits of diversification

- ▶ Diversification reduces portfolio risk as we increase the number of assets
- ▶ As the number of assets increases, the reduction in risk becomes smaller for each added asset
- ▶ $\sigma_P^2 = \frac{1}{N} (\bar{\sigma}_i - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij}$
- ▶ If investing into assets is costly, a widespread diversification might not be cost-effective
- ▶ Investing into fewer assets with **low correlations** (covariances) will be more effective

Costs and benefits of diversification

- ▶ Diversification reduces portfolio risk as we increase the number of assets
- ▶ As the number of assets increases, the reduction in risk becomes smaller for each added asset
- ▶ $\sigma_P^2 = \frac{1}{N} (\bar{\sigma}_i - \bar{\sigma}_{ij}) + \bar{\sigma}_{ij}$
- ▶ If investing into assets is costly, a widespread diversification might not be cost-effective
- ▶ Investing into fewer assets with low correlations (covariances) will be more effective



Copyright © by Andreas Krause

Picture credits:

Cover: Premier regard, Public domain, via Wikimedia Commons, [https://commons.wikimedia.org/wiki/File:DALL-E_-_Financial_markets_\(1\).jpg](https://commons.wikimedia.org/wiki/File:DALL-E_-_Financial_markets_(1).jpg)

Back: Rhododendrites, CC BY-SA 4.0 <https://creativecommons.org/licenses/by-sa/4.0>, via Wikimedia Commons, [https://upload.wikimedia.org/wikipedia/commons/0/04/Manhattan_at_night_south_of_Rockefeller_Center_panorama_\(11263p\).jpg](https://upload.wikimedia.org/wikipedia/commons/0/04/Manhattan_at_night_south_of_Rockefeller_Center_panorama_(11263p).jpg)

Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk