

The background of the slide is a blurred, abstract image of a bridge at night. The bridge's structure is visible as a dark silhouette against a bright, colorful sky filled with streaks of light and glowing spots, resembling a fireworks display or a city skyline at night.

Andreas Krause

Capital Asset Pricing Model

# Portfolio theory

# Portfolio theory

- ▶ Portfolio theory suggests that investors hold a **portfolio of risky assets**

# Portfolio theory

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (**optimal risky portfolio**)

# Portfolio theory

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the **risk-free asset**

# Portfolio theory

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset
- ▶ Based on these investment decisions, we are able to derive an **equilibrium** in which all assets are held

# Portfolio theory

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset
- ▶ Based on these investment decisions, we are able to derive an equilibrium in which all assets are held
- ▶ This equilibrium will restrict the returns of assets as a **too high** return would result in a **too high** weight for this asset

# Portfolio theory

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset
- ▶ Based on these investment decisions, we are able to derive an equilibrium in which all assets are held
- ▶ This equilibrium will restrict the returns of assets as a **too high (low)** return would result in a **too high (low)** weight for this asset

# Portfolio theory

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset
- ▶ Based on these investment decisions, we are able to derive an equilibrium in which all assets are held
- ▶ This equilibrium will restrict the returns of assets as a too high (low) return would result in a too high (low) weight for this asset

# Sharpe ratio

# Sharpe ratio

- The slope of the Capital Market Line in portfolio selection theory is given by

$$s = \frac{\mu_p - r}{\sigma_p}$$

# Sharpe ratio

- ▶ The slope of the Capital Market Line in portfolio selection theory is given by
- $$s = \frac{\mu_p - r}{\sigma_p}$$
- ▶ This is known as the **Sharpe ratio**

# Sharpe ratio

- ▶ The slope of the Capital Market Line in portfolio selection theory is given by  
$$s = \frac{\mu_p - r}{\sigma_p}$$
- ▶ This is known as the Sharpe ratio
- ▶ The optimal portfolio will consist of the **optimal risky portfolio**
- ▶  $\mu_P = \omega^T \mu$

# Sharpe ratio

- ▶ The slope of the Capital Market Line in portfolio selection theory is given by  
$$s = \frac{\mu_p - r}{\sigma_p}$$
- ▶ This is known as the Sharpe ratio
- ▶ The optimal portfolio will consist of the **optimal risky portfolio** and the **risk-free asset**
- ▶  $\mu_P = \omega^T \mu + (1 - \omega^T \iota) r$

# Sharpe ratio

- ▶ The slope of the Capital Market Line in portfolio selection theory is given by  
$$s = \frac{\mu_p - r}{\sigma_p}$$
- ▶ This is known as the Sharpe ratio
- ▶ The optimal portfolio will consist of the **optimal risky portfolio** and the **risk-free asset**
- ▶  $\mu_P = \omega^T \mu + (1 - \omega^T \iota) r$
- ▶  $\sigma_p^2 = \omega^T \Sigma \omega$

# Sharpe ratio

- ▶ The slope of the Capital Market Line in portfolio selection theory is given by  
$$s = \frac{\mu_p - r}{\sigma_p}$$
- ▶ This is known as the Sharpe ratio
- ▶ The optimal portfolio will consist of the optimal risky portfolio and the risk-free asset
- ▶  $\mu_P = \omega^T \mu + (1 - \omega^T \iota) r$
- ▶  $\sigma_p^2 = \omega^T \Sigma \omega$

# Maximizing the Sharpe ratio

# Maximizing the Sharpe ratio

- The Capital Market Line is **tangential** to the efficient frontier

# Maximizing the Sharpe ratio

- ▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the **slope** being maximal

# Maximizing the Sharpe ratio

- ▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal
- ⇒  $\frac{\partial s}{\partial \omega} = 0$

# Maximizing the Sharpe ratio

- ▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$$\Rightarrow \frac{\partial s}{\partial \omega} = 0$$

$$\Rightarrow \boldsymbol{\mu} = r\boldsymbol{\iota} + \frac{\boldsymbol{\Sigma}\boldsymbol{\omega}}{\sigma_P^2} (\mu_P - r)$$

# Maximizing the Sharpe ratio

- ▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal
  - ⇒  $\frac{\partial s}{\partial \omega} = 0$
  - ⇒  $\mu = r\mathbf{1} + \frac{\Sigma\omega}{\sigma_P^2} (\mu_P - r)$
- ▶ We define  $\beta = \frac{\Sigma\omega}{\sigma_P^2}$

# Maximizing the Sharpe ratio

- ▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$$\Rightarrow \frac{\partial s}{\partial \omega} = 0$$

$$\Rightarrow \mu = r\iota + \frac{\Sigma\omega}{\sigma_P^2} (\mu_P - r)$$

- ▶ We define  $\beta = \frac{\Sigma\omega}{\sigma_P^2}$

$$\Rightarrow \mu = r\iota + \beta (\mu_P - r)$$

# Maximizing the Sharpe ratio

- ▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$$\Rightarrow \frac{\partial s}{\partial \omega} = 0$$

$$\Rightarrow \mu = r\iota + \frac{\Sigma\omega}{\sigma_P^2} (\mu_P - r)$$

- ▶ We define  $\beta = \frac{\Sigma\omega}{\sigma_P^2}$

$$\Rightarrow \mu = r\iota + \beta (\mu_P - r)$$

# The CAPM equation

# The CAPM equation

- ▶ The term  $\Sigma\omega$  represents the **covariance** of the assets with the optimal risky portfolio

# The CAPM equation

- ▶ The term  $\Sigma\omega$  represents the covariance of the assets with the optimal risky portfolio
- ▶ The optimal risky portfolio is **identical for all investors**

# The CAPM equation

- ▶ The term  $\Sigma\omega$  represents the covariance of the assets with the optimal risky portfolio
- ▶ The optimal risky portfolio is identical for all investors, it must be the **market portfolio**

# The CAPM equation

- ▶ The term  $\Sigma\omega$  represents the covariance of the assets with the optimal risky portfolio
  - ▶ The optimal risky portfolio is identical for all investors, it must be the market portfolio
- ⇒  $\mu_i = r + \beta_i (\mu_M - r)$

# The CAPM equation

- ▶ The term  $\Sigma\omega$  represents the covariance of the assets with the optimal risky portfolio
- ▶ The optimal risky portfolio is identical for all investors, it must be the market portfolio

$$\Rightarrow \mu_i = r + \beta_i (\mu_M - r)$$

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

# The CAPM equation

- ▶ The term  $\Sigma\omega$  represents the covariance of the assets with the optimal risky portfolio
- ▶ The optimal risky portfolio is identical for all investors, it must be the market portfolio

$$\Rightarrow \mu_i = r + \beta_i (\mu_M - r)$$

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

# Problems with the market portfolio

## Problems with the market portfolio

- The market portfolio should include **all possible investments**

## Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: **stocks**

## Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, **bonds**

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, **real estate**

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, **private equity**

## Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, **hedge funds**

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, **commodities**

## Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, **foreign exchange**

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, **cryptoassets**

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, **human capital**

## Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...
- ▶ Many investments are **not available** to all investors

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...
- ▶ Many investments are not available to all investors, for others **no data** are available

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...
- ▶ Many investments are not available to all investors, for others no data are available
- ▶ For the optimal risky portfolio to be the market portfolio, all investors need to **agree** on the properties of **all assets**

# Problems with the market portfolio

- ▶ The market portfolio should include all possible investments: stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...
- ▶ Many investments are not available to all investors, for others no data are available
- ▶ For the optimal risky portfolio to be the market portfolio, all investors need to agree on the properties of all assets

# Systematic risk

# Systematic risk

- The CAPM only considers the **covariance** of an asset with the market

# Systematic risk

- The CAPM only considers the covariance of an asset with the market, not its **variance** as a risk measure

# Systematic risk

- ▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure
- ▶ The covariance is regarded as the **systematic risk** of an asset

# Systematic risk

- ▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure
- ▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies **with the market** as a whole

# Systematic risk

- ▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure
- ▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole
- ▶ **Unsystematic risk** is the risk **unique to the asset**

# Systematic risk

- ▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure
- ▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole
- ▶ Unsystematic risk, or **idiosyncratic risk**, is the risk unique to the asset

# Systematic risk

- ▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure
- ▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole
- ▶ Unsystematic risk, or idiosyncratic risk, is the risk unique to the asset
- ▶ Idiosyncratic risk can be eliminated through **diversification**

# Systematic risk

- ▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure
- ▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole
- ▶ Unsystematic risk, or idiosyncratic risk, is the risk unique to the asset
- ▶ Idiosyncratic risk can be eliminated through diversification



Copyright © by Andreas Krause

Picture credits:

Cover: Premier regard, Public domain, via Wikimedia Commons, [https://commons.wikimedia.org/wiki/File:DALL-E\\_2\\_Financial\\_markets\\_\(1\).jpg](https://commons.wikimedia.org/wiki/File:DALL-E_2_Financial_markets_(1).jpg)

Back: Rhododendrites, CC BY-SA 4.0 <https://creativecommons.org/licenses/by-sa/4.0/>, via Wikimedia Commons, [https://upload.wikimedia.org/wikipedia/commons/0/04/Manhattan\\_at\\_night\\_south\\_of\\_Rockefeller\\_Center\\_panorama\\_\(11263p\).jpg](https://upload.wikimedia.org/wikipedia/commons/0/04/Manhattan_at_night_south_of_Rockefeller_Center_panorama_(11263p).jpg)

Andreas Krause  
Department of Economics  
University of Bath  
Claverton Down  
Bath BA2 7AY  
United Kingdom

E-mail: [mnsak@bath.ac.uk](mailto:mnsak@bath.ac.uk)