Andreas Krause

Capital Asset Pricing Model

▶ Portfolio theory suggests that investors hold a portfolio of risky assets

Capital Asset Pricing Model

▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio)

▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset

▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset

▶ Based on these investment decisions, we are able to derive an equilibrium in which all assets are held

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset
- ▶ Based on these investment decisions, we are able to derive an equilibrium in which all assets are held
- ▶ This equilibrium will restrict the returns of assets as a too high return would result in a too high weight for this asset

- ▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset
- ▶ Based on these investment decisions, we are able to derive an equilibrium in which all assets are held
- ▶ This equilibrium will restrict the returns of assets as a too high (low) return would result in a too high (low) weight for this asset

▶ Portfolio theory suggests that investors hold a portfolio of risky assets (optimal risky portfolio) and combine this with the risk-free asset

▶ Based on these investment decisions, we are able to derive an equilibrium in which all assets are held

▶ This equilibrium will restrict the returns of assets as a too high (low) return would result in a too high (low) weight for this asset

Sharpe ratio

- The slope of the Capital Market Line in portfolio selection theory is given by
$s = \frac{\mu_p - r}{\sigma_p}$

Sharpe ratio

- ▶ The slope of the Capital Market Line in portfolio selection theory is given by $s = \frac{\mu_p - r}{\sigma_p}$
- ▶ This is known as the Sharpe ratio

- The slope of the Capital Market Line in portfolio selection theory is given by $s = \frac{\mu_p - r}{\sigma_p}$
- This is known as the Sharpe ratio
- The optimal portfolio will consist of the optimal risky portfolio

- $\mu_P = \boldsymbol{\omega}^T \boldsymbol{\mu}$

- The slope of the Capital Market Line in portfolio selection theory is given by $s = \frac{\mu_p - r}{\sigma_p}$
- This is known as the Sharpe ratio
- The optimal portfolio will consist of the optimal risky portfolio and the risk-free asset
- $\mu_P = \boldsymbol{\omega}^T \boldsymbol{\mu} + \left(1 - \boldsymbol{\omega}^T \boldsymbol{\iota}\right) r$

▶ The slope of the Capital Market Line in portfolio selection theory is given by
$s = \frac{\mu_p - r}{\sigma_p}$

▶ This is known as the Sharpe ratio

▶ The optimal portfolio will consist of the optimal risky portfolio and the risk-free asset

▶ $\mu_P = \boldsymbol{\omega}^T \boldsymbol{\mu} + \left(1 - \boldsymbol{\omega}^T \boldsymbol{\iota}\right) r$

▶ $\sigma_p^2 = \boldsymbol{\omega}^T \boldsymbol{\Sigma} \boldsymbol{\omega}$

Sharpe ratio

▶ The slope of the Capital Market Line in portfolio selection theory is given by $s = \frac{\mu_p - r}{\sigma_p}$

▶ This is known as the Sharpe ratio

▶ The optimal portfolio will consist of the optimal risky portfolio and the risk-free asset

▶ $\mu_P = \boldsymbol{\omega}^T \boldsymbol{\mu} + \left(1 - \boldsymbol{\omega}^T \boldsymbol{\iota}\right) r$

▶ $\sigma_p^2 = \boldsymbol{\omega}^T \boldsymbol{\Sigma} \boldsymbol{\omega}$

Capital Asset Pricing Model

▶ The Capital Market Line is tangential to the efficient frontier

▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$\Rightarrow \frac{\partial s}{\partial \boldsymbol{\omega}} = 0$

▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$\Rightarrow$ $\frac{\partial s}{\partial \boldsymbol{\omega}} = 0$

$\Rightarrow$ $\boldsymbol{\mu} = r\boldsymbol{\iota} + \frac{\boldsymbol{\Sigma}\boldsymbol{\omega}}{\sigma_P^2}\left(\mu_P - r\right)$

Capital Asset Pricing Model

▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$\Rightarrow \frac{\partial s}{\partial \boldsymbol{\omega}} = 0$

$\Rightarrow \boldsymbol{\mu} = r\boldsymbol{\iota} + \frac{\boldsymbol{\Sigma}\boldsymbol{\omega}}{\sigma_P^2}\left(\mu_P - r\right)$

▶ We define $\boldsymbol{\beta} = \frac{\boldsymbol{\Sigma}\boldsymbol{\omega}}{\sigma_P^2}$

► The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$\Rightarrow \frac{\partial s}{\partial \boldsymbol{\omega}} = 0$

