Andreas Krause

Optimal portfolios

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Portfolio choice

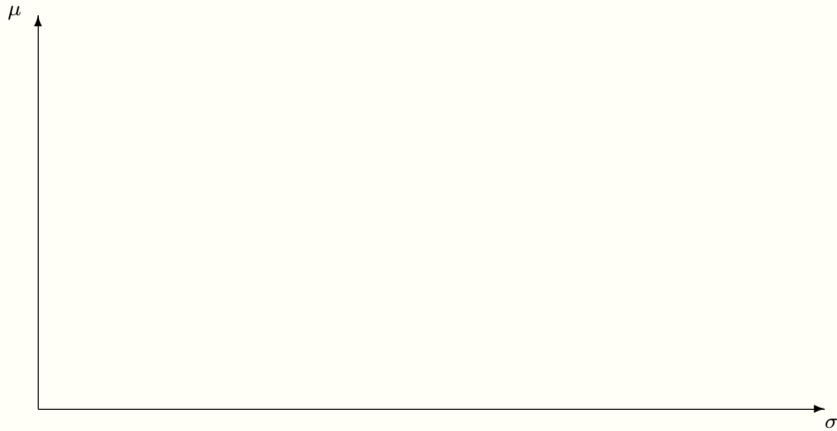- Assets can be characterised by their expected returns and risk
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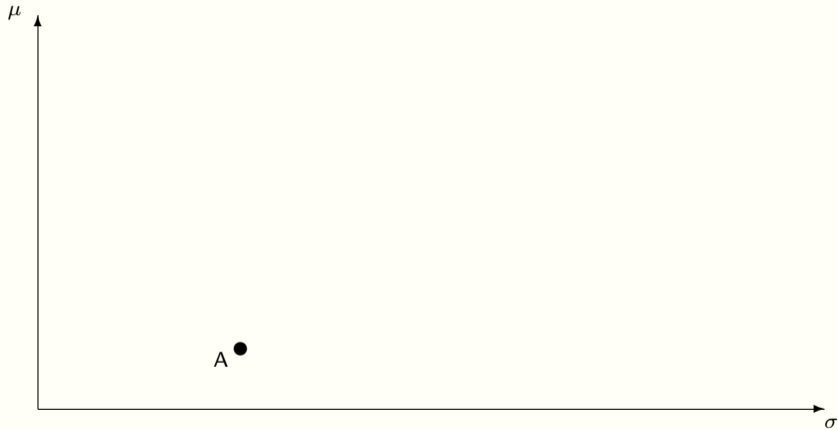
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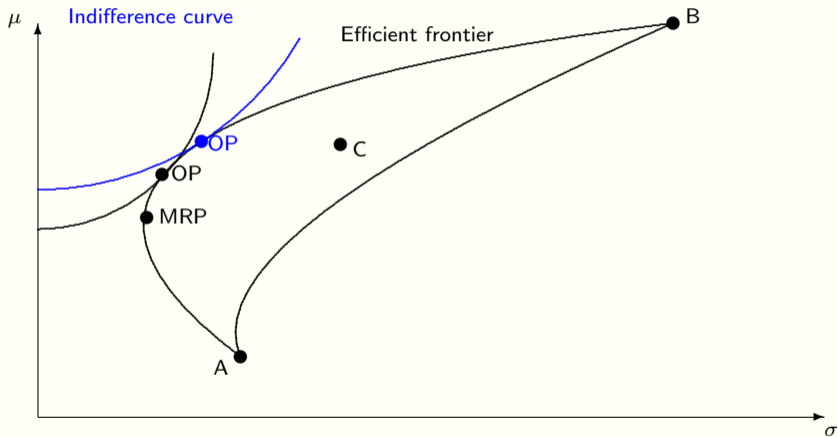
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Optimal portfolios with risky assets

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Optimal risky portfolio

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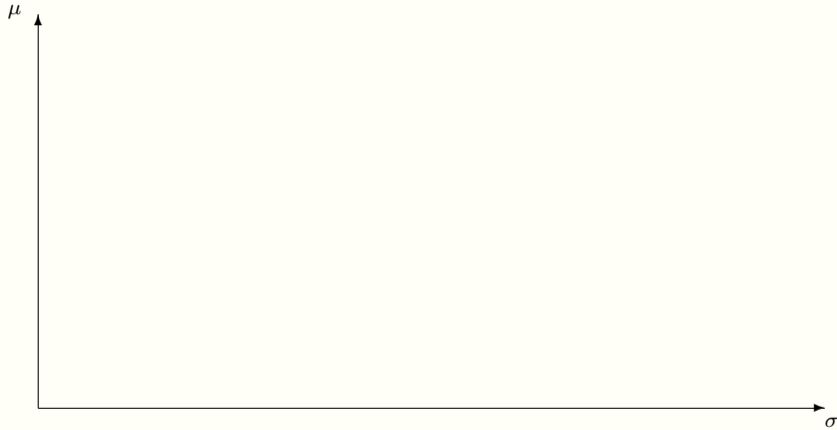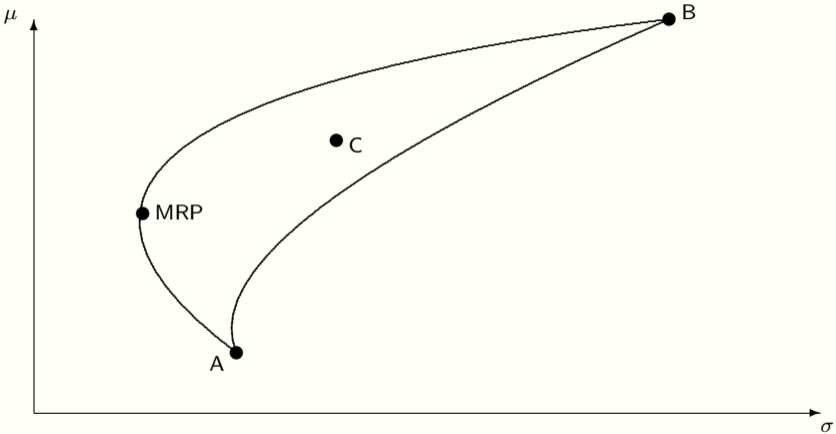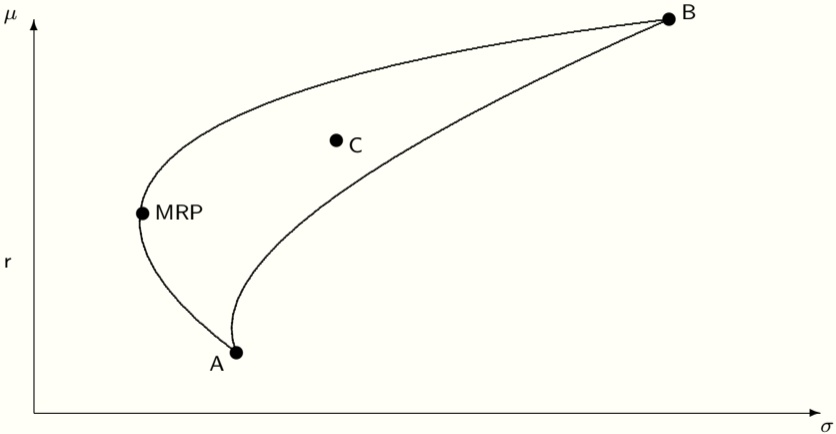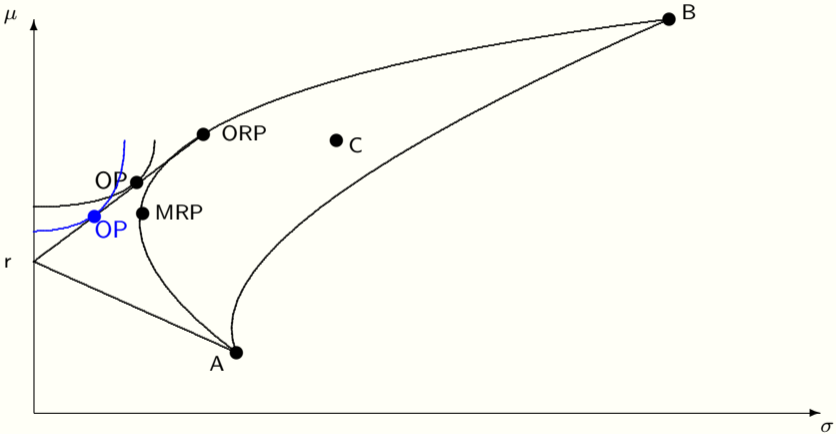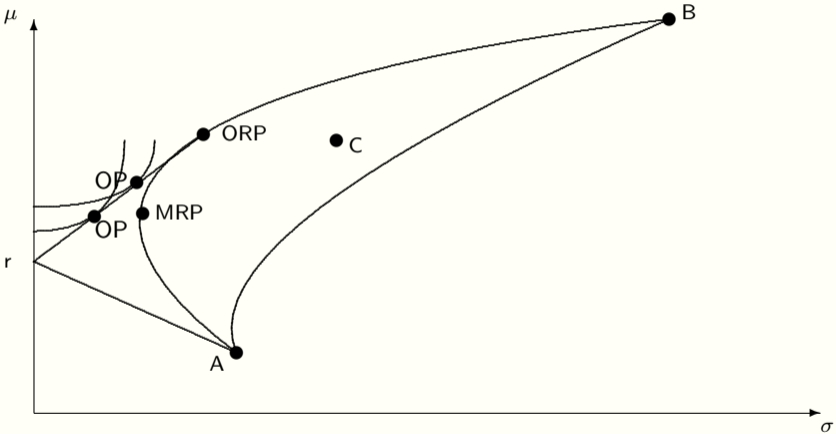
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