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Capital Asset Pricing Model

Portfolio theory

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset
- ▶ Based on these investment decisions, we are able to derive an equilibrium in which all assets are held
- ▶ This equilibrium will restrict the returns of assets as a too high (low) return would result in a too high (low) weight for this asset

Sharpe ratio

- ▶ The slope of the Capital Market Line in portfolio selection theory is given by
$$s = \frac{\mu_p - r}{\sigma_p}$$
- ▶ This is known as the Sharpe ratio
- ▶ The optimal portfolio will consist of the optimal risky portfolio and the risk-free asset
- ▶ $\mu_P = \omega^T \mu + (1 - \omega^T \iota) r$
- ▶ $\sigma_p^2 = \omega^T \Sigma \omega$

Maximizing the Sharpe ratio

- ▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$$\Rightarrow \frac{\partial s}{\partial \omega} = 0$$

$$\Rightarrow \mu = r\iota + \frac{\Sigma\omega}{\sigma_P^2} (\mu_P - r)$$

- ▶ We define $\beta = \frac{\Sigma\omega}{\sigma_P^2}$

$$\Rightarrow \mu = r\iota + \beta (\mu_P - r)$$

The CAPM equation

- ▶ The term $\Sigma\omega$ represents the covariance of the assets with the optimal risky portfolio
- ▶ The optimal risky portfolio is identical for all investors, it must be the market portfolio

$$\Rightarrow \mu_i = r + \beta_i (\mu_M - r)$$
$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...
- ▶ Many investments are not available to all investors, for others no data are available
- ▶ For the optimal risky portfolio to be the market portfolio, all investors need to agree on the properties of all assets

Systematic risk

- ▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure
- ▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole
- ▶ Unsystematic risk, or idiosyncratic risk, is the risk unique to the asset
- ▶ Idiosyncratic risk can be eliminated through diversification



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