



Andreas Krause

Optimal portfolios

- We explore the properties of efficient portfolios using two assets and how efficient portfolios of more than two assets are determined.
- We will also introduce a risk-free asset to complement the risky assets in a portfolio.

Portfolio choice

- We will first look at how the properties of a portfolio emerge from that of the individual assets.
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 - Each asset can have its own outcome (expected return),
 - as well as its own variance (risk). We will require information on this for each asset we consider in our portfolio.
- ▶ Of importance will also be the correlation between all these assets, which will also have to be determined. It is common to use past returns to obtain such information for stocks, but individual assessments can also be used to obtain this information.
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 - The outcome of the portfolio (expected return) is given by the weight of each asset in the portfolio. As a normalisation we use the investment into the specific asset, divided by the total investment, which gives the weight of the asset in the portfolio.
 - We multiply this weight by the expected return of the asset. This gives us the weighted average of the expected returns, which is the expected return of the portfolio.
- ▶ *Formula.* The weight and expected returns are vectors, the superscript T denotes the transpose.
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 - We do a similar approach to determine the risk (variance) of the portfolio. We use the weights of the asset in the portfolio.
 - In addition, we need the variance-covariance matrix. This matrix has as its main diagonal the variance of the returns and the off-diagonal elements are the covariances between the assets; the matrix is symmetric as the covariance between assets i and j is the same as the covariance between assets j and i .
- ▶ *Formula.* For two assets, this formula is equivalent to $\sigma_P^2 = \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + 2\omega_1\omega_2\sigma_{12}$, with σ_{12} being the covariance of the two assets.
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Portfolio choice

- ▶ Assets can be characterised by their **expected returns**

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- ▶ Assets can be characterised by their expected returns and risk
- ▶ A portfolio of assets will also require information on the **correlation** between assets

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- ▶ A portfolio of assets will also require information on the correlation between assets
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- ▶ $\mu_P = \omega^T$

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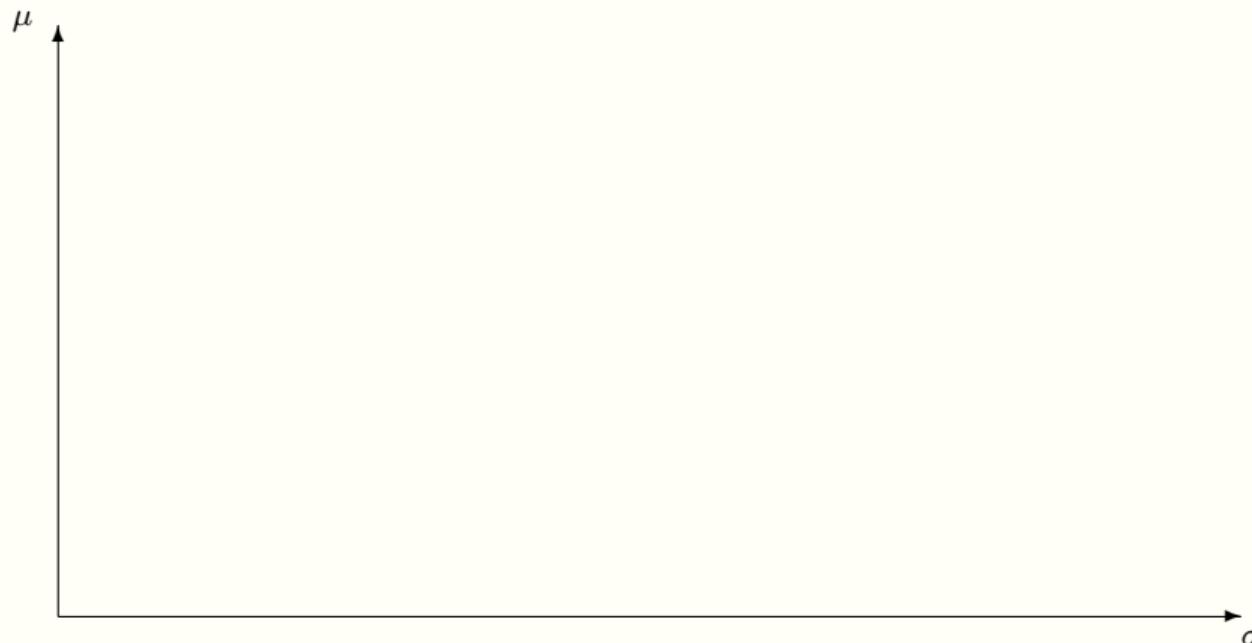
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Portfolios with two assets

- Looking at the possible portfolios between two assets, we will focus on the impact that different correlations have on the risks of the portfolio.
- We look at the expected return and rather than the variance, the standard deviation, its square root, is used as risk measure; this use of the risk measure is done by convention.
- We consider the first asset to have a specific expected return and standard deviation.
- The second asset here has a higher return and a higher risk.
- We can show that if the two assets are perfectly correlated, all possible portfolios are located on a straight line between them.
- The minimum risk portfolio in this case will be the asset with the lower risk, here asset A .
- We can now slowly decrease the correlation between the two assets. We see how the possible portfolios move towards the left; the minimum risk portfolio (MRP) moves to the left as well, implying a reduced risk, but it also moves up, providing a higher return.
- We can carry on reducing the correlation until we reach a correlation of zero.
- The minimum risk portfolio will have a significantly lower risk than either of the two assets.
- We can now reduce the correlation even further by looking at negative correlations. The portfolios move ever further to the left and the minimum risk portfolio has ever smaller risks.
- Once we reach a perfectly negative correlation, we can show that the portfolios are on a straight line to the vertical axis.
- The minimum risk portfolio has no risk in this case.
- A lower correlation moves the efficient frontier, which would be the part above the minimum risk portfolio, to the upper left. This implies that a lower correlation gives a higher utility and individuals would prefer assets with lower correlations.

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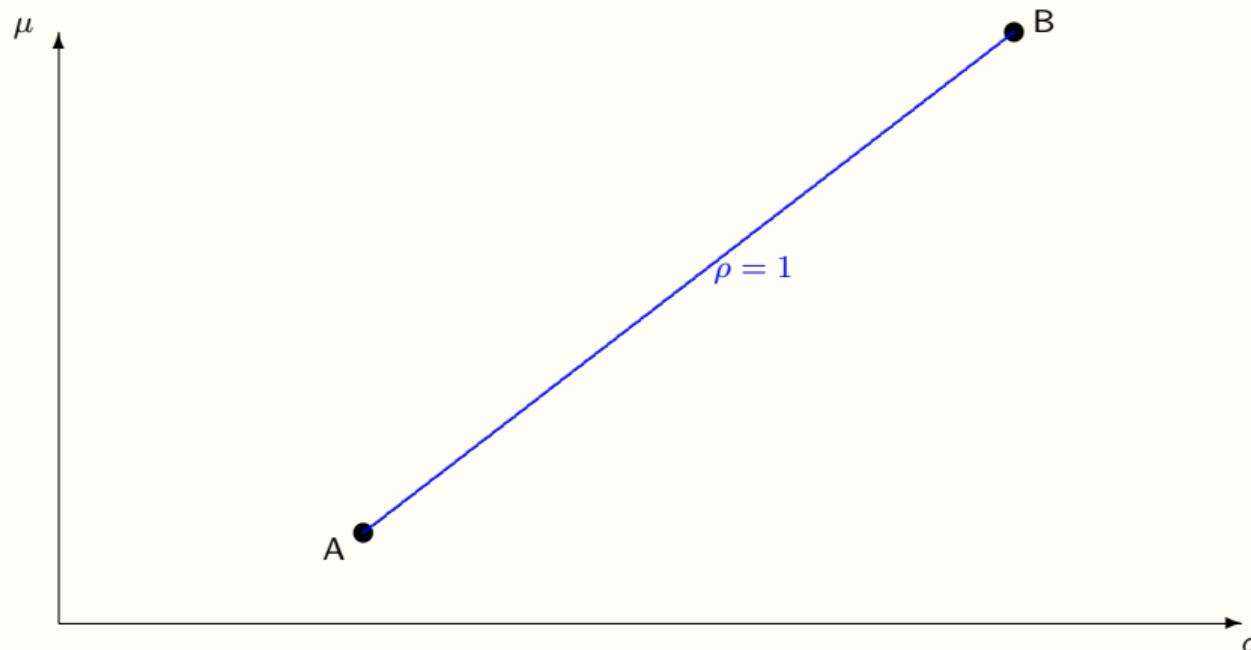


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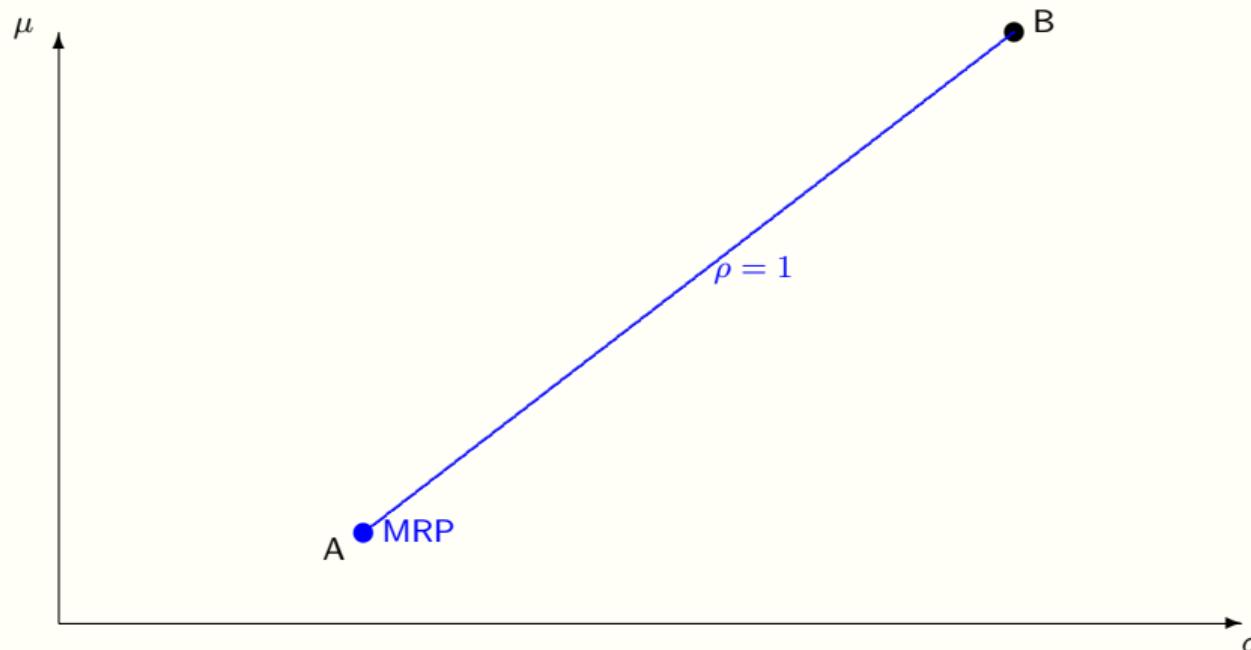


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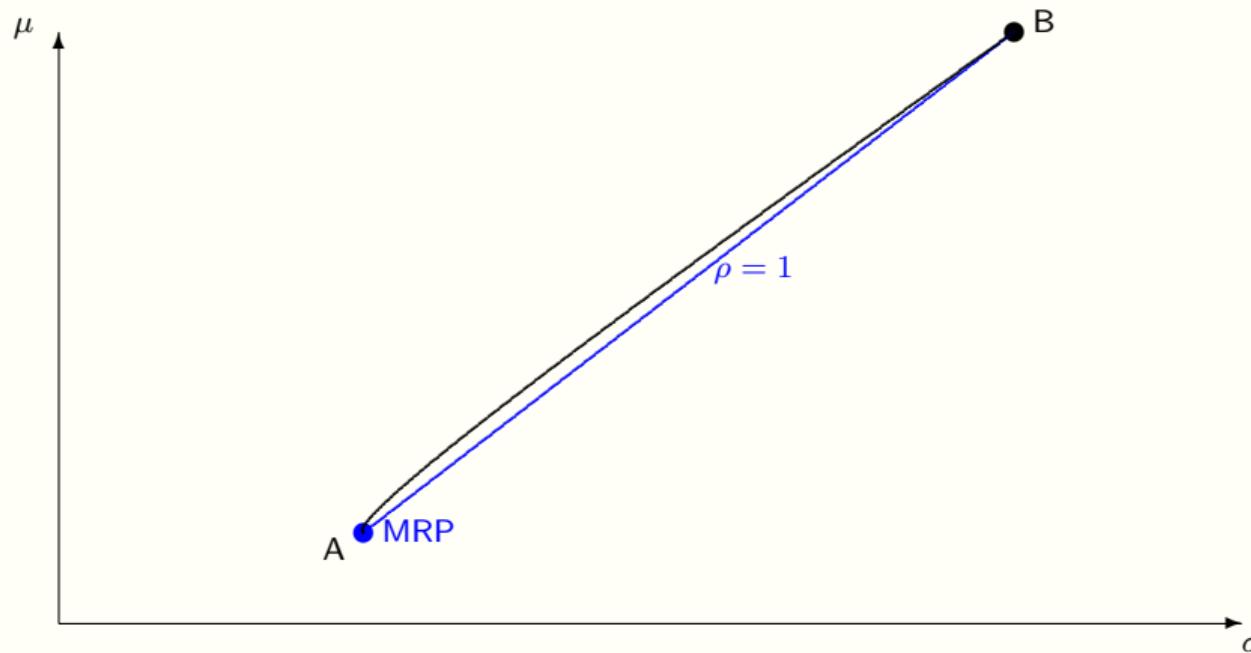


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Portfolios with two assets

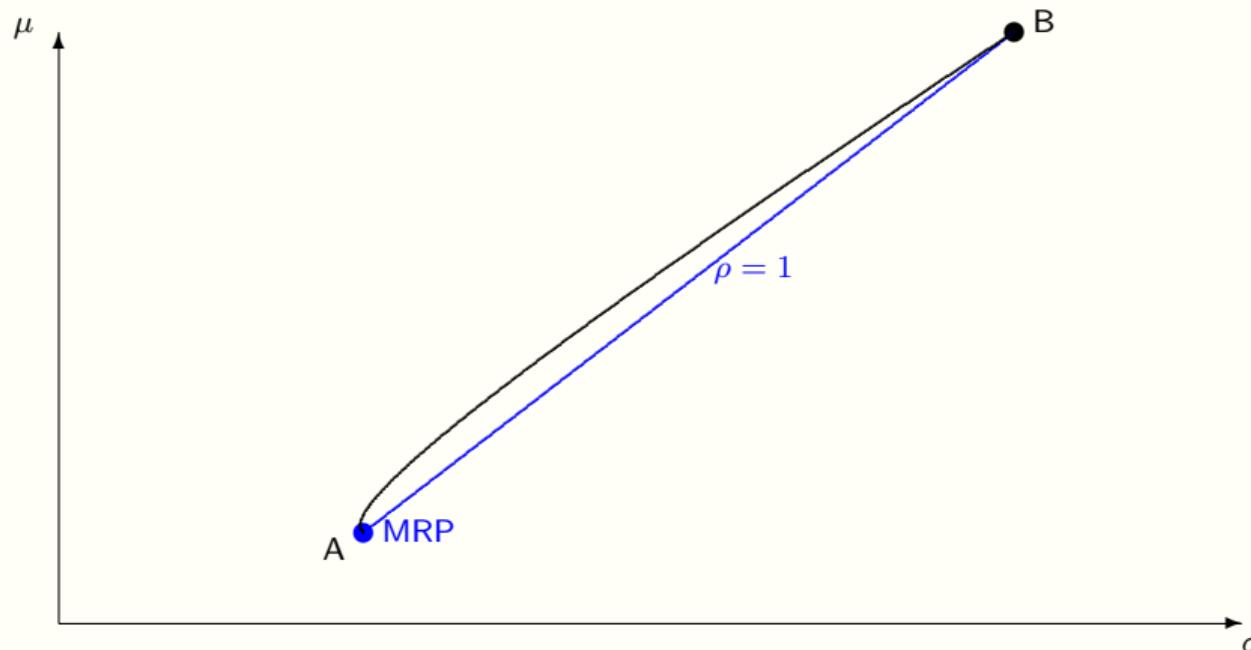


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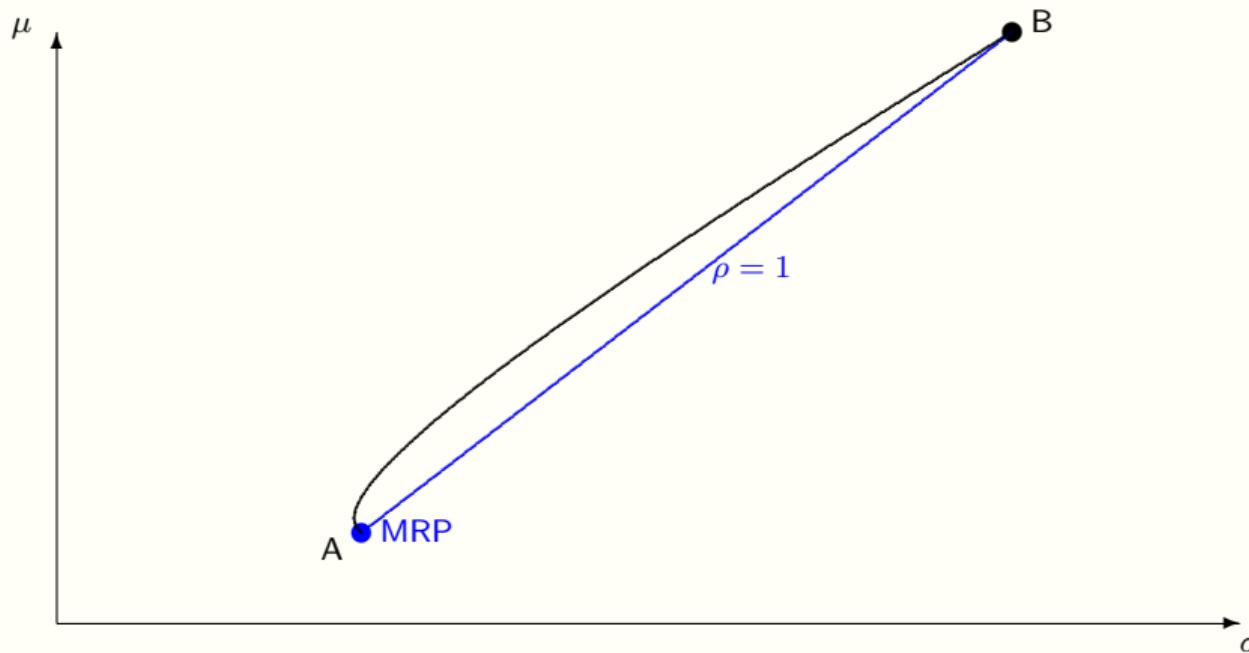


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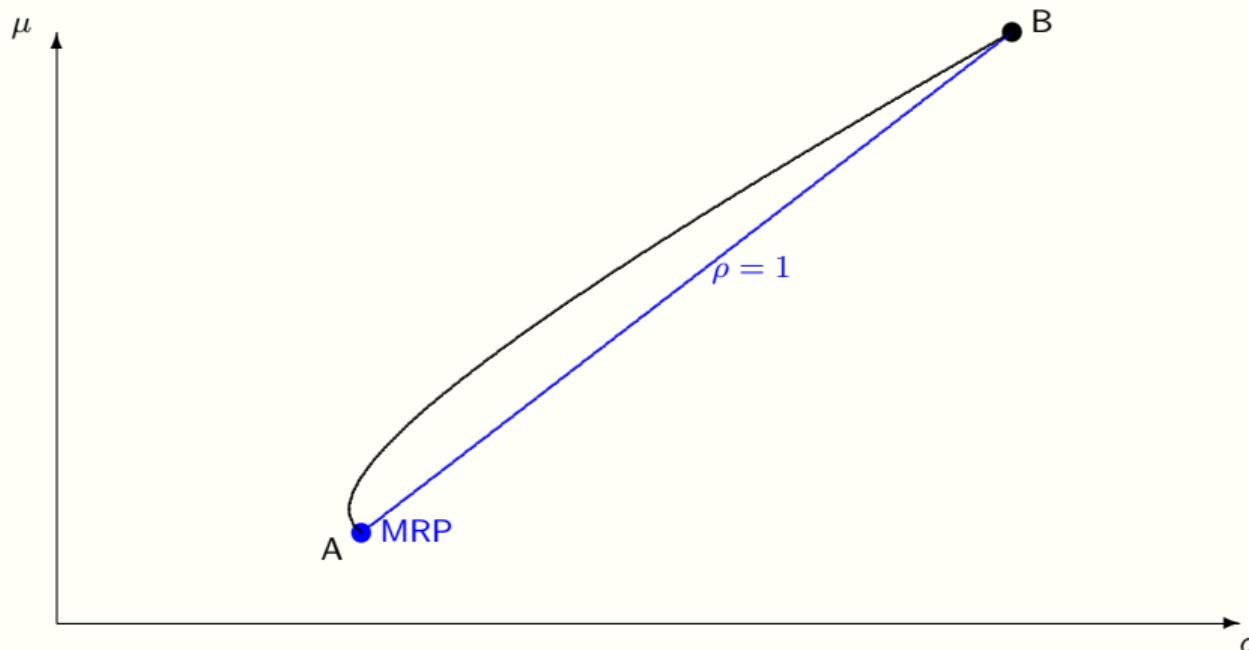


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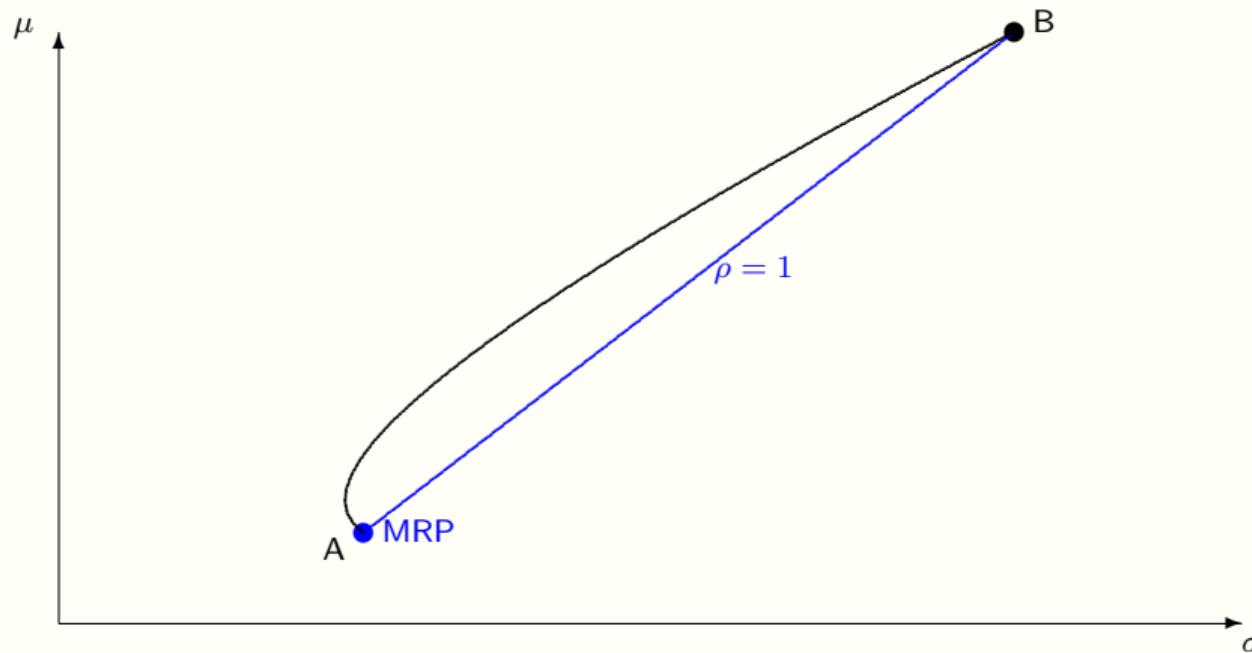


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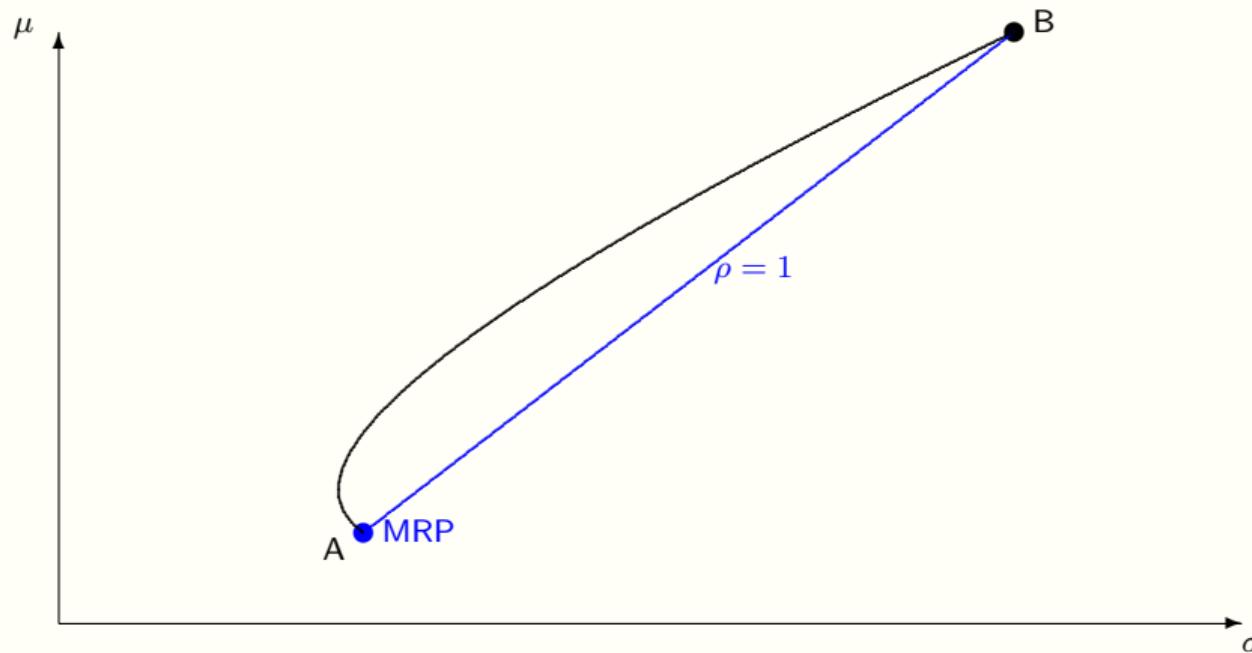


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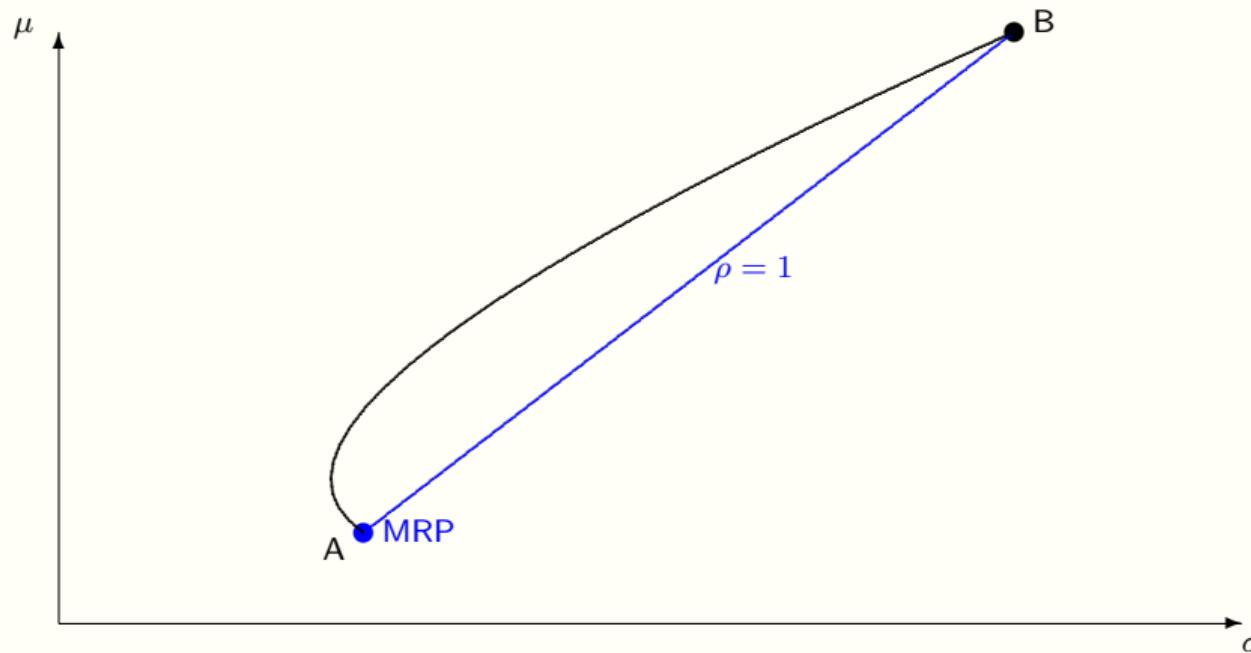


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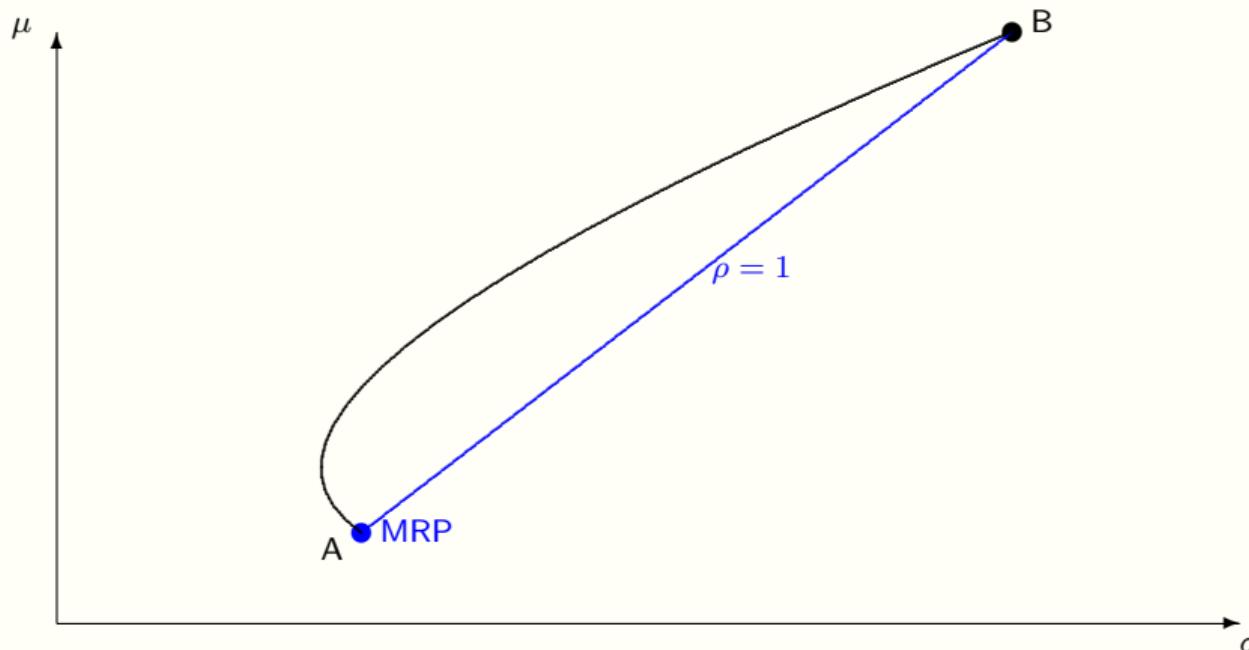


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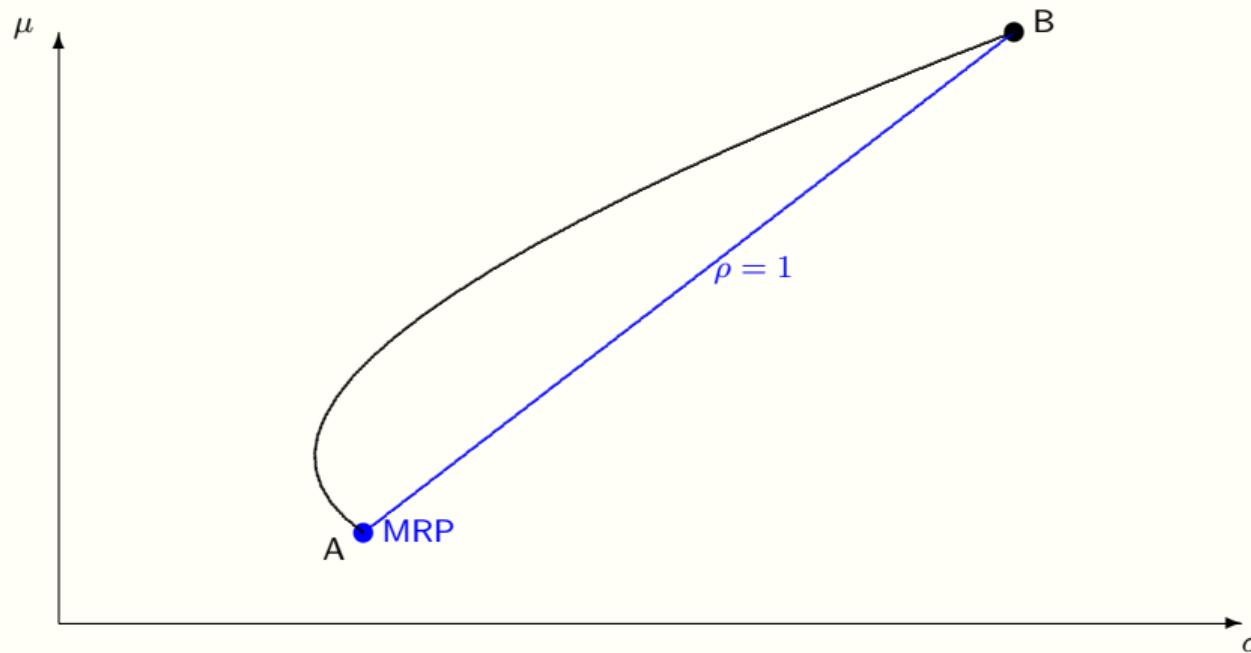


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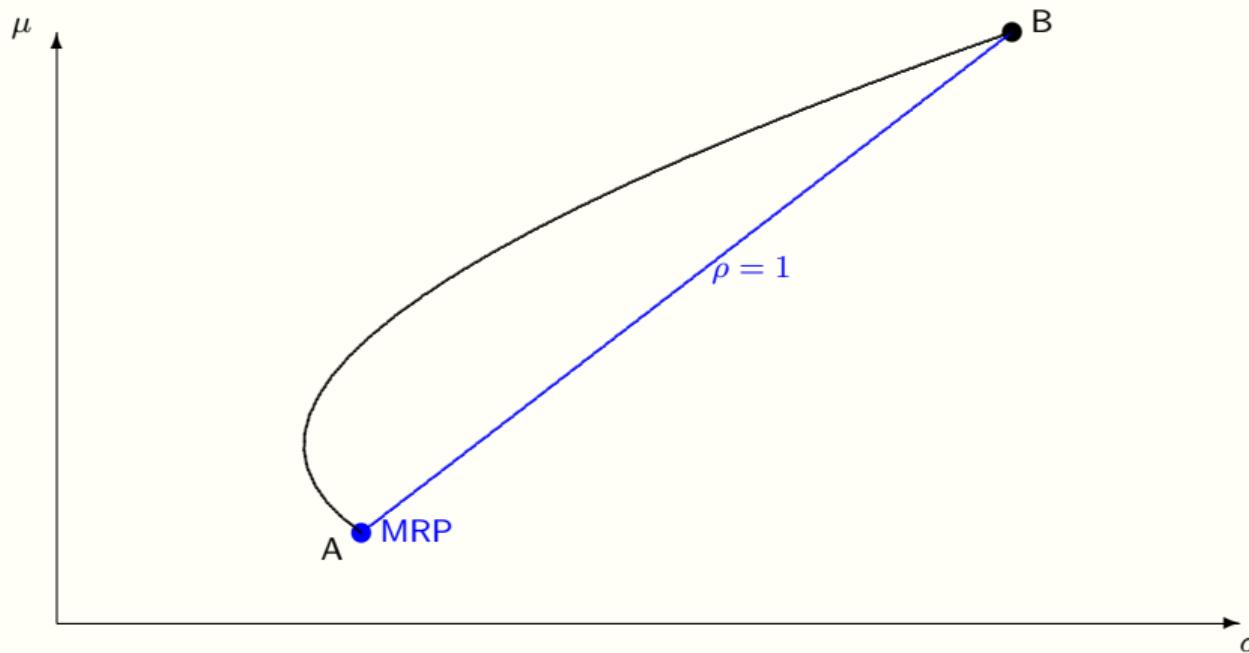


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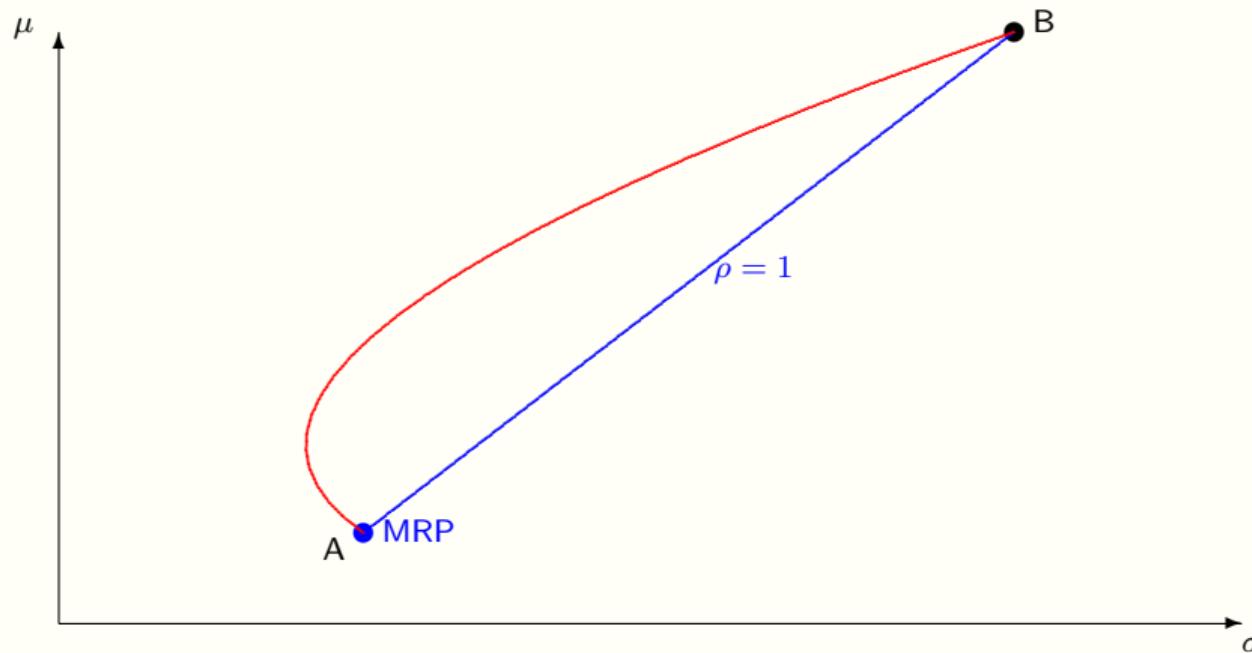


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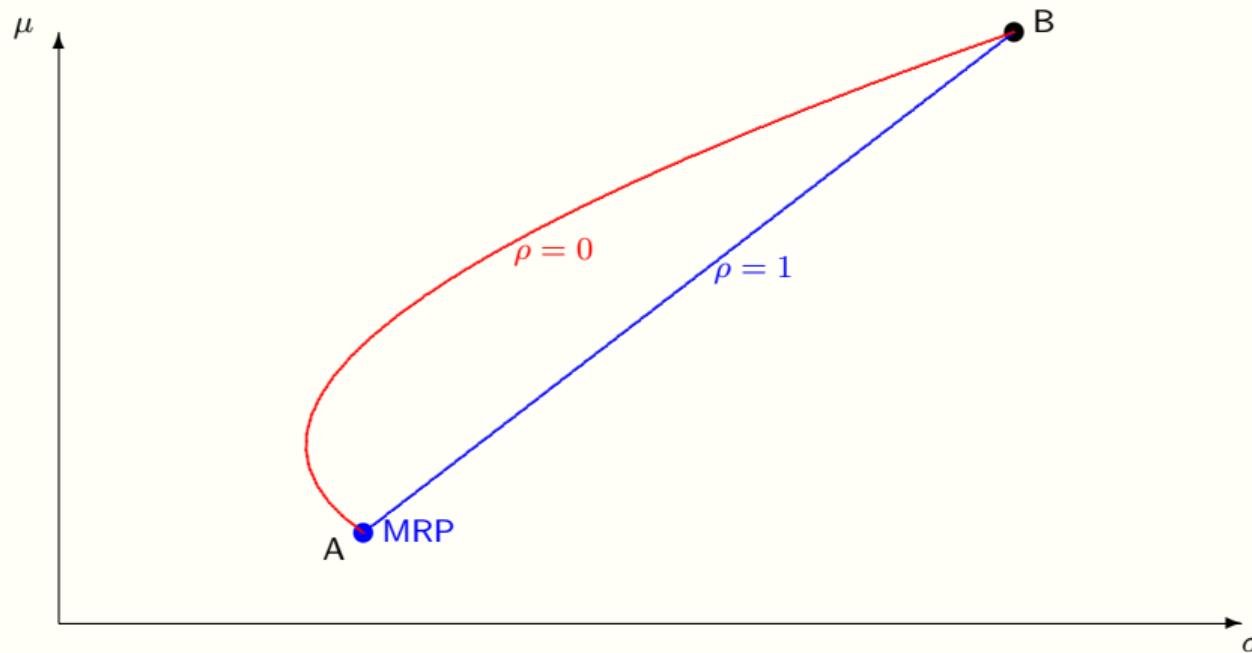


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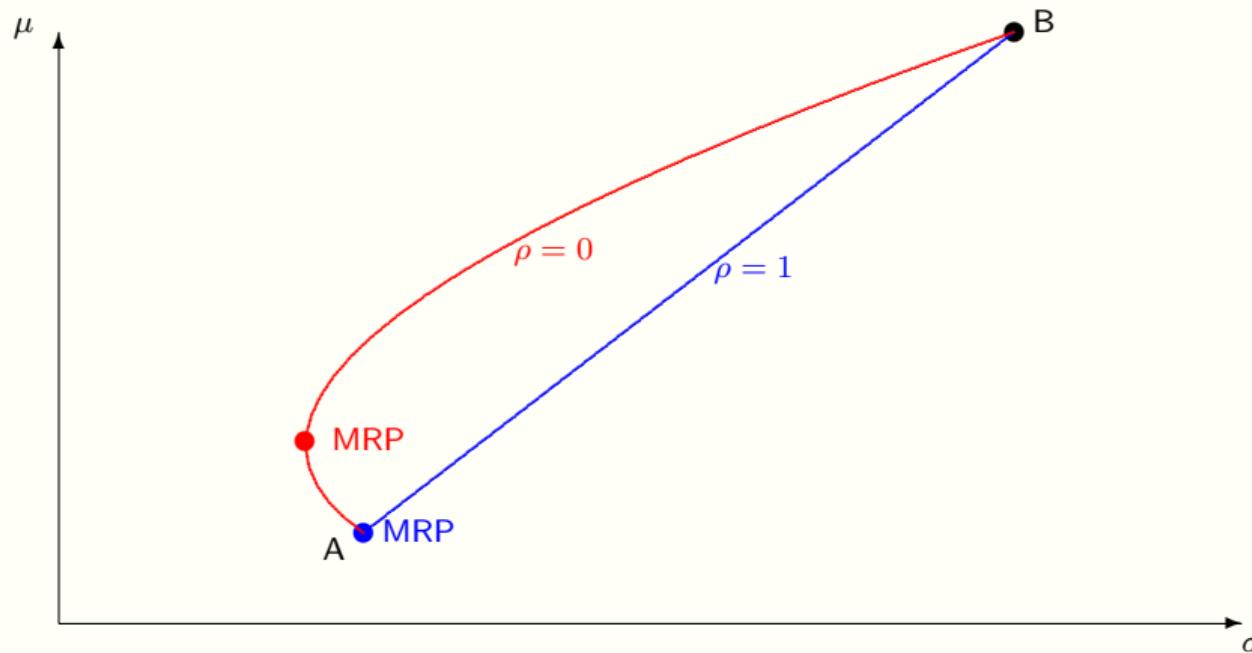


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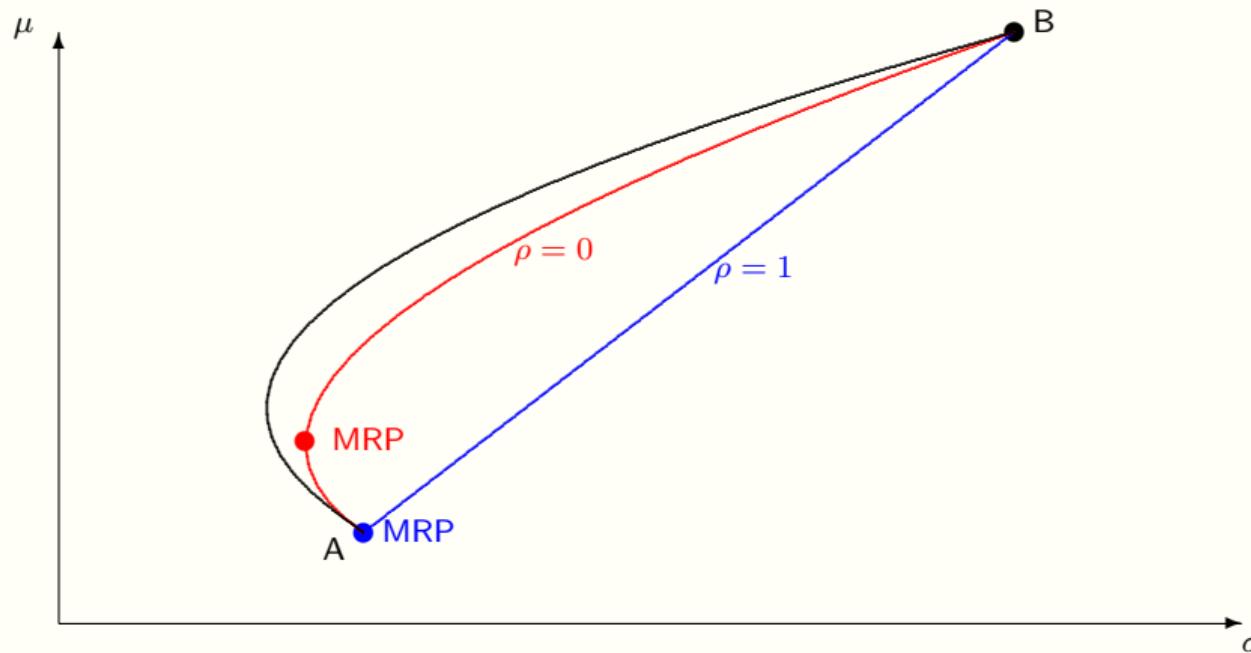


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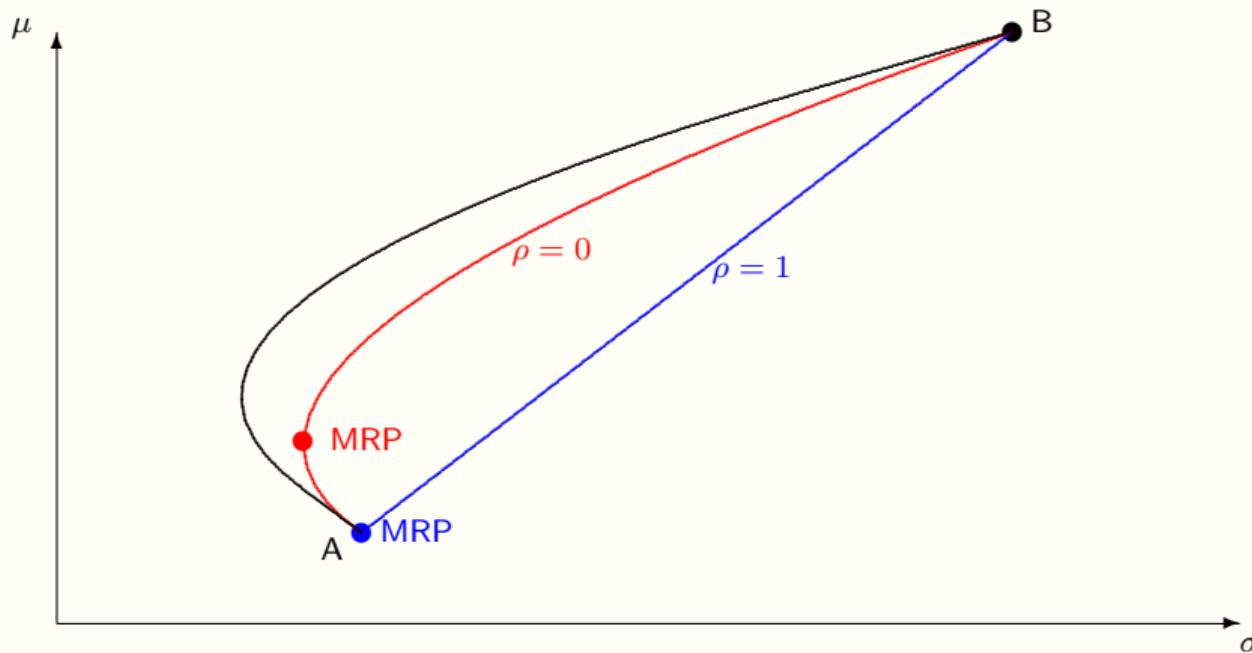


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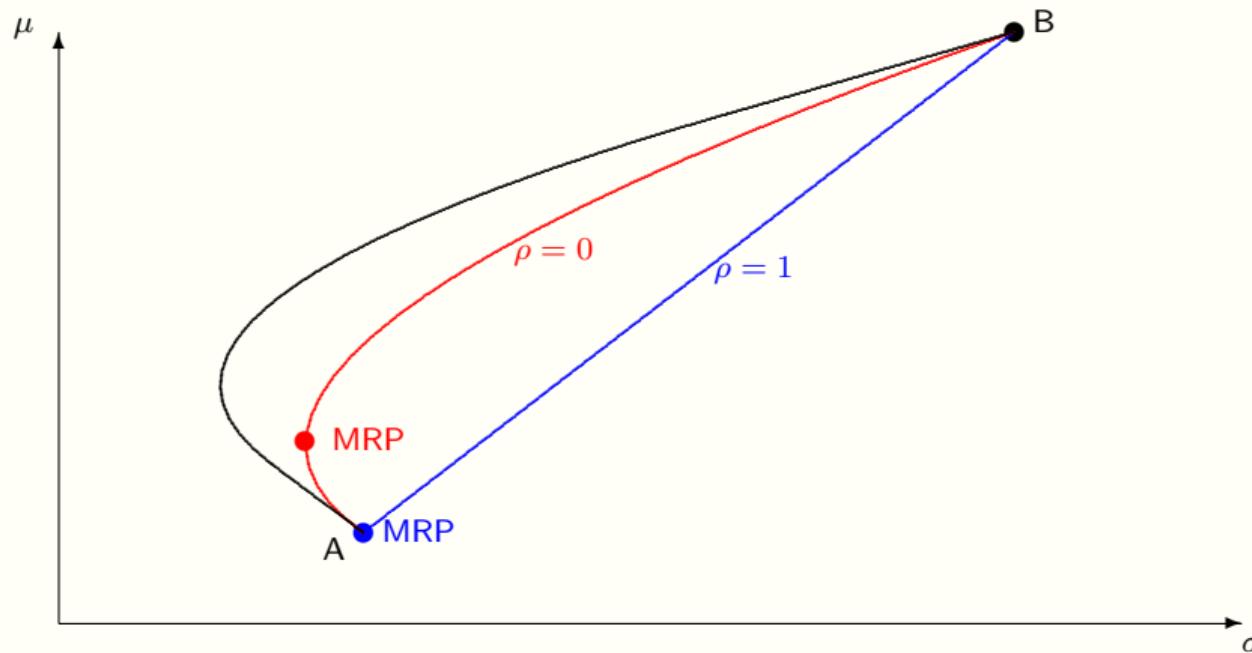


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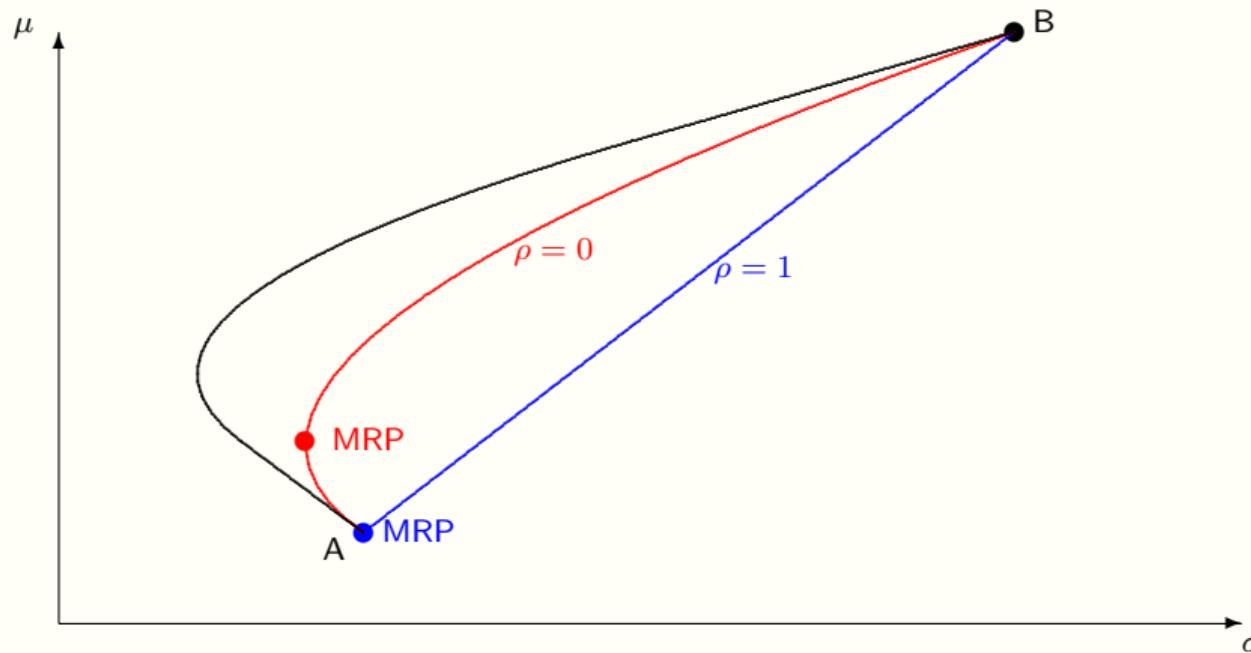


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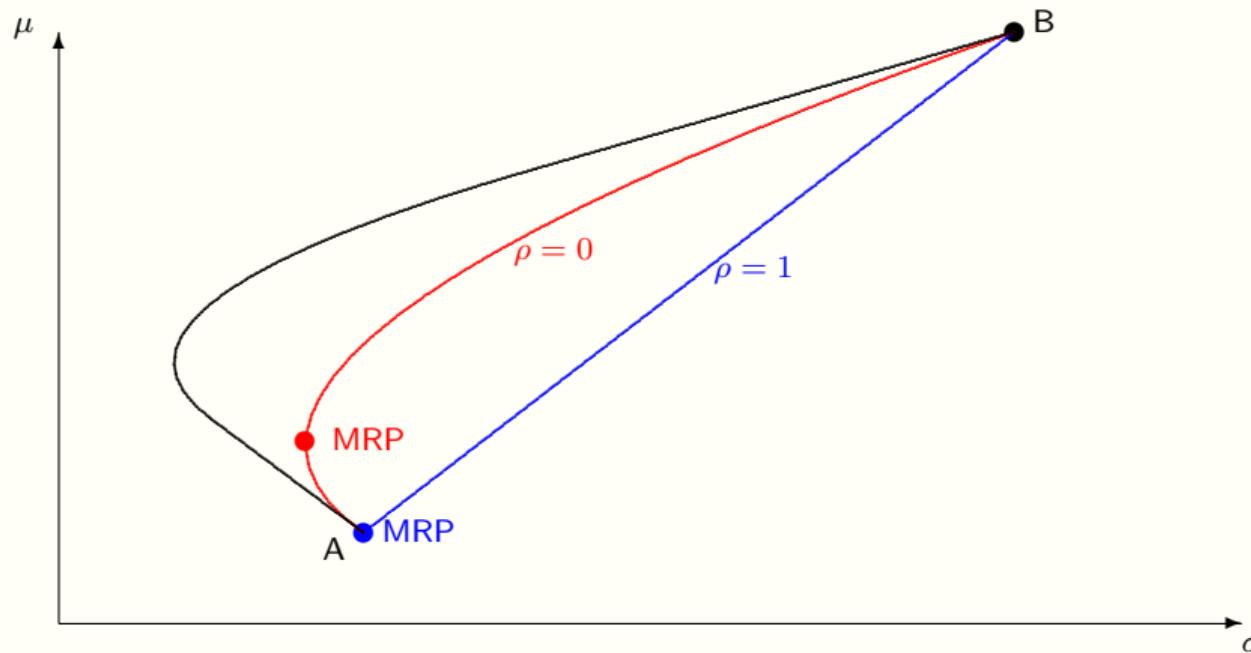


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Portfolios with two assets

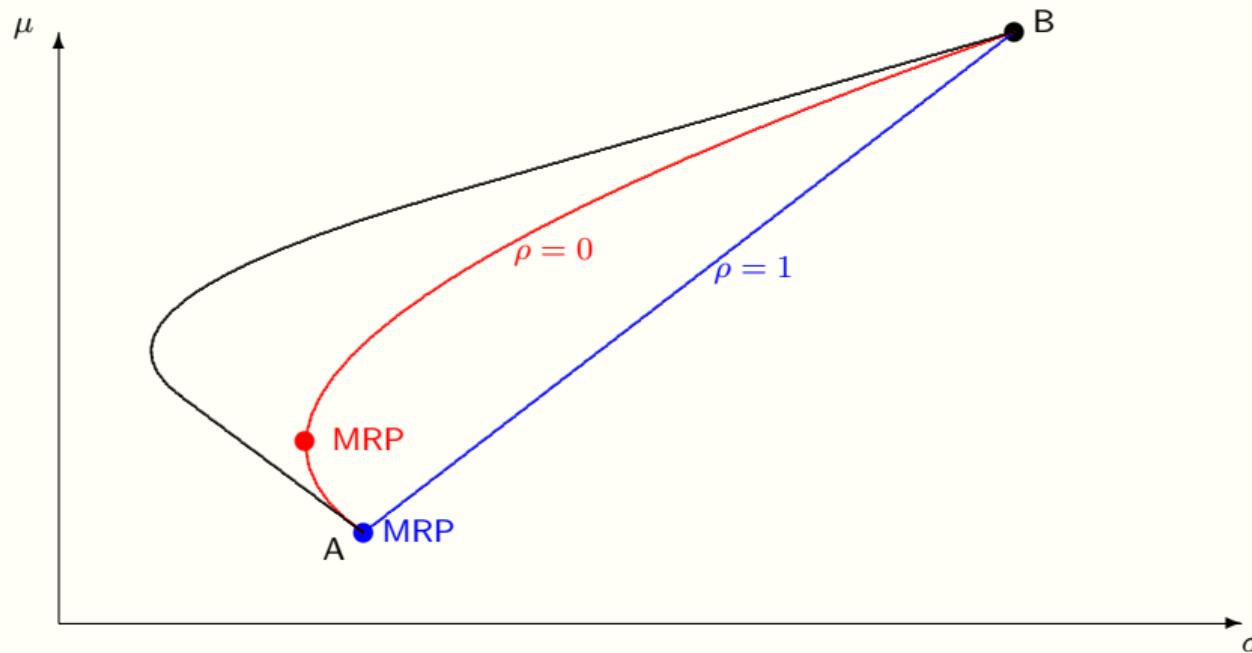


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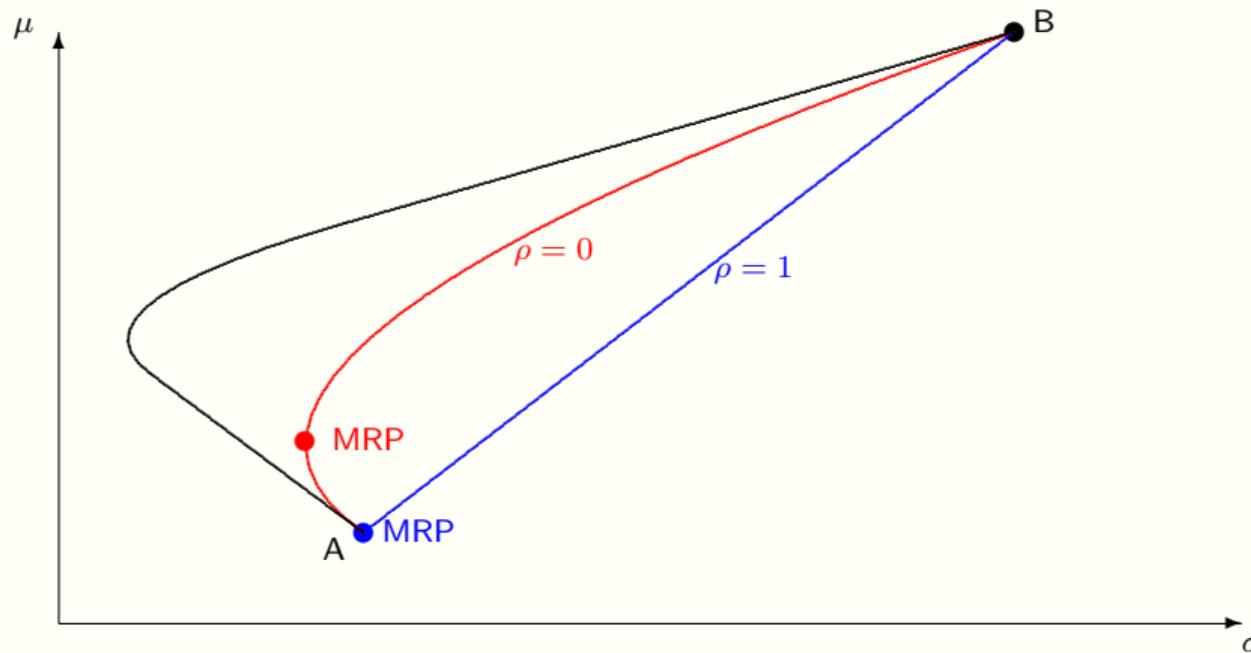


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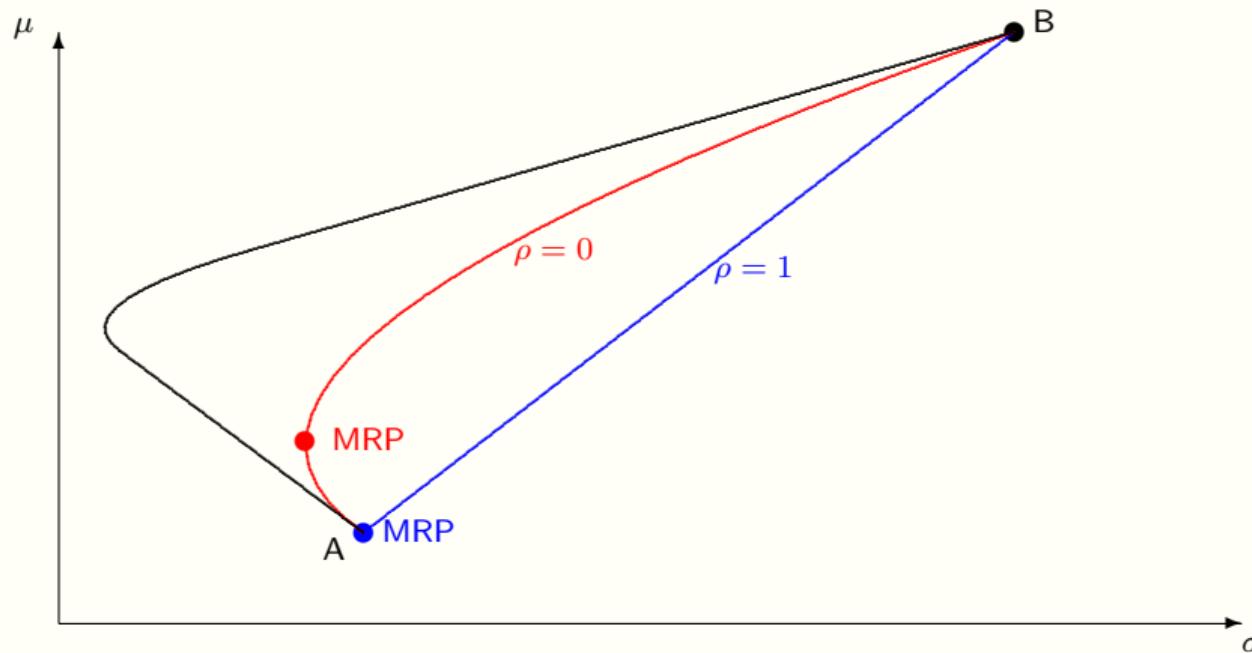


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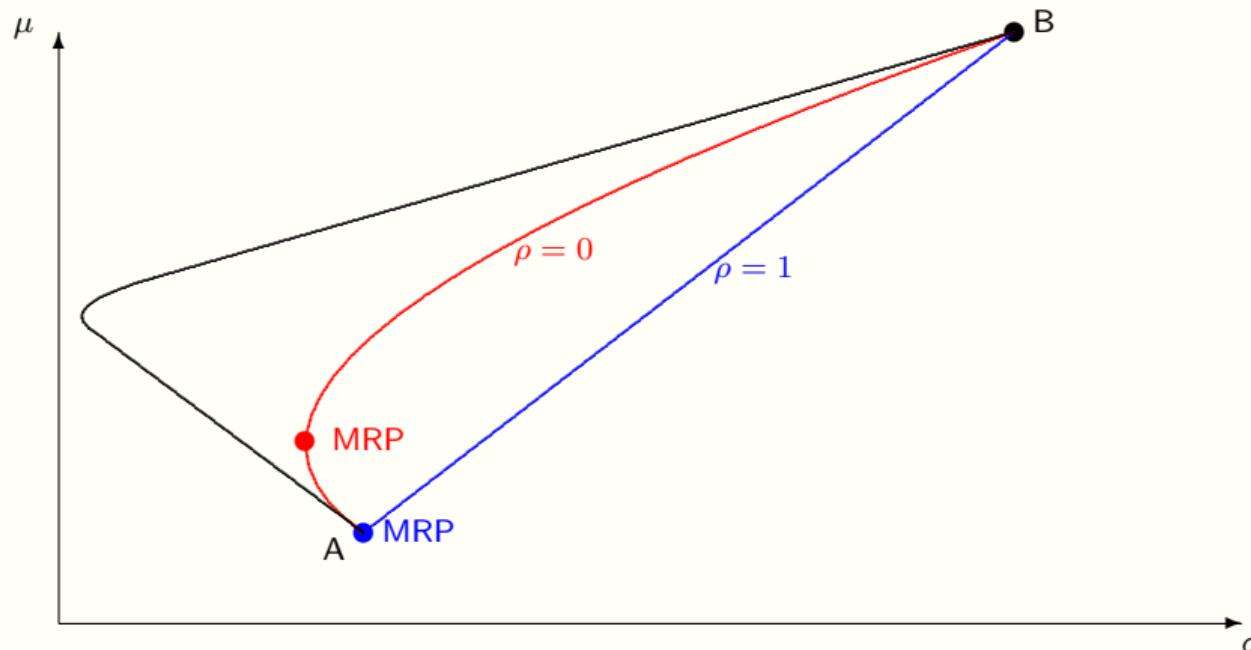


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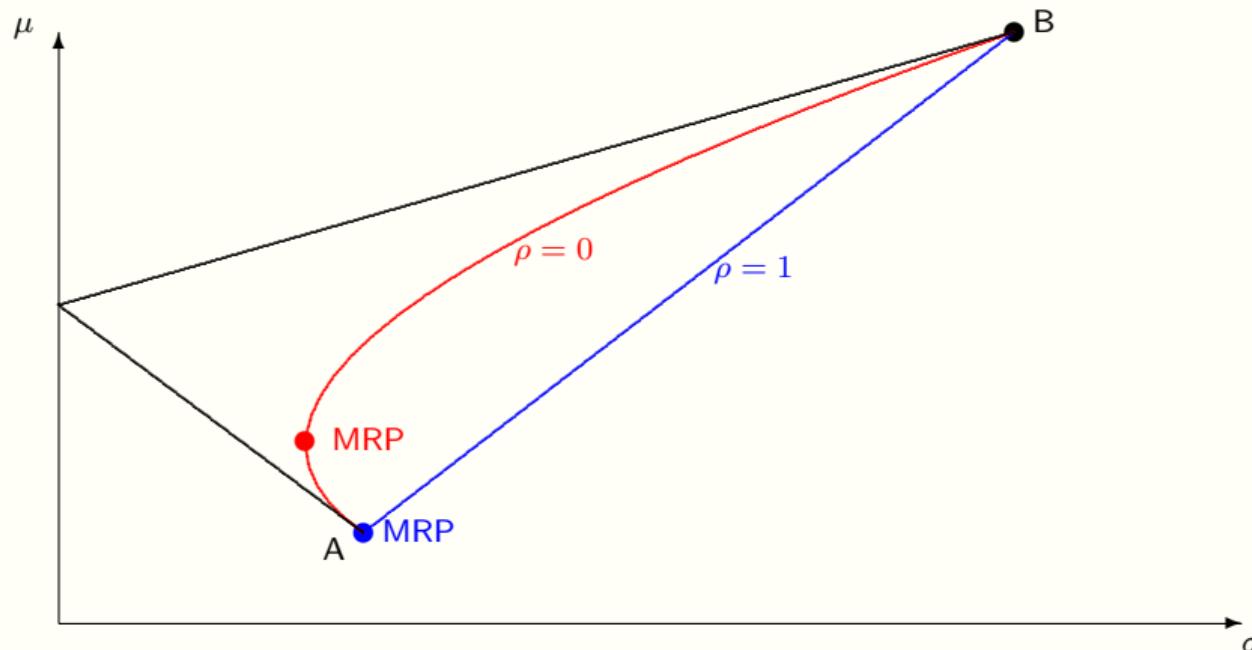


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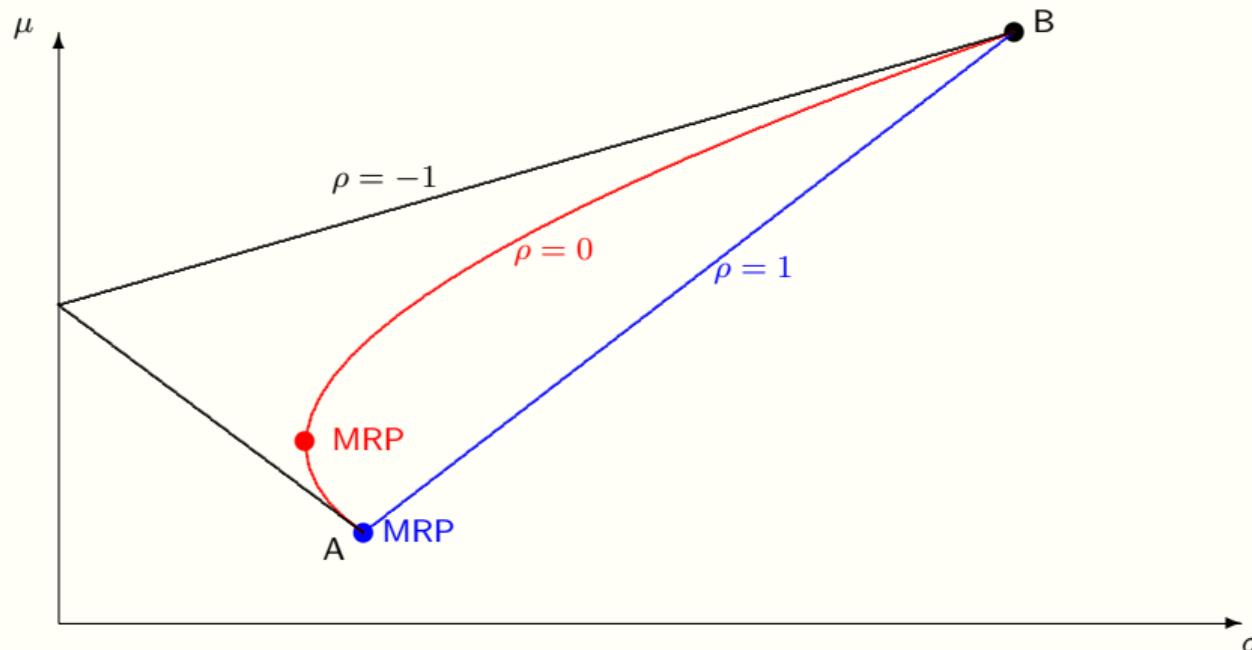


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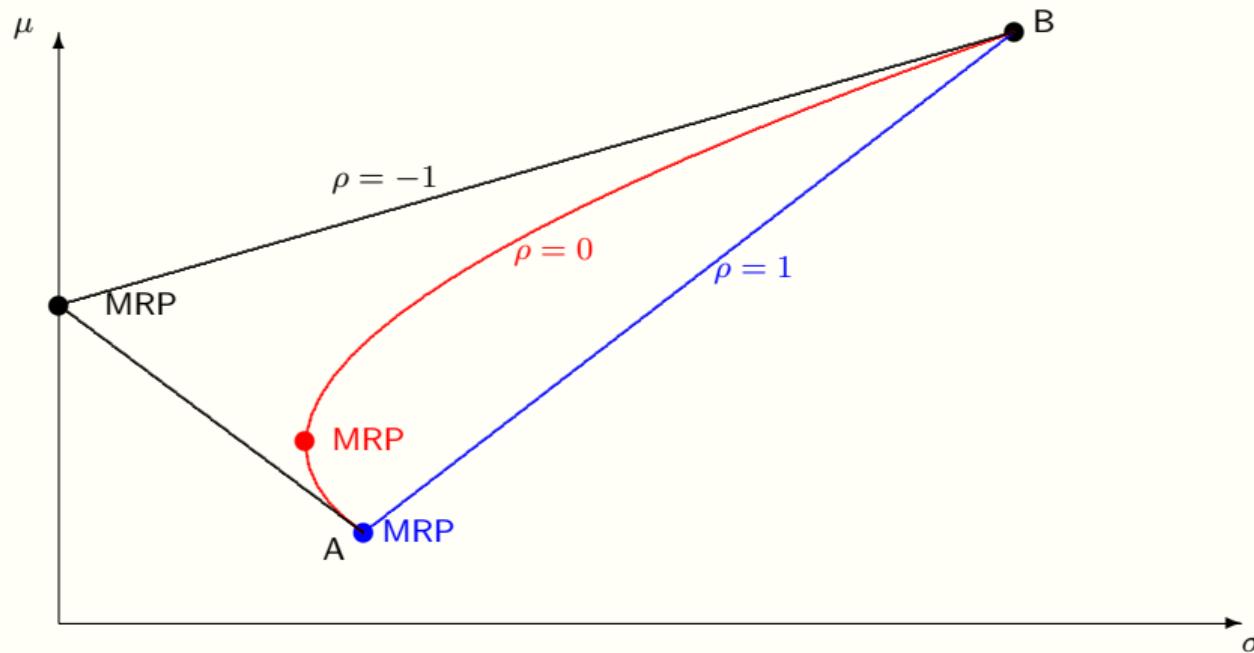


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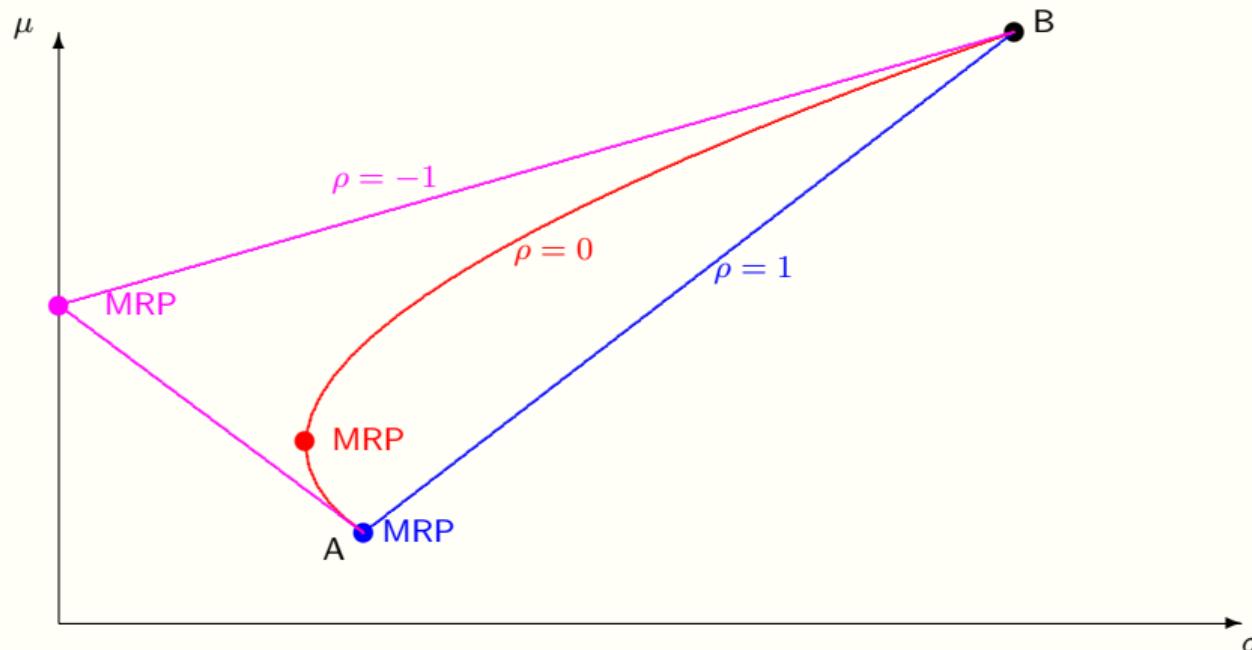


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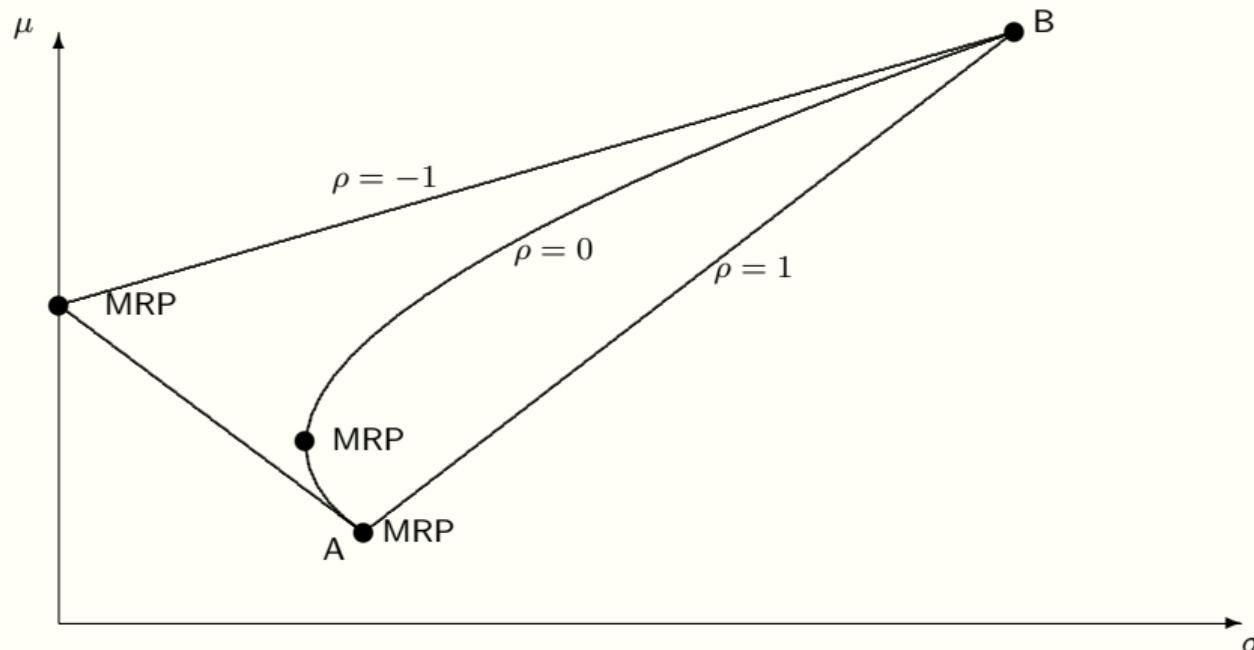


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Correlation of assets and portfolio risk

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- We can now explore the importance of correlations for the risk of a portfolio.
- We have seen that a lower correlation reduces the risks of portfolios by moving the opportunity set (the line as we only have two assets), to the left.
- To understand this relationship, assume that assets are highly correlated. In this case their movements will be mostly going in the same direction.
- ⇒ If both assets move in the same direction, then the portfolio will move a similar magnitude each time the assets themselves move. This means that the risk of the portfolio is similar to the risk of the assets.
- If correlations are low, then on many occasions the two assets will move in opposite directions. The consequence is that when adding the two movements in the portfolio, they partially will cancel each other out.
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Correlation of assets and portfolio risk

- ▶ Lower correlations between assets **reduces risks** for a given return

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Correlation of assets and portfolio risk

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- ▶ If asset returns are highly correlated, they mainly move in the **same direction**

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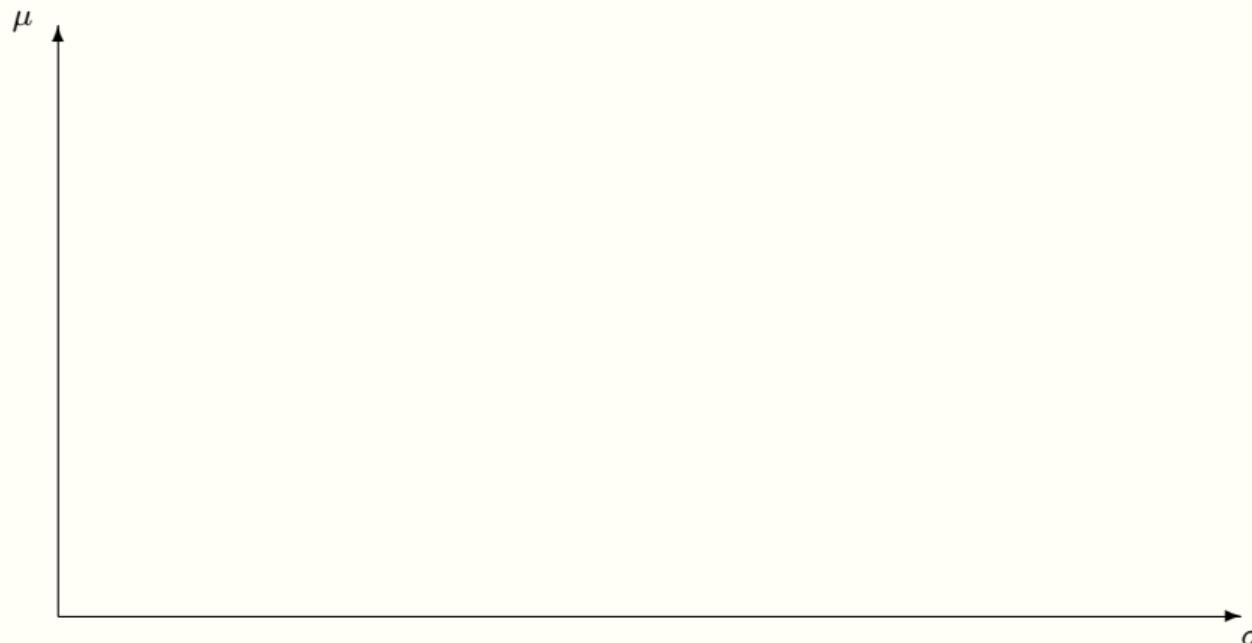
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- If returns are perfectly negatively correlated, then we can obtain a portfolio where these movements cancel each other out completely. With perfectly negatively correlated returns, the signs of these returns will be opposite on all occasions. It is then merely a case to obtain the magnitude of these movements from their respective variance to ensure the returns cancel each other out perfectly. If the portfolio has no risk, the return it generates should be the risk-free rate.
- We can now expand our analysis from two assets to portfolios of more assets.

Portfolio with more assets

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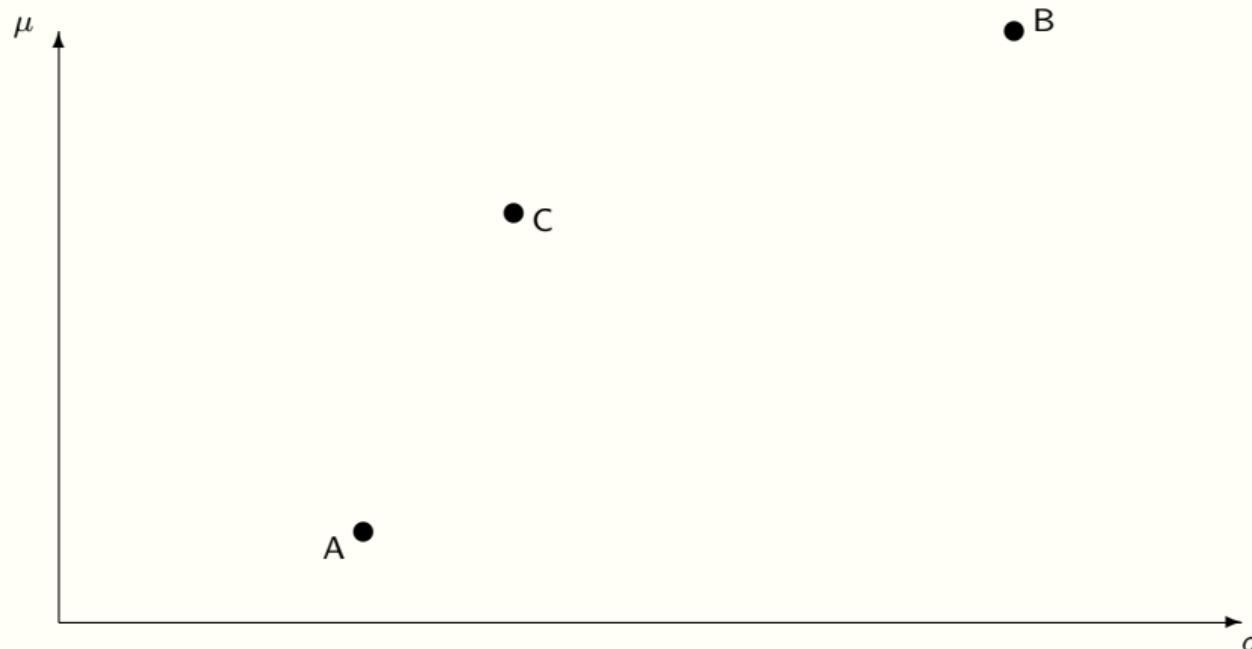
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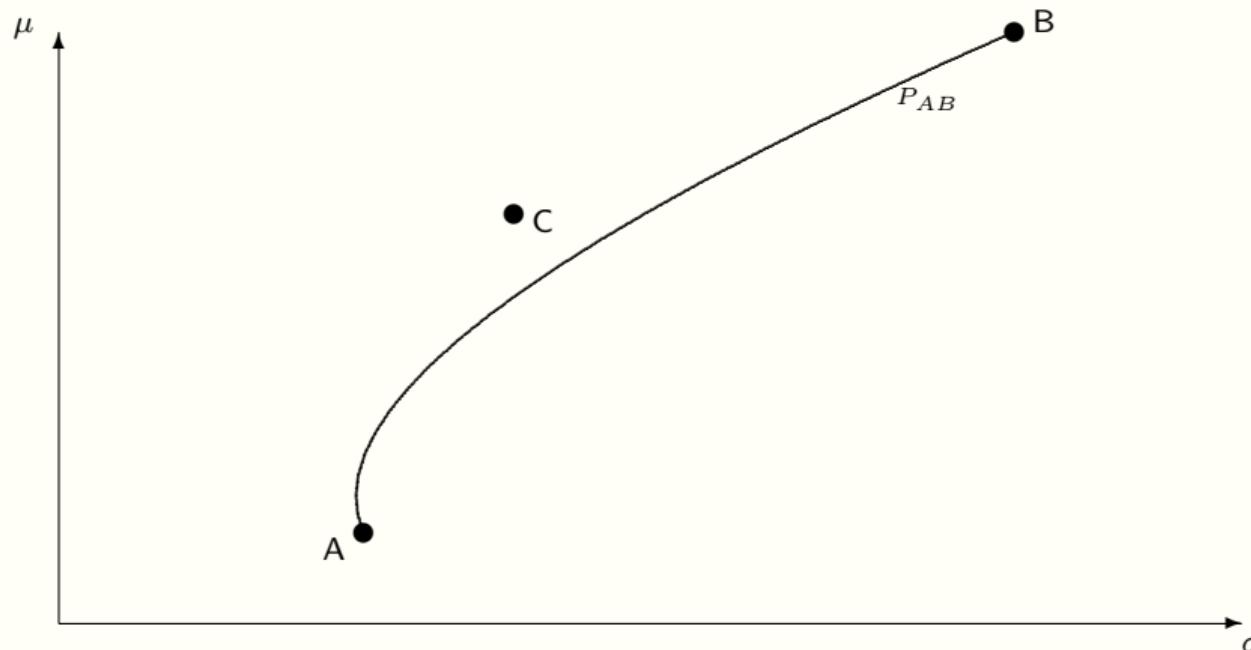
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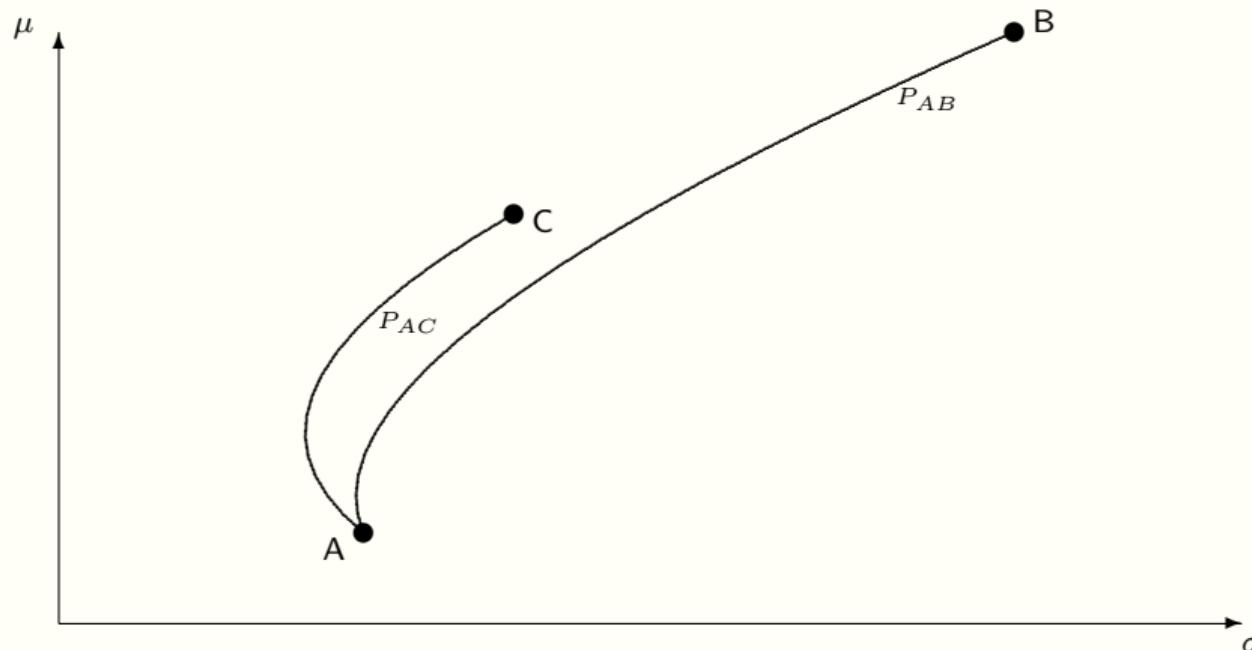
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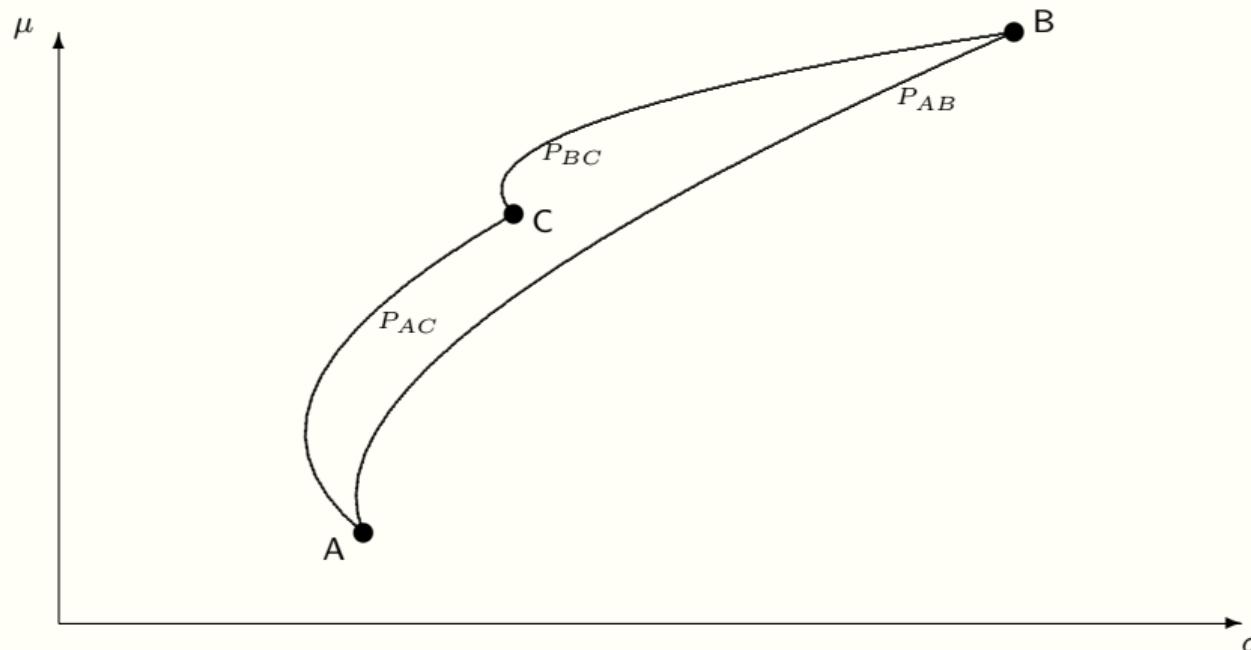
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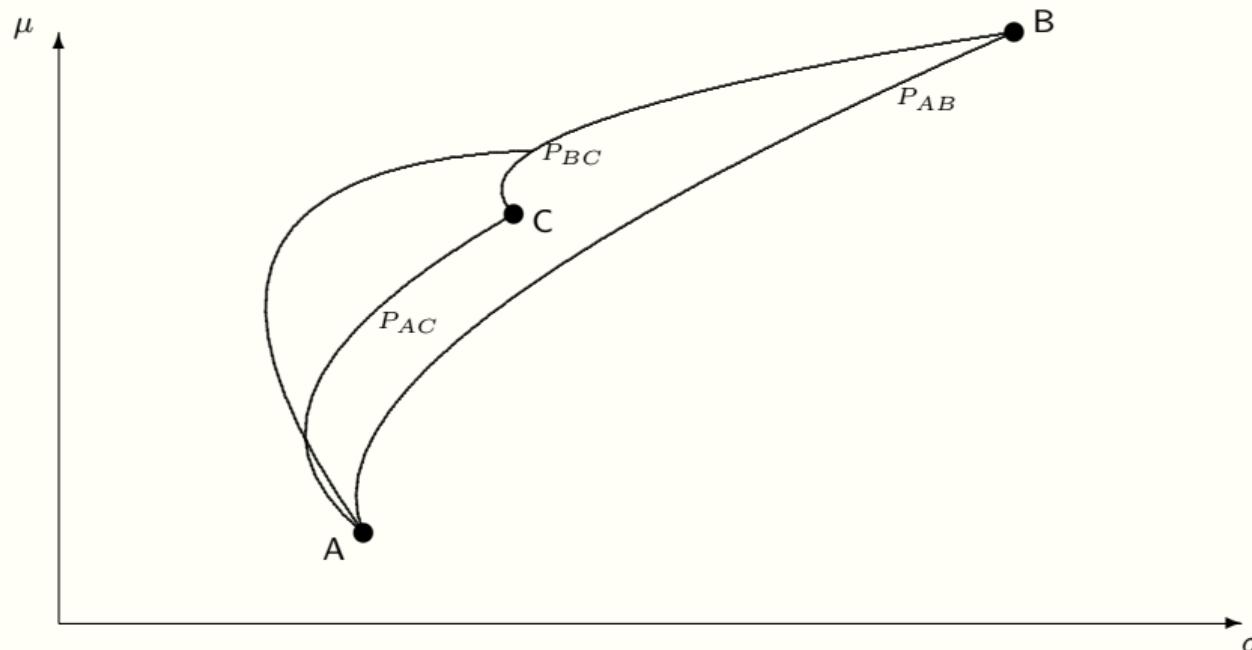
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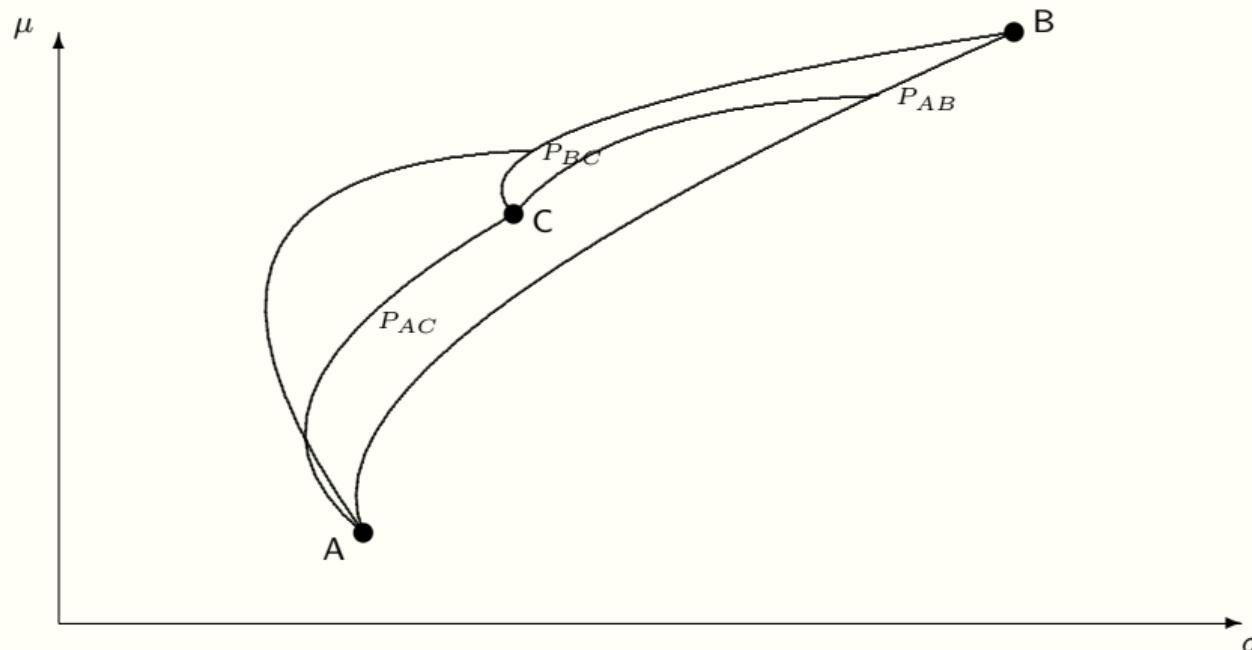
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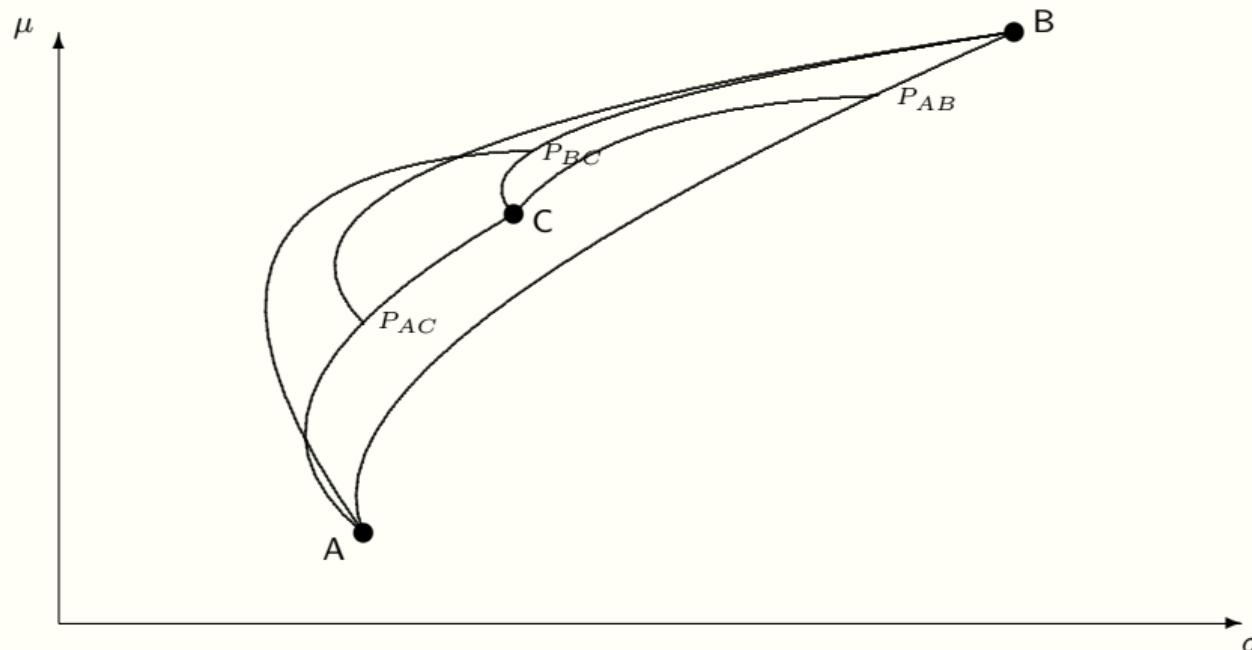
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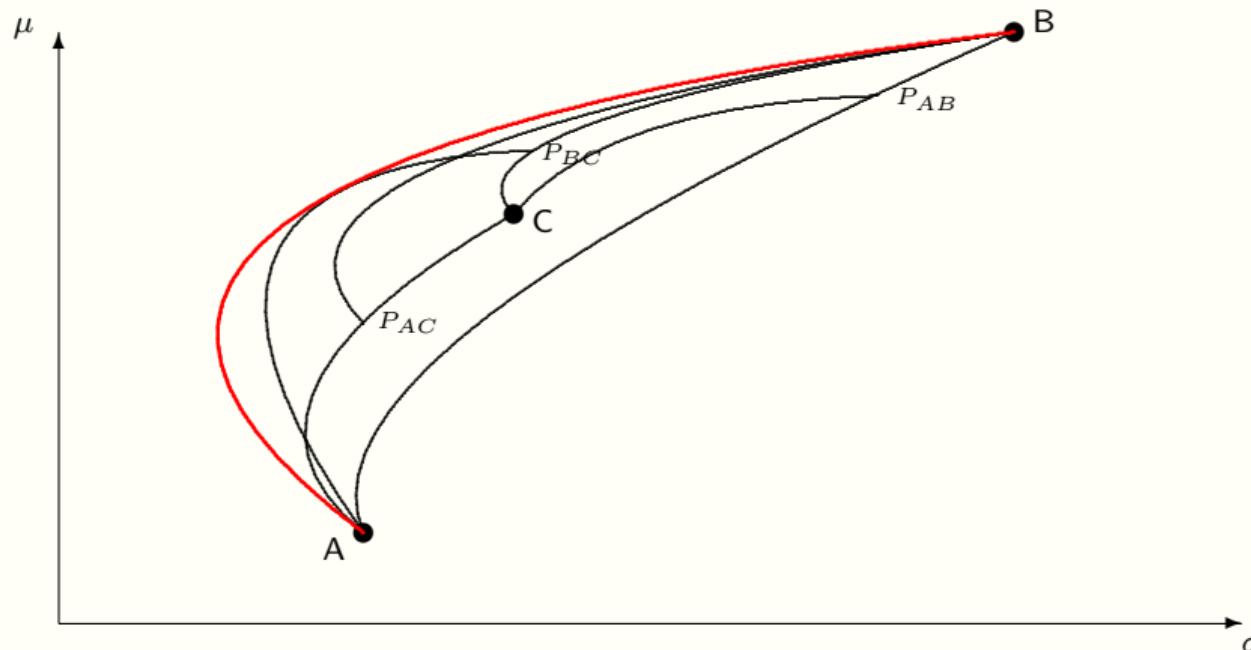
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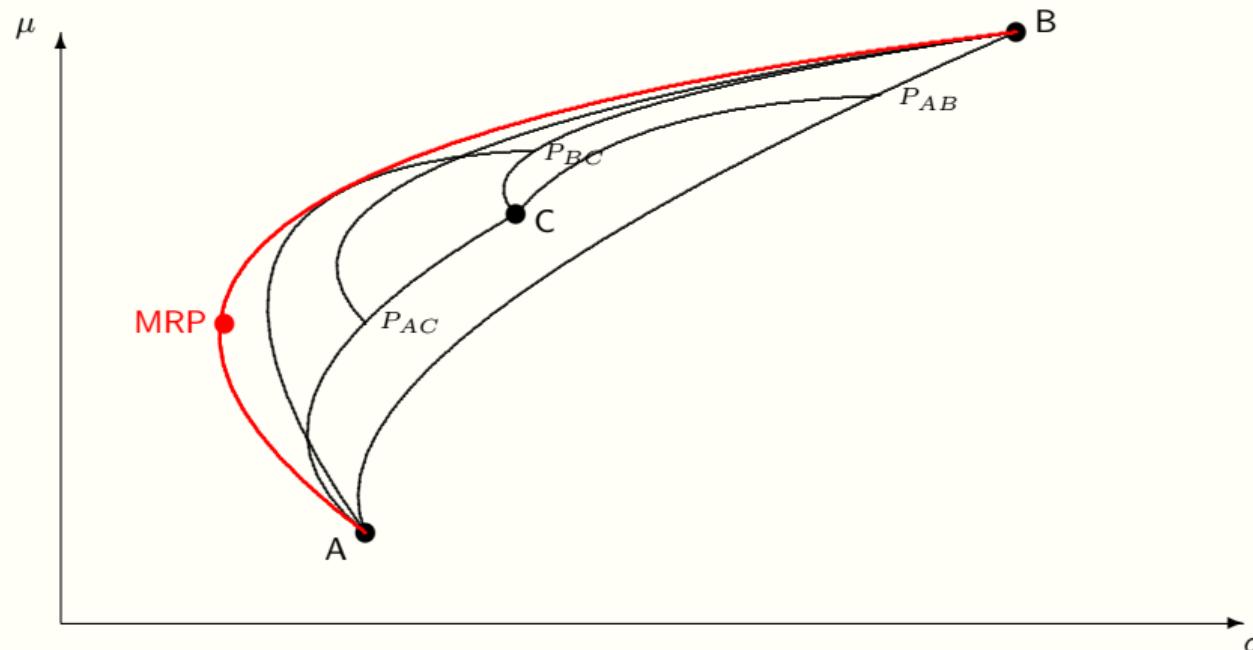
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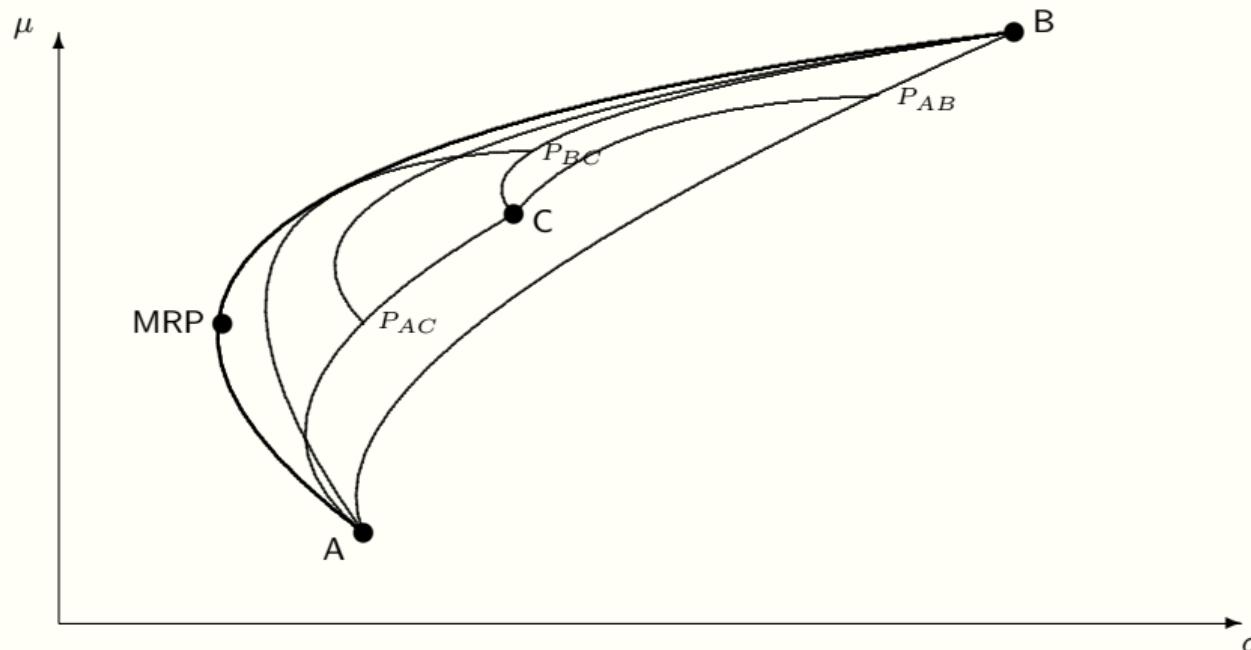
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The effect of diversification

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- We can use these results to assess the impact diversification has on individuals.
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- ⇒ Diversification **reduces** portfolio risk

The effect of diversification

- We can use these results to assess the impact diversification has on individuals.
- We have seen that increasing the number of assets reduces the risk of the efficient frontier.
- Risks cannot increase when having more assets as one option is to assign an additional asset a weight of zero, implying that it is not included in the portfolio and hence it will not affect the outcome. If increasing the weight of this additional asset does reduce risks, then the resulting portfolio is not efficient and the efficient frontier remains unchanged. If it reduces the risks, it will move the efficient frontier to the upper left and improve it the utility.
- ⇒ **Diversification is what it is called when holding several assets in a portfolio and more diversification is when holding more assets. We see that diversification will reduce the risk of the portfolio and increase the utility of investors.**
- Having seen that diversification is beneficial, we can now determine the optimal portfolio an investor should choose.

The effect of diversification

- ▶ More assets can reduce the risks of the efficient portfolios for a given return
- ▶ Risks cannot increase as additional assets could be given a weight of zero
- ⇒ Diversification reduces portfolio risk

The effect of diversification

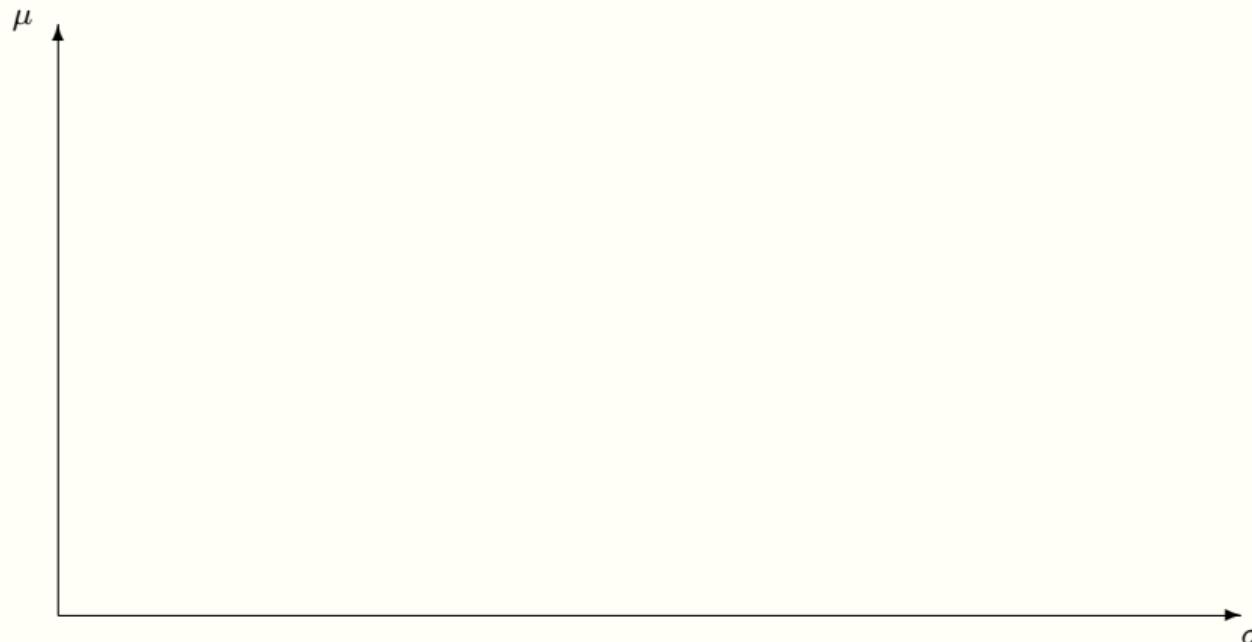
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The optimal portfolio

The optimal portfolio

- We can now determine graphically the optimal portfolio in our case of having three assets.
- We again consider the expected return and risk, in the form of the standard deviation, of assets and portfolios.
- We consider the same three assets as previously.
- The assets included form the opportunity set that consists of the risk-return combinations of all possible portfolios included in the bordered area. The minimum risk portfolio (MRP) is also indicated.
- The part of the border like above the MRP until asset *B* is the efficient frontier.
- To determine the optimal portfolio on the efficient frontier, we need to use the utility function. We know that the indifference curves have a positive slope and that the utility increases the higher the indifference curve is located. The highest possible point for the indifference point is when it is tangential to the efficient frontier.
- The optimal portfolio (OP) is then located at this tangential point.
- If we consider a different individual who is less risk averse, then its indifference curve will be less steep. The optimal portfolio will still be located where the indifference curve is tangential to the efficient frontier.
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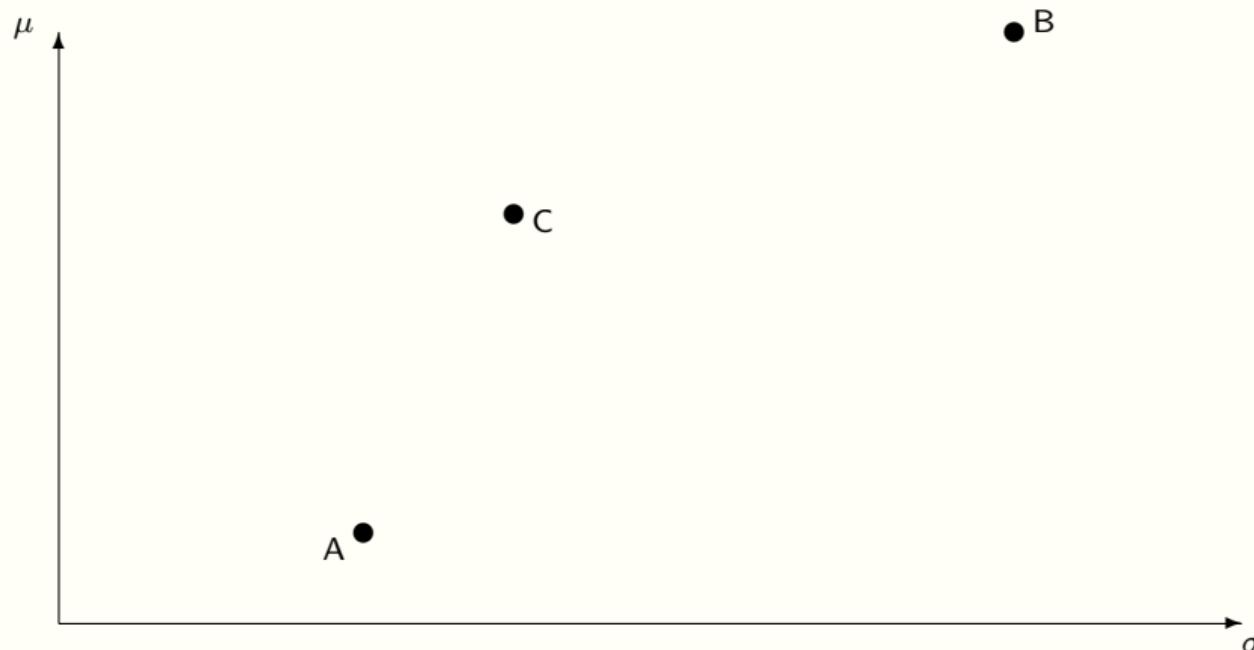
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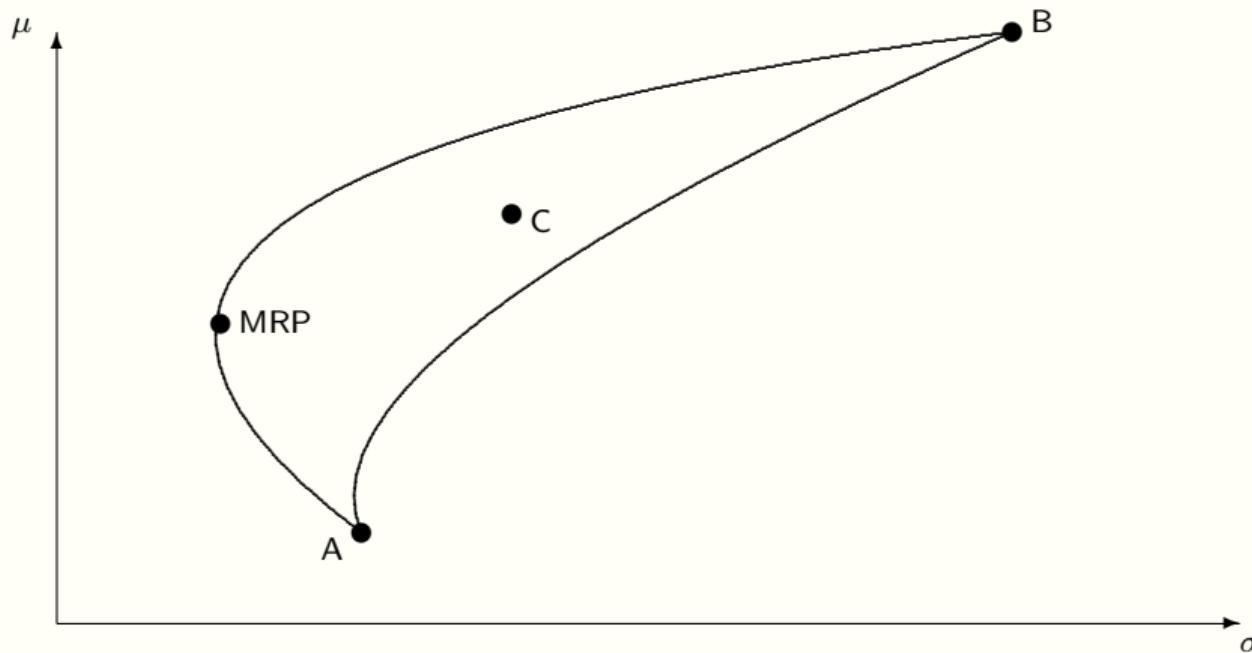
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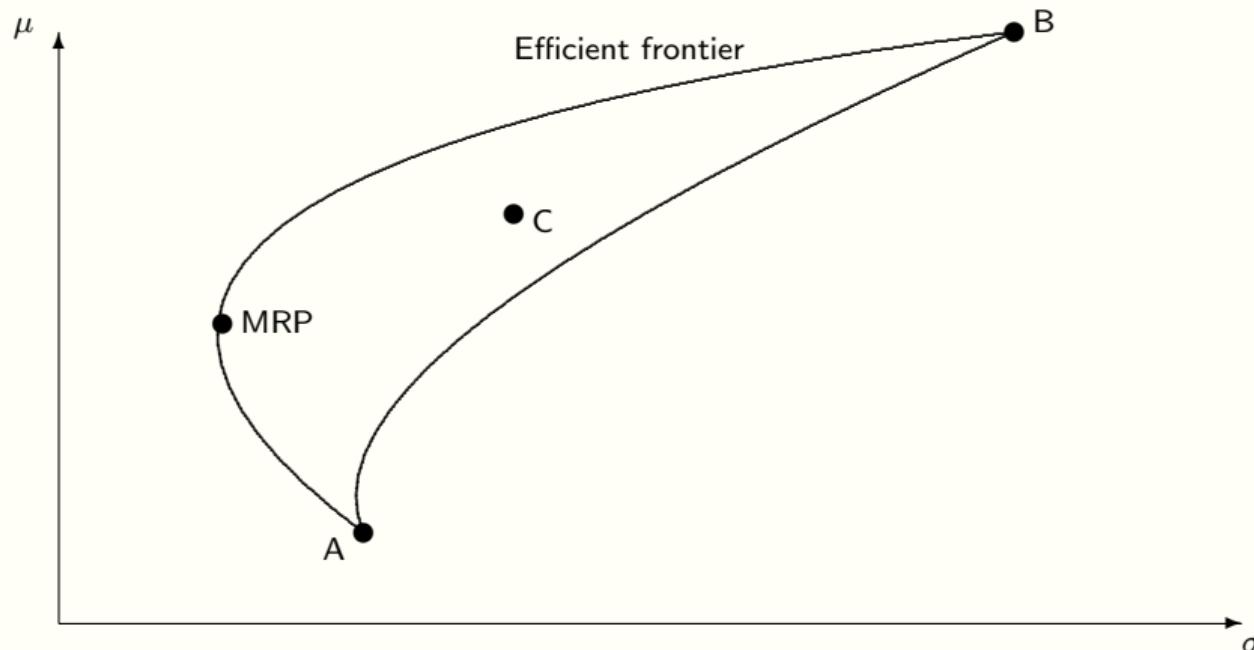
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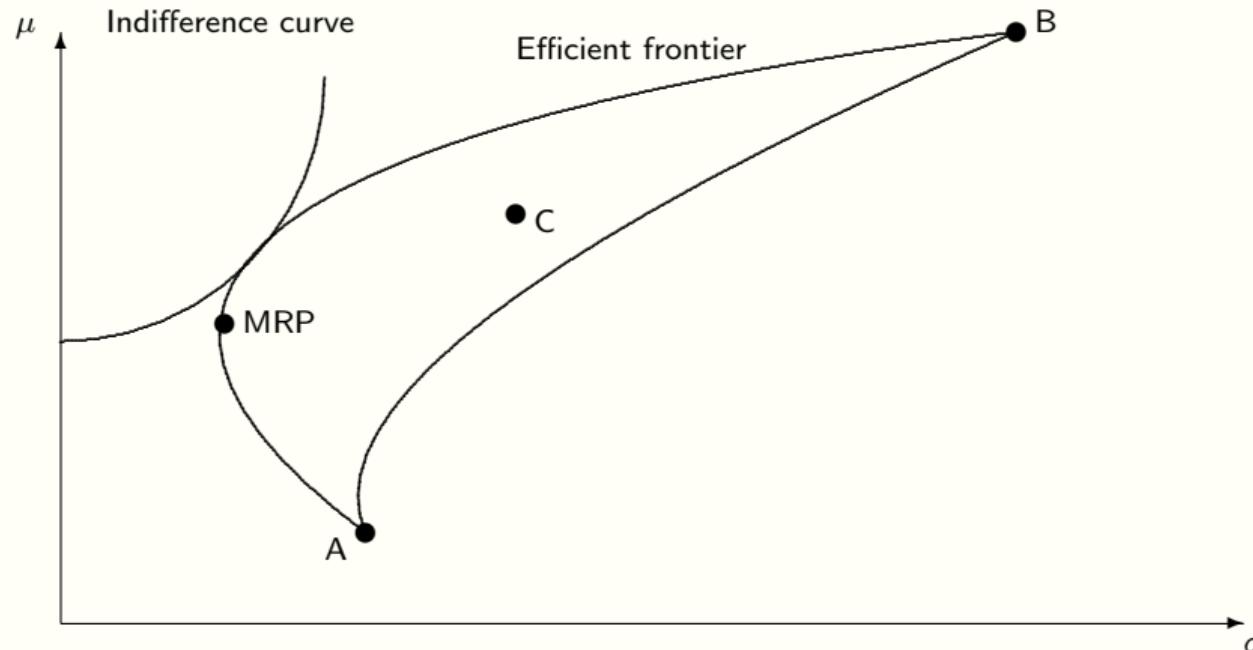
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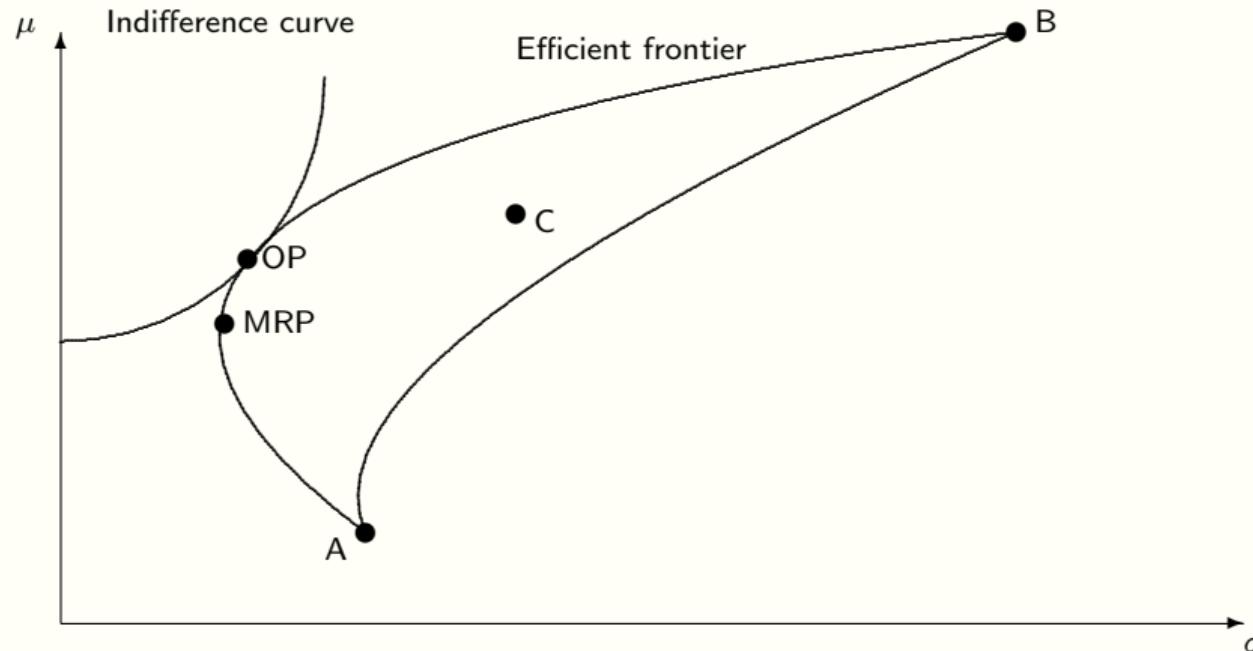
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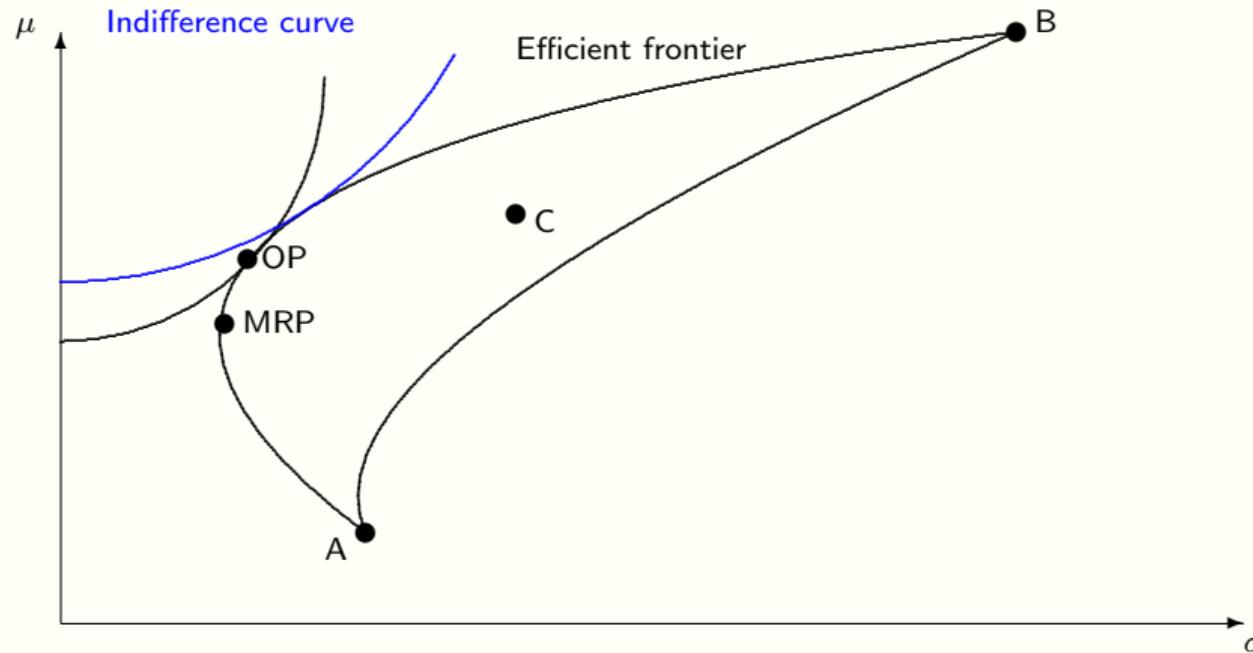
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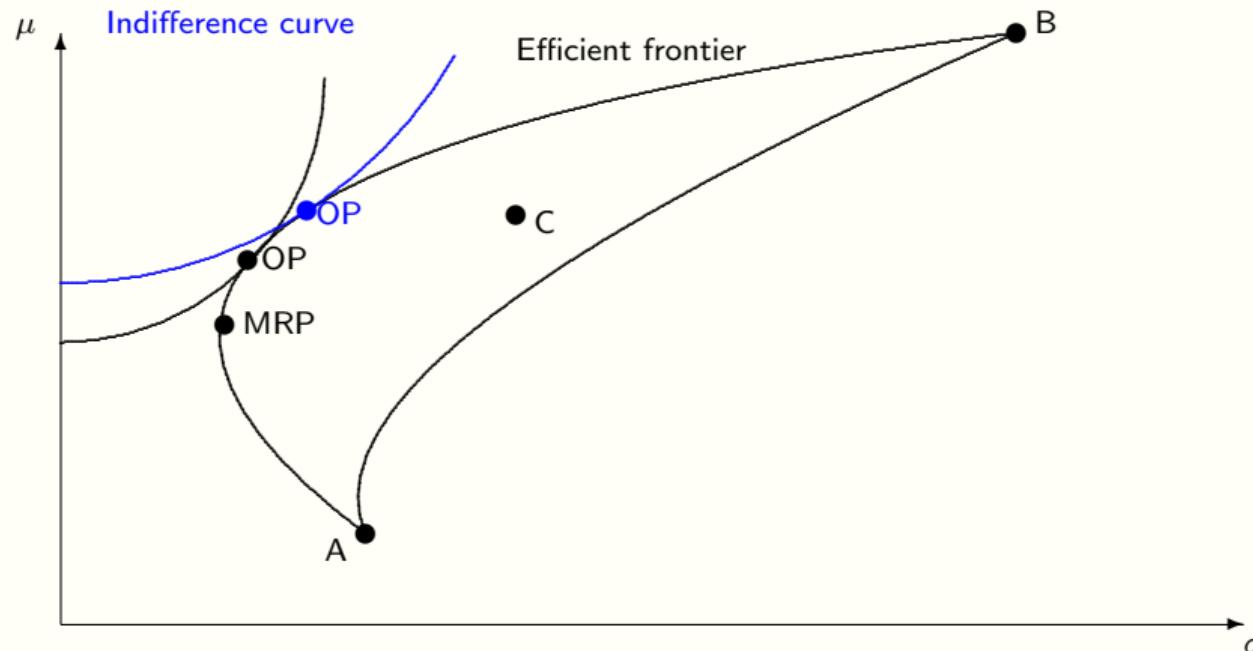
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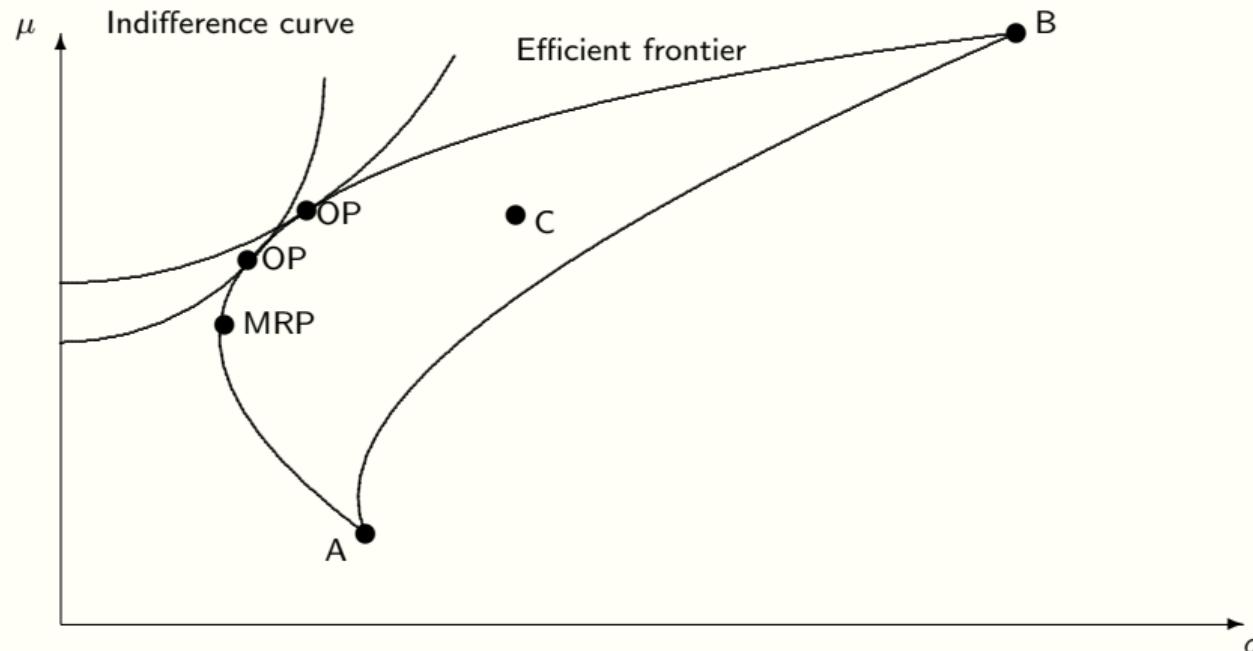
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Optimal portfolios with risky assets

- We can now summarize the key result on how to find the optimal portfolio.
- We determine the point where the slope of the efficient frontier is the same as the slope of the indifference curve. This is another way of saying they are tangential.
- The higher the risk aversion, the lower the risk of the optimal portfolio, and hence the optimal portfolio will move towards the minimum risk portfolio.
- As we have determined the optimal portfolio as one of the efficient portfolios, we can reverse this and state that any optimal portfolio will be efficient.
- Thus far we have assumed that all assets in our portfolio are risky. Of course, we could also choose an asset that is risk-free.

Optimal portfolios with risky assets

- ▶ The optimal portfolio is where the indifference curve is **tangential** to the efficient frontier

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Optimal portfolios with risky assets

- ▶ The optimal portfolio is where the indifference curve is tangential to the efficient frontier
- ▶ Higher risk aversion implies that the optimal portfolio is **closer** to the minimum risk portfolio

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Optimal portfolios with risky assets

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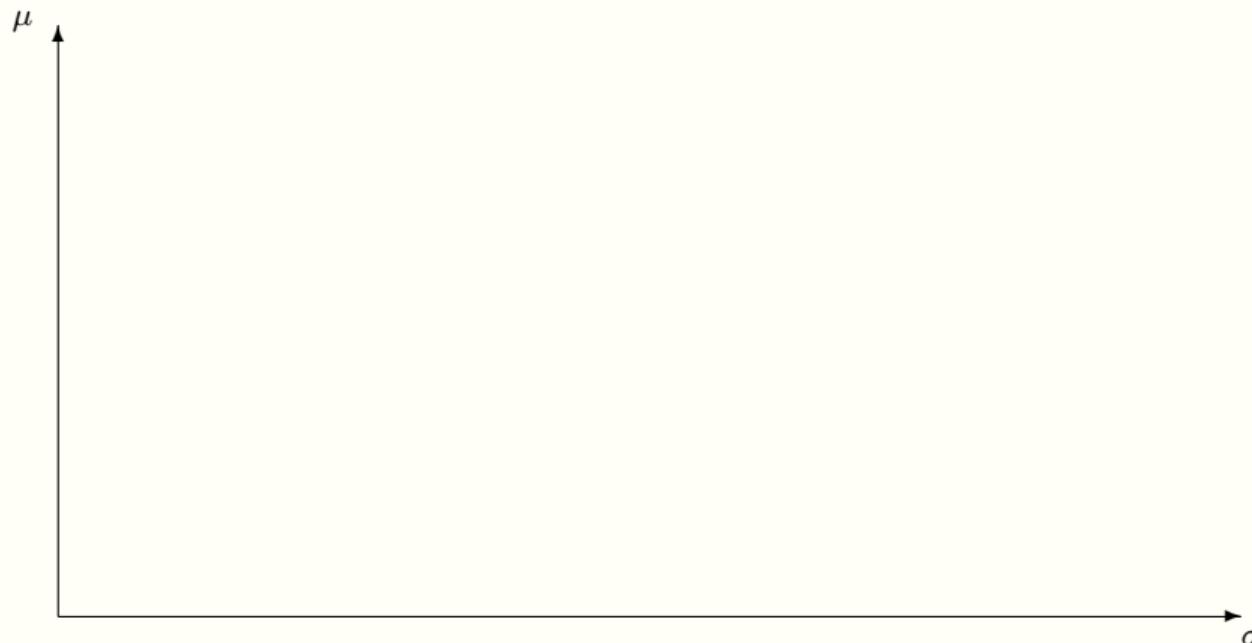
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Portfolios with a risk-free asset

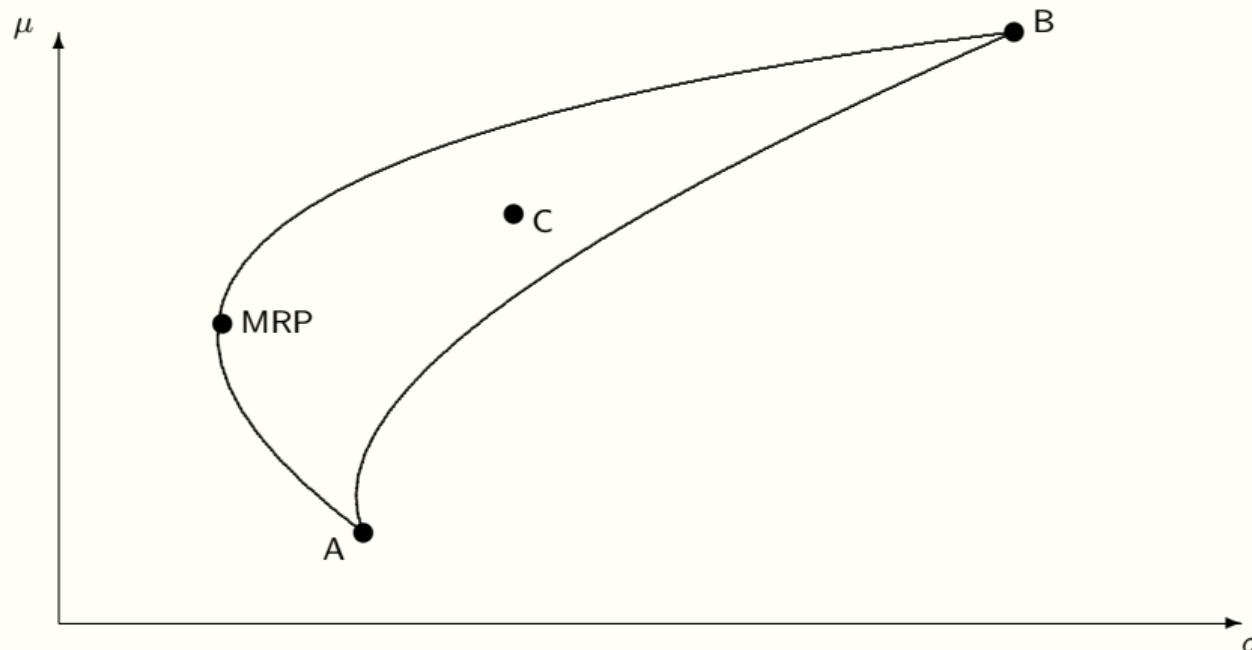
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- We again consider the expected return and risk, in the form of the standard deviation, of assets and portfolios.
- We consider the same three risky assets and the opportunity set of these three assets remains unchanged.
- We now introduce a fourth asset, the risk-free asset. This asset will have a return equal to the risk-free return and will be located on the vertical axis as its standard deviation is zero. As it is risk-free, its covariance with all other assets is also zero.
- We can show that combining the risk-free asset with any other risky asset, will lead to the portfolios being located on a straight line between the risky asset and the risk-free asset. This is similar to the case where correlations were perfectly positive or perfectly negative, but has a different reason; here the correlation between the risk-free asset and the risky asset is zero. This straight line emerges due to the risk of the risk-free asset being zero. We have added the opportunity set with this risk-free asset. The lowest portfolios are on the line from the risk-free asset to asset *A* and no lower point can be reached. The highest point that can be reached is the point where the line from the risk-free asset touches the opportunity set of the risky assets as indicated here. No higher point can be reached. Of course, any point between these two straight line can be achieved as well.
- The point at which the upper line touches the opportunity set of risky assets is called the optimal risky portfolio (ORP).
- The efficient frontier of this portfolio consisting of risky assets and the risk-free asset will not be the line connecting the risk-free asset and the ORP as these are the highest upper left points that can be reached. Beyond that the efficient frontier will follow the efficient frontier of the risky assets only. The optimal portfolio will now be again determined with help of the indifference curve being the one that is highest.
- The resulting tangential point is the optimal portfolio (OP). We now can determine the components of this optimal portfolio. As it is on the line between the risk-free asset and the ORP, it will be a combination of these two components; the constituents will be the risk-free asset and the ORP. In this context we can treat a portfolio as if it was a single asset; this is due to us only being concerned about the properties of the assets, regardless of how they have been achieved. Therefore, the ORP is the optimal portfolio, but only the part which is comprised of risky assets, hence its name. The optimal portfolio is a combination of the ORP and the risk-free asset.
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Portfolios with a risk-free asset



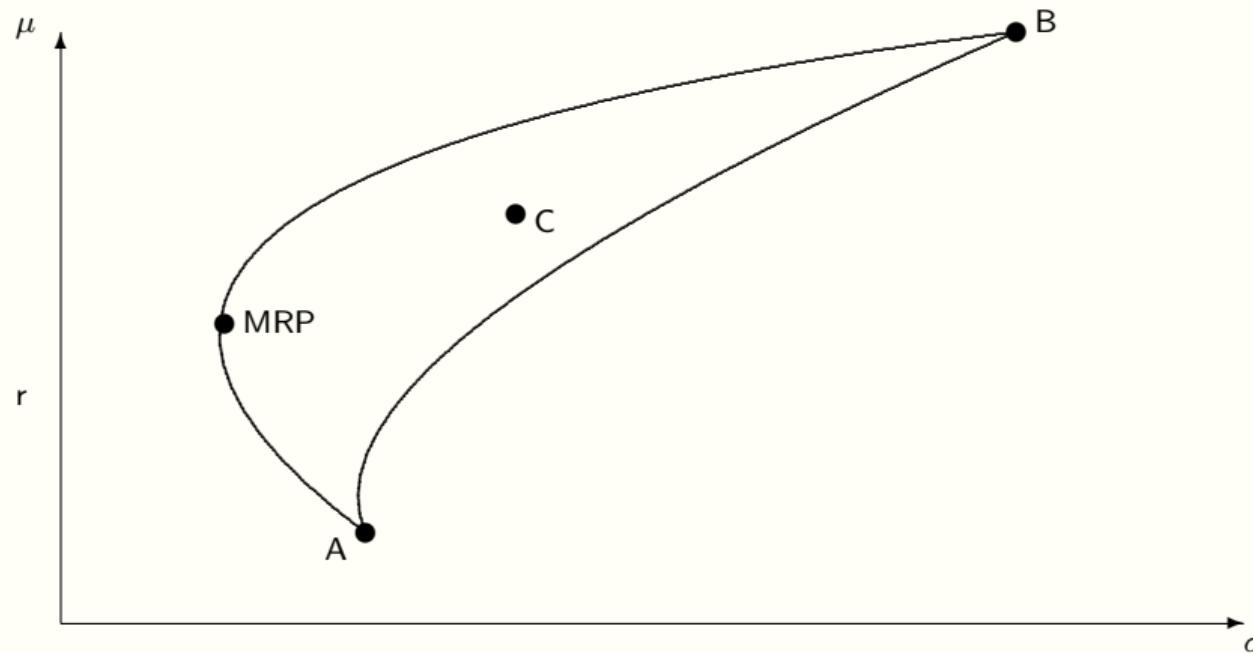
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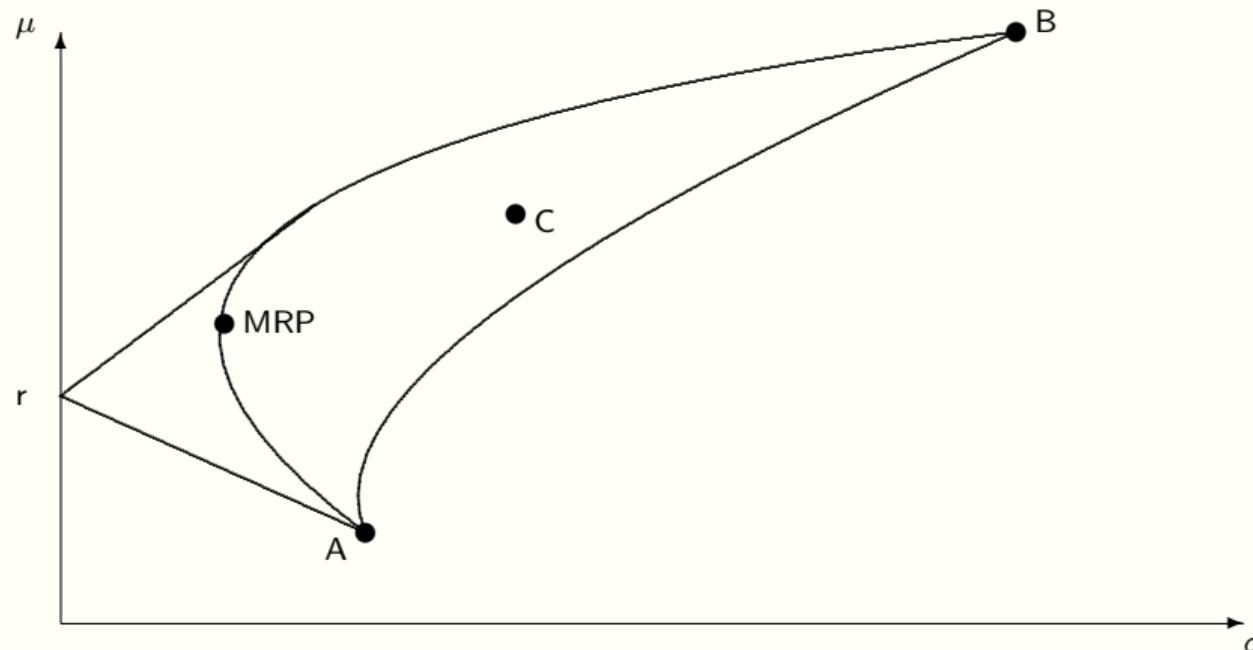
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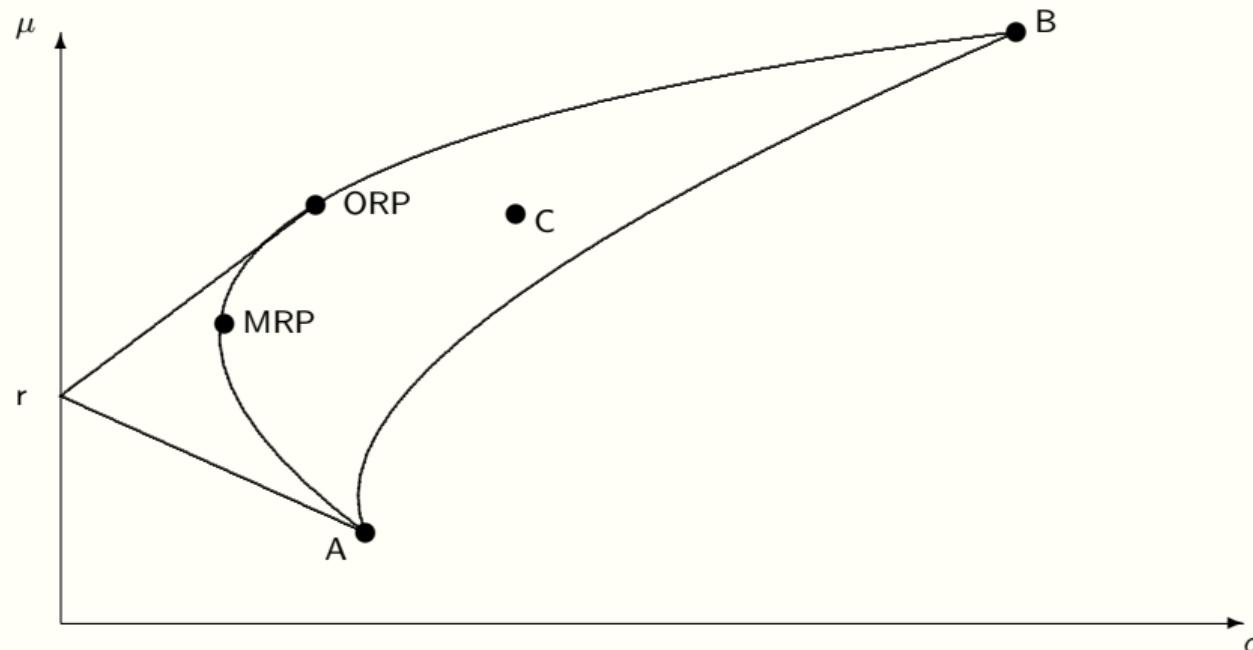
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Portfolios with a risk-free asset



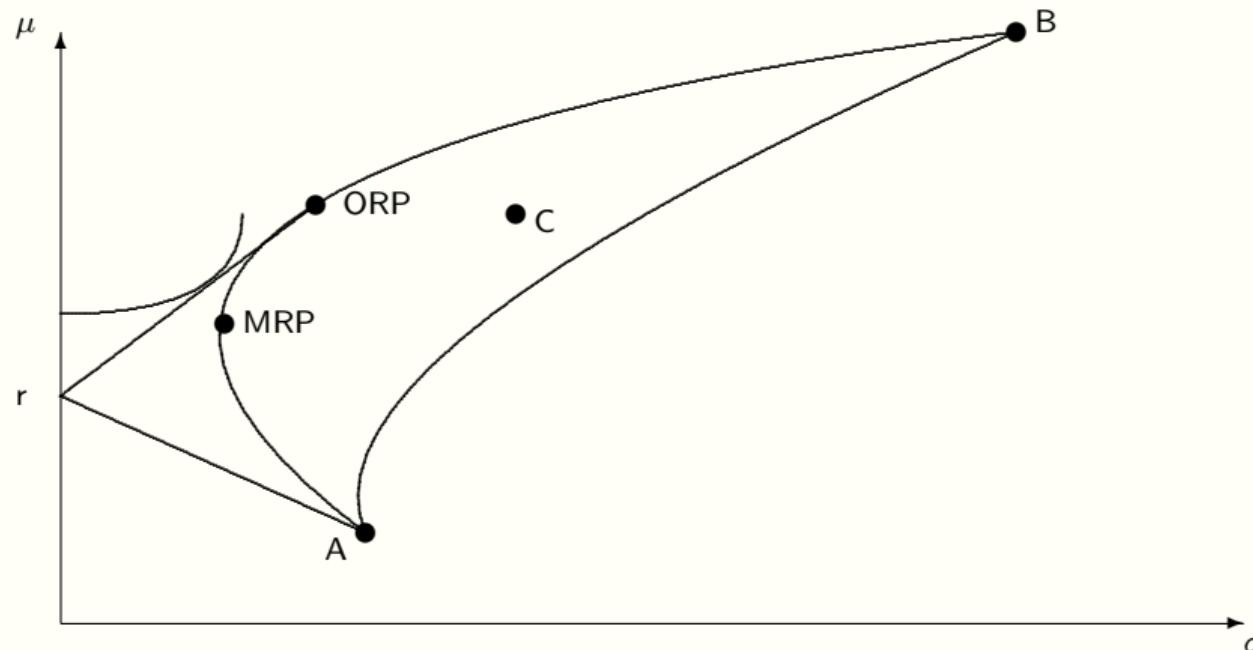
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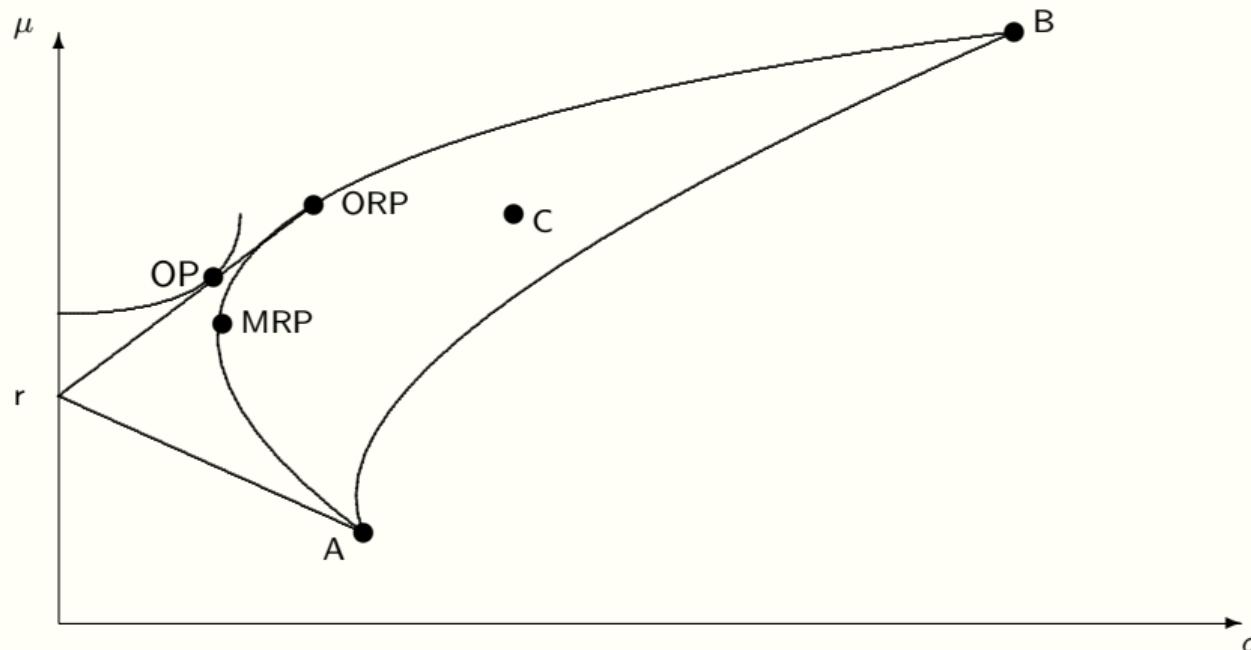
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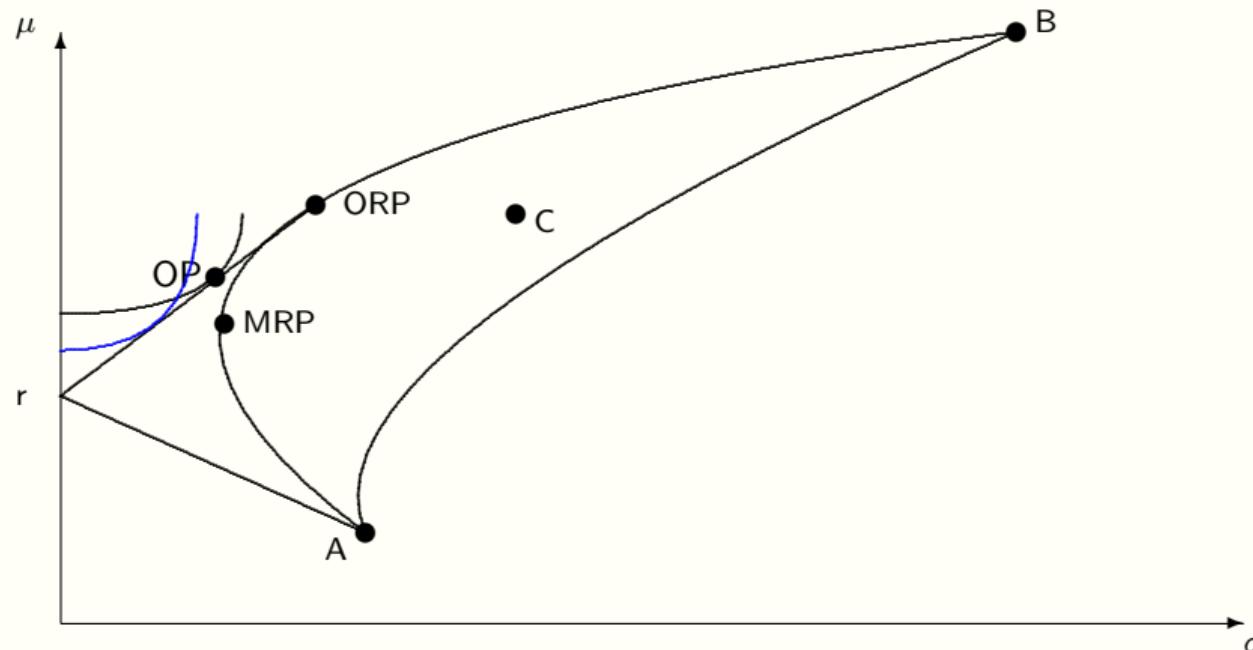
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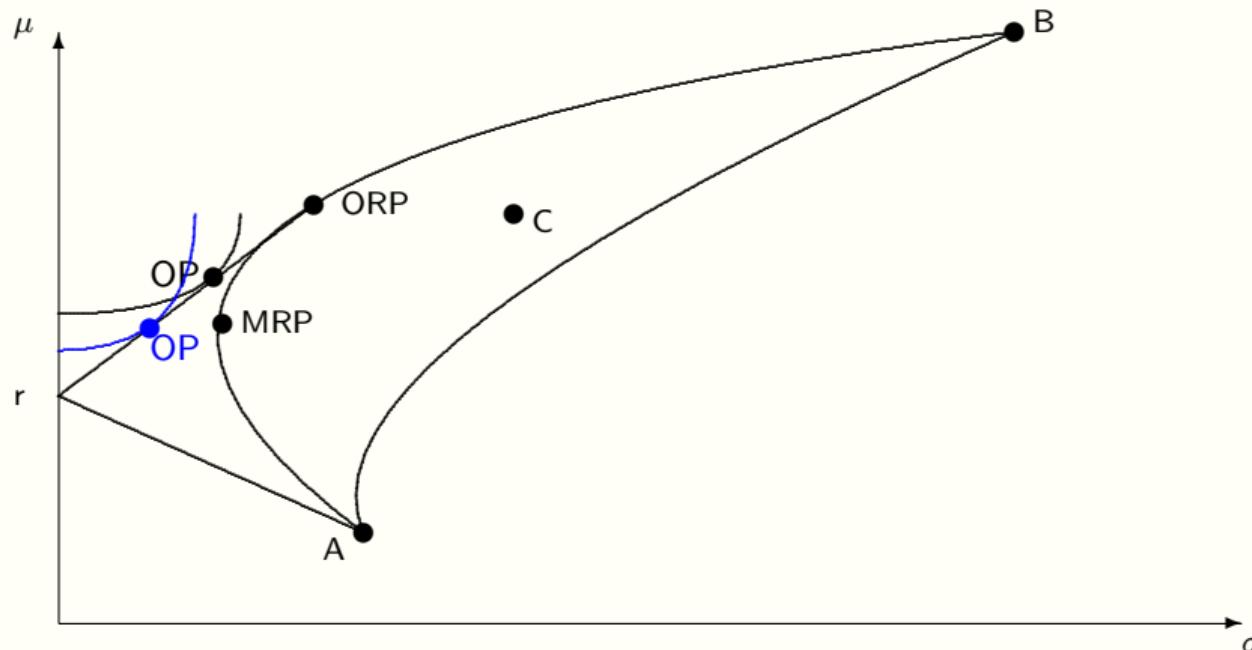
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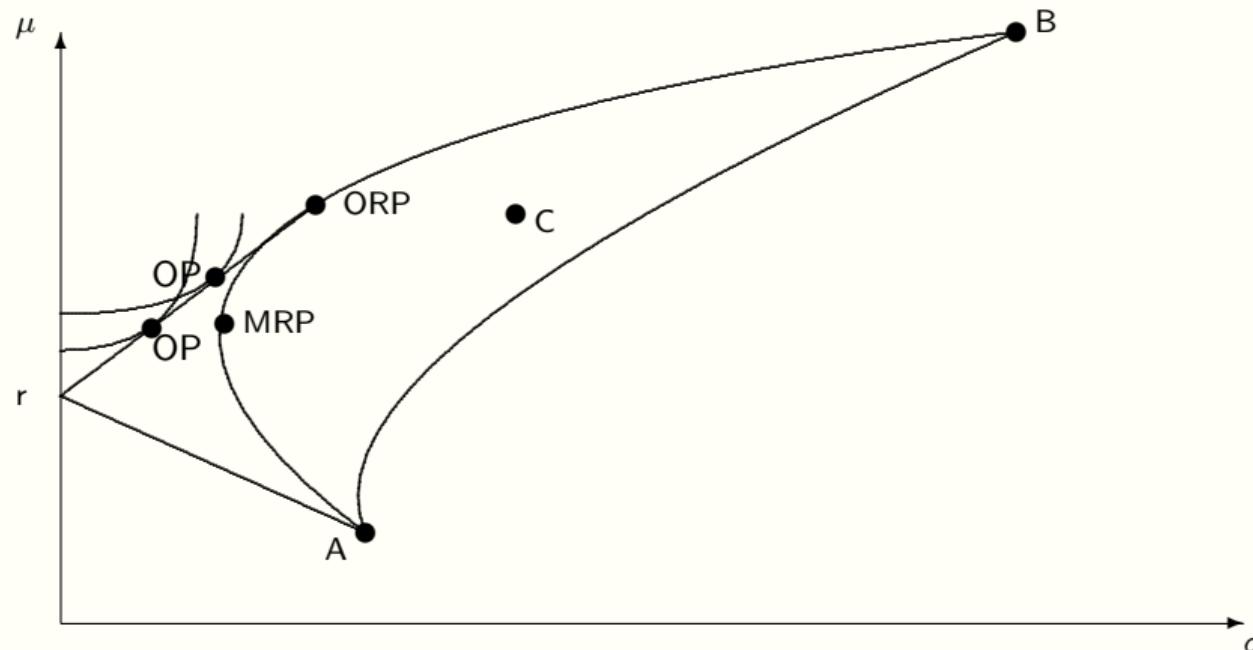
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Optimal risky portfolio

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 - ▶ • By definition, a risk-free asset has no risk, hence a variance or standard deviation of zero.
 - As it is risk-free it also does not move with any risky asset, giving it a correlation of zero with all assets.
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- The ORP only depends on the characteristics of the risky assets, their expected returns, standard deviation, and correlations, as well as the risk-free rate (the return of the risk-free asset).
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- ▶ A risk-free asset has no risk and no covariance with any asset
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- We can use these points to obtain one of the most fundamental results in portfolio theory.