$\Rightarrow \boldsymbol{\mu} = r\boldsymbol{\iota} + \frac{\boldsymbol{\Sigma}\boldsymbol{\omega}}{\sigma_P^2}\left(\mu_P - r\right)$

► We define $\boldsymbol{\beta} = \frac{\boldsymbol{\Sigma}\boldsymbol{\omega}}{\sigma_P^2}$

$\Rightarrow \boldsymbol{\mu} = r\boldsymbol{\iota} + \boldsymbol{\beta}\left(\mu_P - r\right)$

▶ The Capital Market Line is tangential to the efficient frontier, this is equivalent to the slope being maximal

$\Rightarrow$ $\frac{\partial s}{\partial \boldsymbol{\omega}} = 0$

$\Rightarrow$ $\boldsymbol{\mu} = r\boldsymbol{\iota} + \frac{\boldsymbol{\Sigma}\boldsymbol{\omega}}{\sigma_P^2}\left(\mu_P - r\right)$

▶ We define $\boldsymbol{\beta} = \frac{\boldsymbol{\Sigma}\boldsymbol{\omega}}{\sigma_P^2}$

$\Rightarrow$ $\boldsymbol{\mu} = r\boldsymbol{\iota} + \boldsymbol{\beta}\left(\mu_P - r\right)$

▶ The term $\Sigma\omega$ represents the covariance of the assets with the optimal risky portfolio

The CAPM equation

- The term $\Sigma\omega$ represents the covariance of the assets with the optimal risky portfolio
- The optimal risky portfolio is identical for all investors

- The term $\boldsymbol{\Sigma\omega}$ represents the covariance of the assets with the optimal risky portfolio
- The optimal risky portfolio is identical for all investors, it must be the market portfolio

- The term $\Sigma\boldsymbol{\omega}$ represents the covariance of the assets with the optimal risky portfolio
- The optimal risky portfolio is identical for all investors, it must be the market portfolio

$\Rightarrow$ $\mu_i = r + \beta_i \left(\mu_M - r\right)$

▶ The term $\boldsymbol{\Sigma\omega}$ represents the covariance of the assets with the optimal risky portfolio

▶ The optimal risky portfolio is identical for all investors, it must be the market portfolio

$\Rightarrow$ $\mu_i = r + \beta_i \left(\mu_M - r \right)$

$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$

▶ The term $\boldsymbol{\Sigma\omega}$ represents the covariance of the assets with the optimal risky portfolio

▶ The optimal risky portfolio is identical for all investors, it must be the market portfolio

$\Rightarrow$ $\mu_i = r + \beta_i \left(\mu_M - r \right)$

$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$

▶ The market portfolio should include all possible investments

▶ The market portfolio should include all possible investments, stocks

▶ The market portfolio should include all possible investments, stocks, bonds

► The market portfolio should include all possible investments, stocks, bonds, real estate

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets

► The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital

► The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...

▶ Many investments are not available to all investors

Problems with the market portfolio

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...

▶ Many investments are not available to all investors, for others no data are available

Problems with the market portfolio

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, …

▶ Many investments are not available to all investors, for others no data are available

▶ For the optimal risky portfolio to be the market portfolio, all investors need to agree on the properties of all assets

▶ The market portfolio should include all possible investments, stocks, bonds, real estate, private equity, hedge funds, commodities, foreign exchange, cryptoassets, human capital, ...

▶ Many investments are not available to all investors, for others no data are available

▶ For the optimal risky portfolio to be the market portfolio, all investors need to agree on the properties of all assets

▶ The CAPM only considers the covariance of an asset with the market

▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure

▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure

▶ The covariance is regarded as the systematic risk of an asset

Capital Asset Pricing Model

▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure

▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole

▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure

▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole

▶ Unsystematic risk   is the risk unique to the asset

▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure

▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole

▶ Unsystematic risk, or idiosyncratic risk, is the risk unique to the asset

▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure

▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole

▶ Unsystematic risk, or idiosyncratic risk, is the risk unique to the asset

▶ Idiosyncratic risk can be eliminated through diversification

Systematic risk

▶ The CAPM only considers the covariance of an asset with the market, not its variance as a risk measure

▶ The covariance is regarded as the systematic risk of an asset and measures how much it varies with the market as a whole

▶ Unsystematic risk, or idiosyncratic risk, is the risk unique to the asset

▶ Idiosyncratic risk can be eliminated through diversification

Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk