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Optimal portfolios

Portfolio choice

- Assets can be characterised by their expected returns and risk
- A portfolio of assets will also require information on the correlation between assets
- Expected returns of a portfolio are given by the weights of each asset and their individual expected returns
- $\mu_P = \boldsymbol{\omega}^T \boldsymbol{\mu}$
- The risk of a portfolio is given by the weights of each asset and the variance-covariance matrix
- $\sigma_P^2 = \boldsymbol{\omega}^T \boldsymbol{\Sigma} \boldsymbol{\omega}$
- A portfolio of assets reduces risk
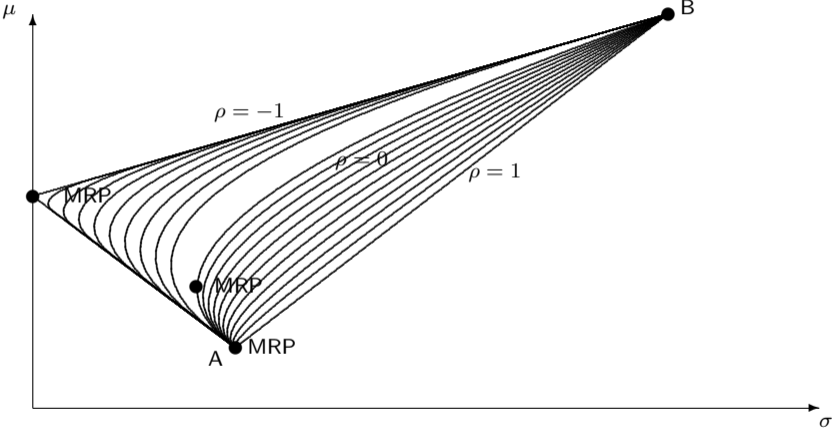
Correlation of assets and portfolio risk

► Lower correlations between assets reduces risks for a given return

► If asset returns are highly correlated, they mainly move in the same direction

⇒ Portfolio returns move widely

► With low correlations, the returns are often having different signs

⇒ Portfolio returns are moving less

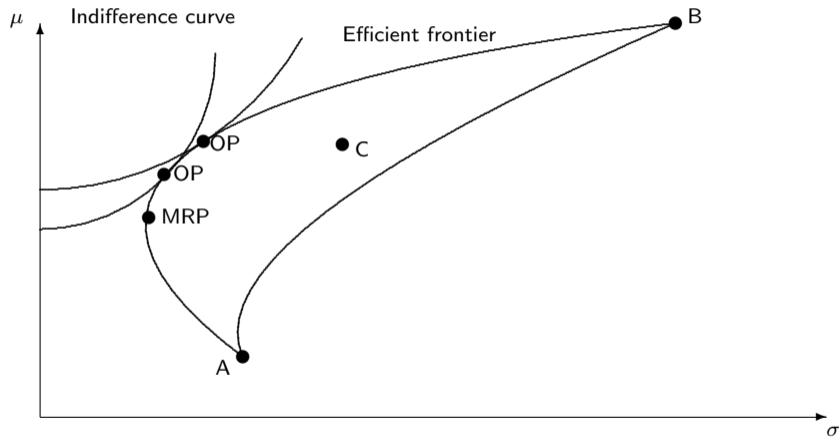► If returns are perfectly negatively correlated, returns can be risk-free
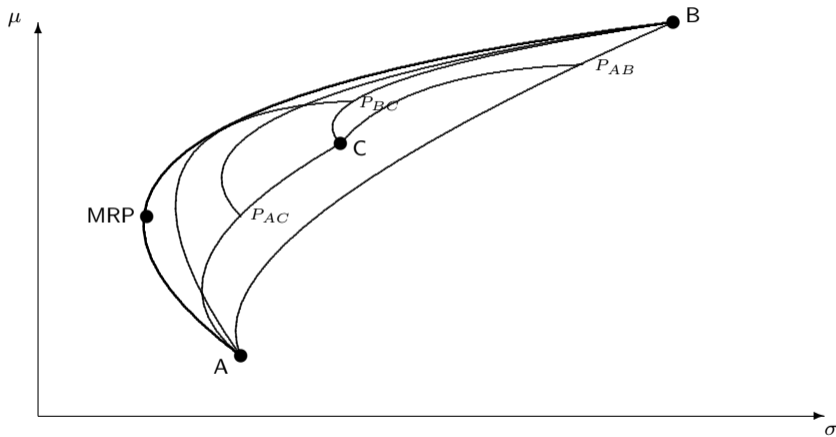
The effect of diversification

▶ More assets can reduce the risks of the efficient portfolios for a given return
▶ Risks cannot increase as additional assets could be given a weight of zero
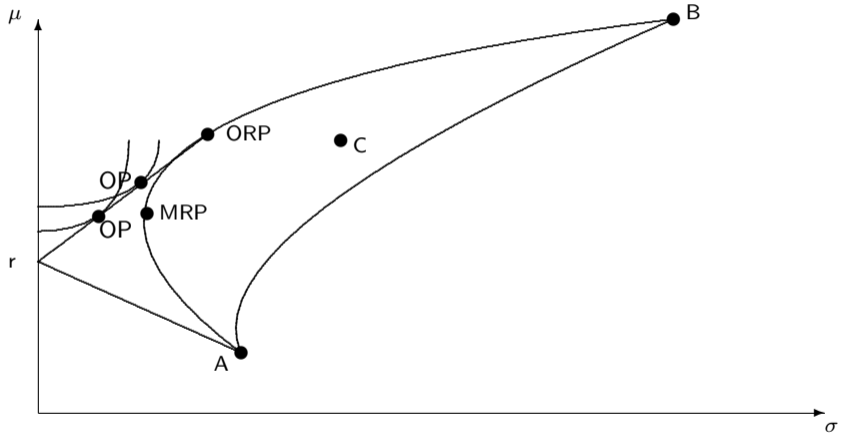⇒ Diversification reduces portfolio risk

Optimal portfolios

Optimal portfolios with risky assets

▶ The optimal portfolio is where the indifference curve is tangential to the efficient frontier
▶ Higher risk aversion implies that the optimal portfolio is closer to the minimum risk portfolio
▶ Any optimal portfolio will be efficient
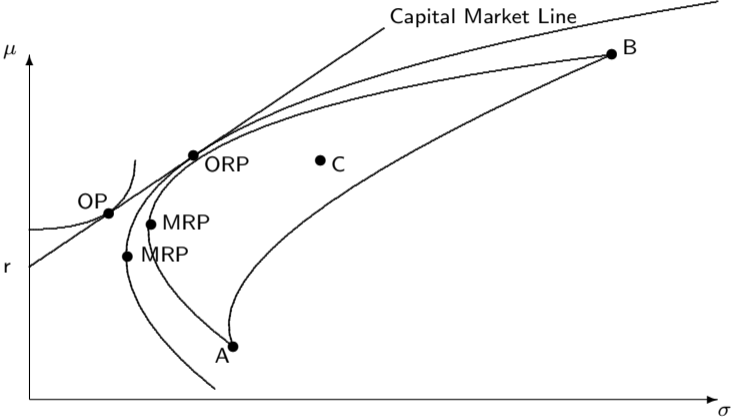
Optimal portfolios

Optimal risky portfolio

- ▶ A risk-free asset has no risk and no covariance with any asset
- ⇒ The efficient frontier with a risk-free assets is a straight line
- ▶ The efficient frontier is the line that is tangential to the efficient frontier of the risky assets
- ▶ This tangential point gives us the optimal risky portfolio
- ▶ The optimal risky portfolio does not depend on the preferences of individuals, but only on the characteristics of the assets

Separation theorem

▶ The optimal portfolio is a combination of optimal risky portfolio and the risk-free asset

▶ The more risk averse an individual is, the higher the weight of the risk-free asset

⇒ All individuals combine the optimal risky portfolio and the risk-free assets

▶ Only the weights of the risk-free asset and the optimal risky portfolio depends on individual preferences

- Short-sales are occurring if an asset is sold without holding it, giving a negative weight
- You obtain a loan in form of the asset, sell it for cash, then have to repay the asset in the future, buying it back for cash
- If the asset has increased in value, a loss accrues as it has to be bought back at a higher price
- If the asset has decreased in value, a profit accrues as it has to be bought back at a lower price
- Weights in the portfolio can now be negative, which increases the opportunity set and improves the efficient frontier
- The portfolio gives a higher utility than without short sales

Summary

- The optimal portfolio consists of a risky portfolio, identical for all investors, and the risk-free asset
- Only the combinations between these two components vary across individuals
- Using expected returns, variances and covariances, the optimal risky portfolio can be determined accurately

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