Optimal risky portfolio

- ▶ A risk-free asset has no risk and no covariance with any asset
- ⇒ The efficient frontier with a risk-free assets is a straight line
- ▶ The efficient frontier is the line that is tangential to the efficient frontier of the risky assets
- ▶ This tangential point gives us the optimal risky portfolio
- ▶ The optimal risky portfolio does **not** depend on the preferences of individuals

- We can now summarise the implications of introducing a risk-free asset into our portfolio.
 - ▶
 - By definition, a risk-free asset has no risk, hence a variance or standard deviation of zero.
 - As it is risk-free it also does not move with any risky asset, giving it a correlation of zero with all assets.
- ⇒ We can show mathematically that the efficient frontier between a risk-free asset and a risky asset is a straight line.
 - ▶ The efficient frontier in the case of having a risk-free and risky assets, would be the straight line that is tangential to the efficient frontier of the risky assets.
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Separation theorem

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- We have now established one of the key results in portfolio theory, the fact we can select portfolios in two steps.
- We have seen that the optimal portfolio is a combination of the risk-free asset with a specific portfolio of risky asset, the optimal risky portfolio.
- We have seen that all investors use this combination of the risk-free asset with the same ORP, what varies with the degree of risk aversion are the relative weights of these two components. A higher risk aversion results in a larger weight of the risk-free asset.
- ⇒ Hence all investors combine the same two components and thus the portfolio selection process can be separated into two steps. Firstly, determine the optimal risky portfolio; secondly, combine this optimal risky portfolio with the risk-free asset. That portfolio selection can be decomposed into these two steps is known as the separation theorem.
- It is only the weight of the risk-free asset and the ORP that depend on the risk aversion of investors, not the composition of the ORP itself, which is identical for all investors, as long as they agree on the properties of assets.
- Thus far we made the implicit assumption that the weights of all assets in a portfolio have to be positive, or zero. We will now introduce the possibility of having negative weights.

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- The optimal portfolio is a combination of **optimal risky portfolio** and the **risk-free asset**

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Optimal portfolio with short sales

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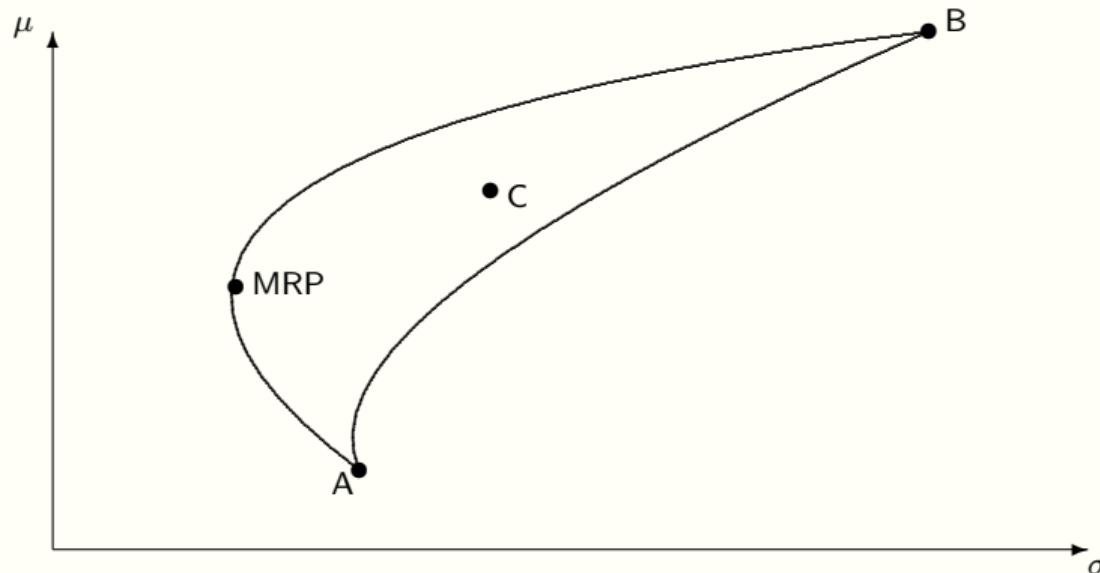
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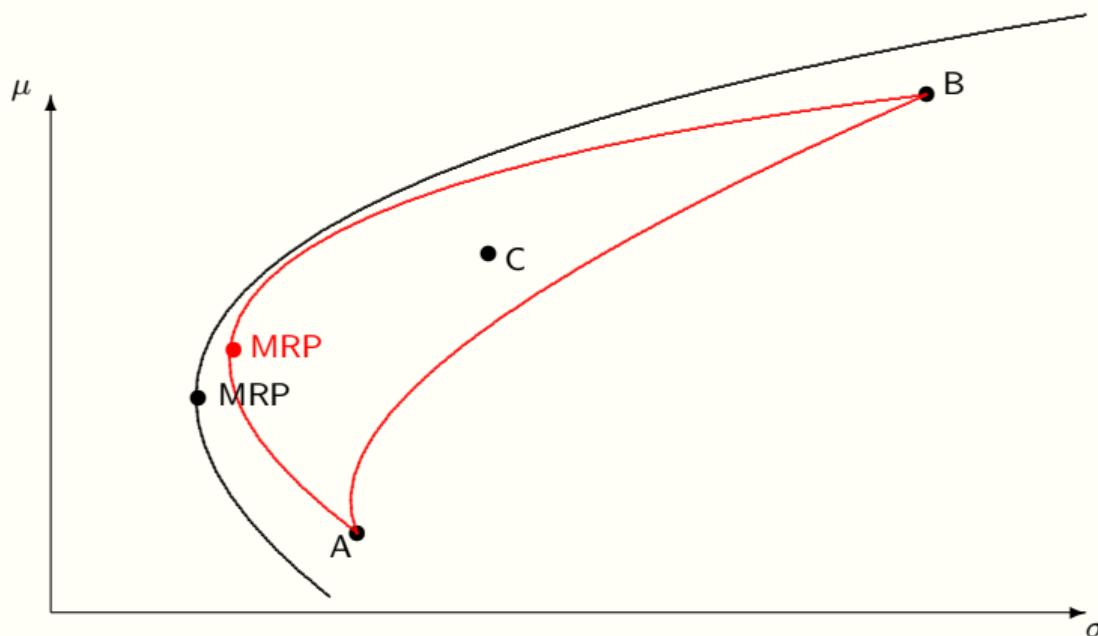
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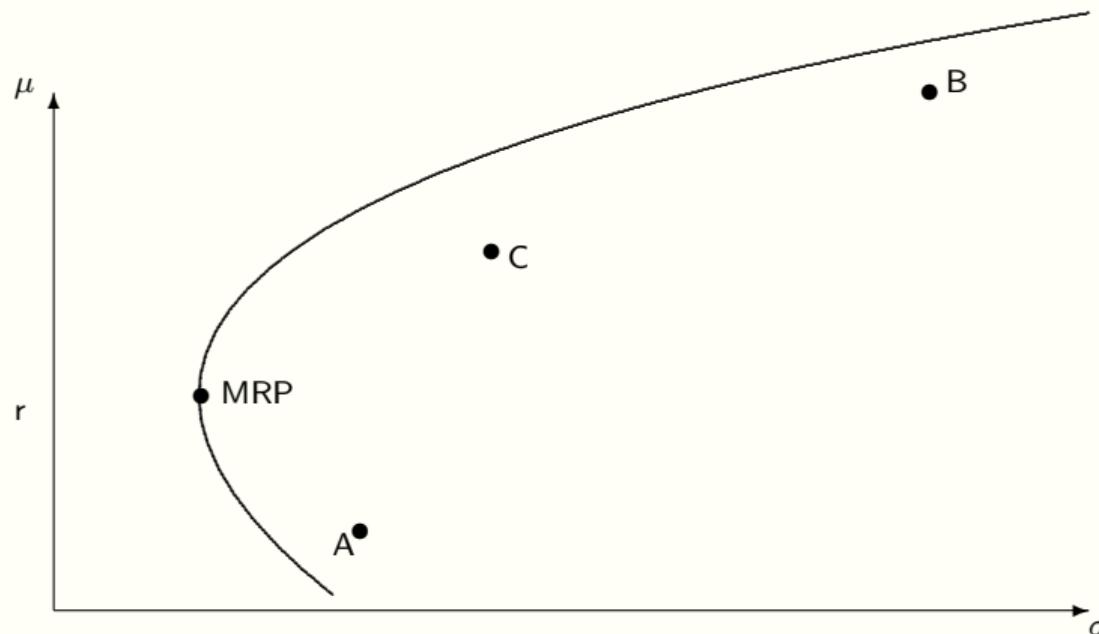
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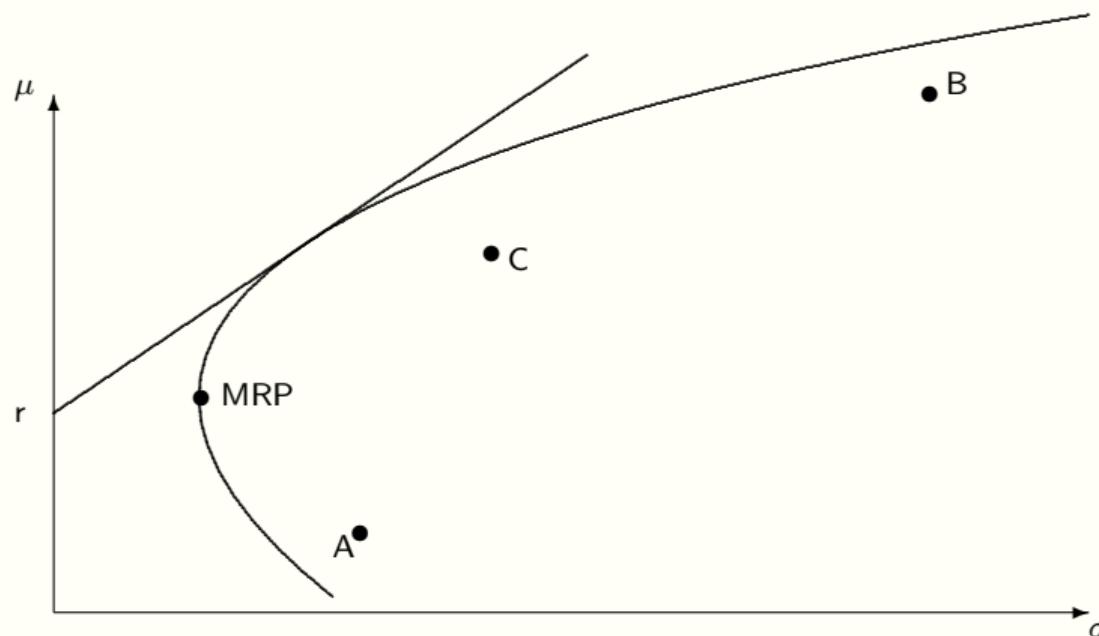
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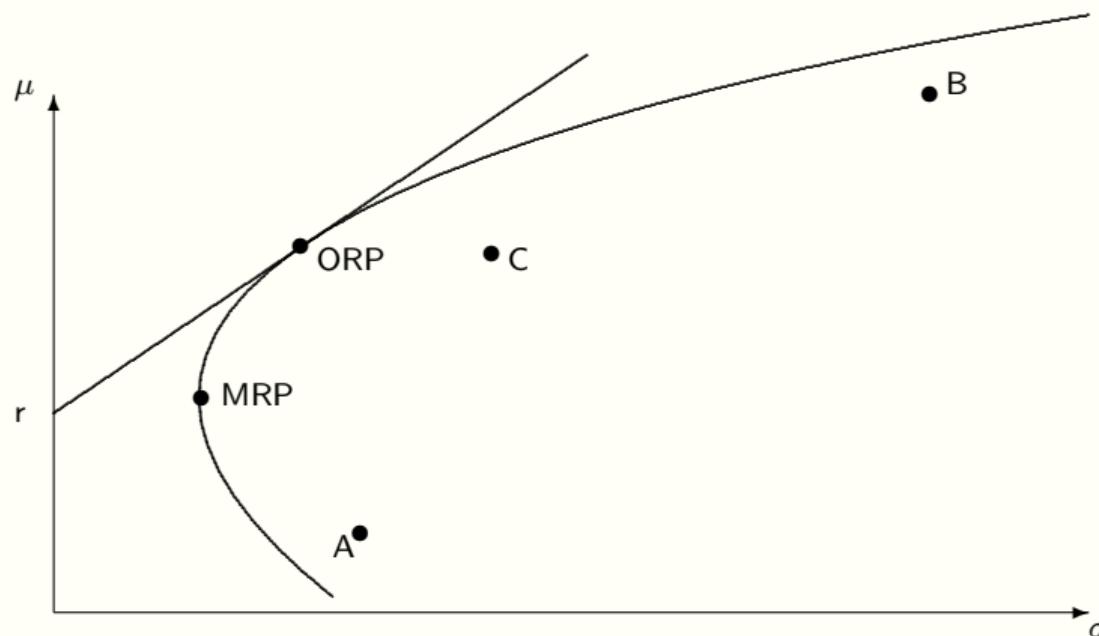
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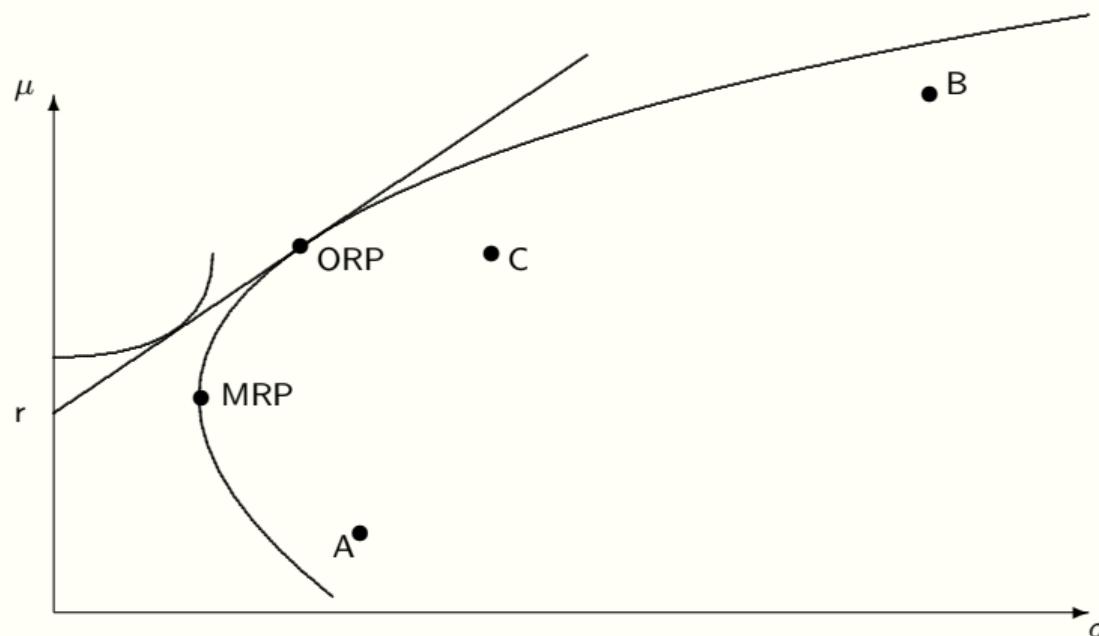
Optimal portfolio with short sales



Optimal portfolio with short sales

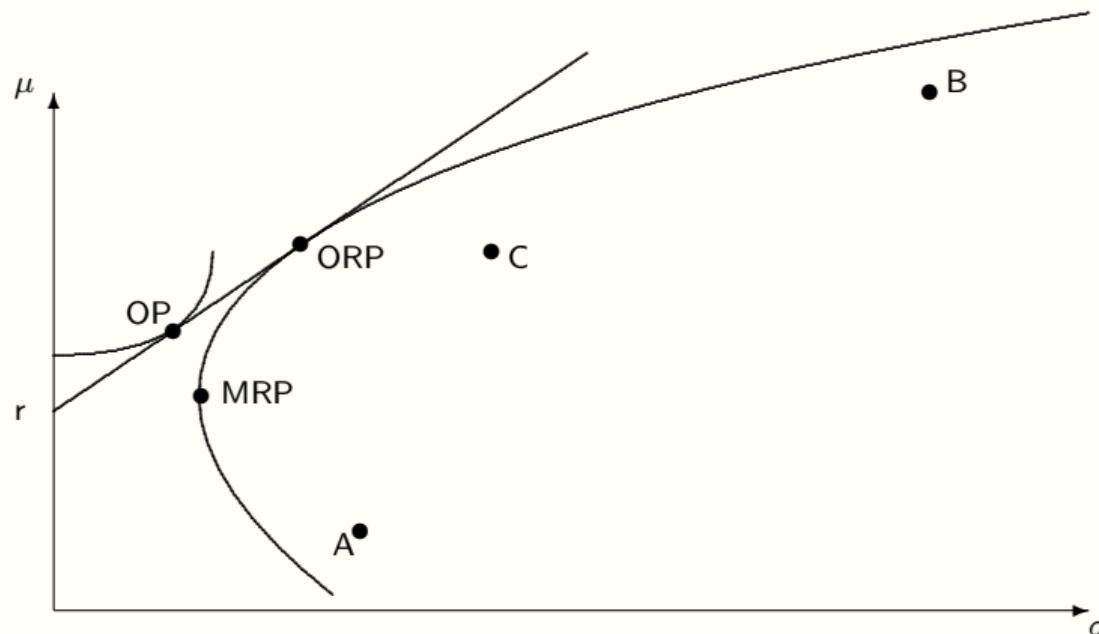
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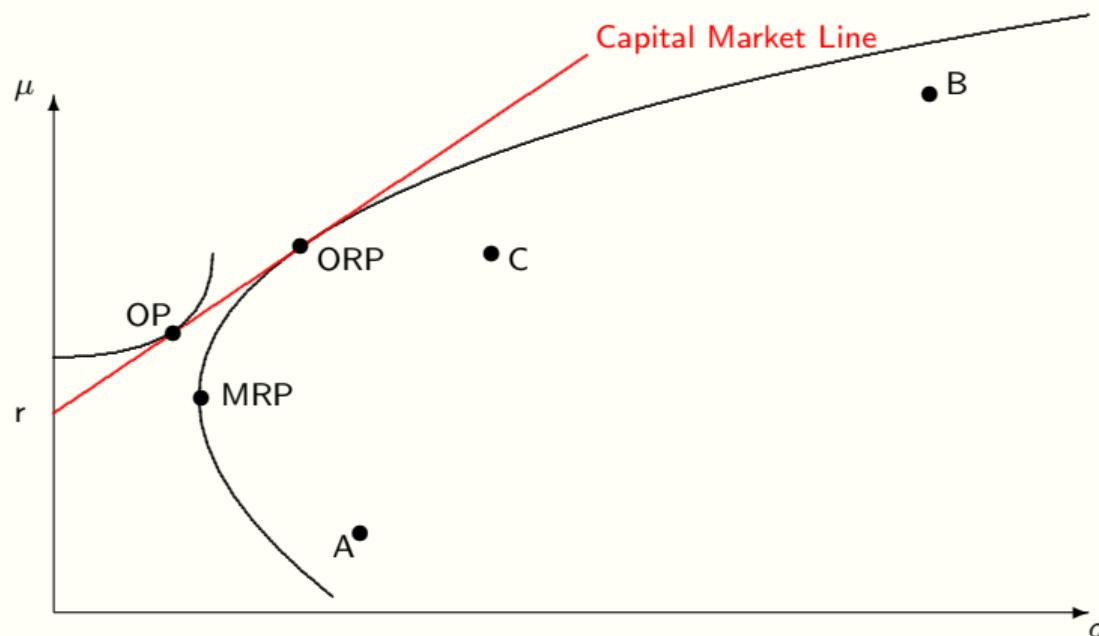
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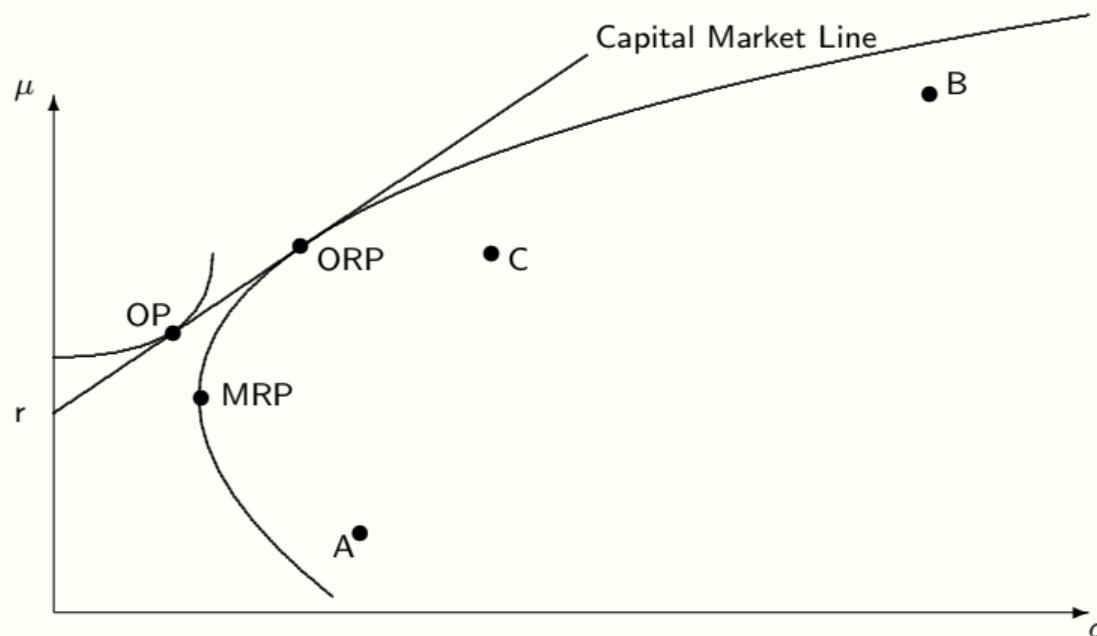
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The effect of short sales

→ We can now Summarize the key results of portfolio selection in the presence of short sales.

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 - Short sales occur is an investor sells an asset it does not hold.
 - This is equivalent to having negative weights on these assets.
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 - Short sales can be interpreted as a loan in the form of a (risky) asset.
 - After having obtained the loan, the asset is sold for cash and the proceeds invested into other assets.
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 - With such short sales, weights of assets can be negative.
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The effect of short sales

- ▶ Short-sales are occurring if an asset is sold without holding it, giving a negative weight
- ▶ You obtain a **loan** in form of the asset

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- ▶ Short-sales are occurring if an asset is sold without holding it, giving a negative weight
- ▶ You obtain a loan in form of the asset, **sell it** for cash

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The effect of short sales

- ▶ Short-sales are occurring if an asset is sold without holding it, giving a negative weight
- ▶ You obtain a loan in form of the asset, sell it for cash, then have to **repay the asset** in the future

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- ▶ Short-sales are occurring if an asset is sold without holding it, giving a negative weight
- ▶ You obtain a loan in form of the asset, sell it for cash, then have to repay the asset in the future, **buying it back** for cash

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- ▶ You obtain a loan in form of the asset, sell it for cash, then have to repay the asset in the future, buying it back for cash
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 - With such short sales, weights of assets can be negative.
 - This in itself, even neglecting the negative correlation of such positions, increases the opportunity set.
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- While short sales are increasing the utility of investors, in reality the ability to make short sales will be limited. As they are loans to the investor, the availability of such loans will not be universal and the amount that can be borrowed will also be strictly controlled. Thus realistically, for most investors short sales are either not available at all or form a small part of the positions in a portfolio.

The effect of short sales

- ▶ Short-sales are occurring if an asset is sold without holding it, giving a negative weight
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Summary

- We can now summarize the key results of portfolio selection theory.
- We have found that the optimal portfolio will consist of the risk-free asset and a risky portfolio that is identical for all investors, the optimal risky portfolio. The optimal risky portfolio is identical for all investors as long as they agree on the characteristics of all assets.
- The combination of risk-free and optimal risky portfolio will differ between investors and will depend on their risk aversion.
- The optimal risky portfolio can be determined objectively, without reference to any utility function, by estimating expected returns, variances and correlations.
- Portfolio selection seems a straight forward process. It is however difficult to determine the risk aversion of investors and the variances, returns and correlations of assets are also not stable over time, making their estimation difficult.

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