Andreas Krause

Capital Asset Pricing Model

Portfolio theory

- Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset
- Based on these investment decisions, we are able to derive an equilibrium in which all assets are held
- This equilibrium will restrict the returns of assets as a too high (low) return would result in a too high (low) weight for this asset

Sharpe ratio

- The slope of the Capital Market Line in portfolio selection theory is given by $s = \frac{\mu_p - r}{\sigma_p}$
- This is known as the Sharpe ratio
- The optimal portfolio will consist of the optimal risky portfolio and the risk-free asset
- $\mu_P = \boldsymbol{\omega}^T \boldsymbol{\mu} + \left(1 - \boldsymbol{\omega}^T \boldsymbol{\iota}\right) r$
- $\sigma_p^2 = \boldsymbol{\omega}^T \boldsymbol{\Sigma} \boldsymbol{\omega}$

Maximizing the Sharpe ratio

- The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$\Rightarrow \frac{\partial s}{\partial \boldsymbol{\omega}} = 0$

$\Rightarrow \boldsymbol{\mu} = r\boldsymbol{\iota} + \frac{\boldsymbol{\Sigma\omega}}{\sigma_P^2}(\mu_P - r)$

- We define $\boldsymbol{\beta} = \frac{\boldsymbol{\Sigma\omega}}{\sigma_P^2}$

$\Rightarrow \boldsymbol{\mu} = r\boldsymbol{\iota} + \boldsymbol{\beta}(\mu_P - r)$

The CAPM equation

- The term $\mathbf{\Sigma\omega}$ represents the covariance of the assets with the optimal risky portfolio
- The optimal risky portfolio is identical for all investors, it must be the market portfolio

$$\Rightarrow \mu_i = r + \beta_i \left(\mu_M - r\right)$$

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

Problems with the market portfolio

- The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...
- Many investments are not available to all investors, for others no data are available
- For the optimal risky portfolio to be the market portfolio, all investors need to agree on the properties of all assets

Systematic risk

- The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure
- The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole
- Unsystematic risk, or idiosyncratic risk, is the risk unique to the asset
- Idiosyncratic risk can be eliminated through diversification

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Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk