Andreas Krause

Mean-Variance criterion

Preferences for outcomes

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Preferences for outcomes

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Choice between two alternatives

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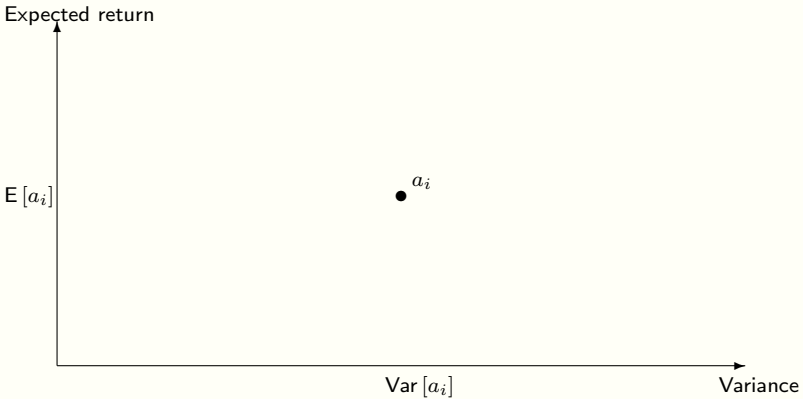

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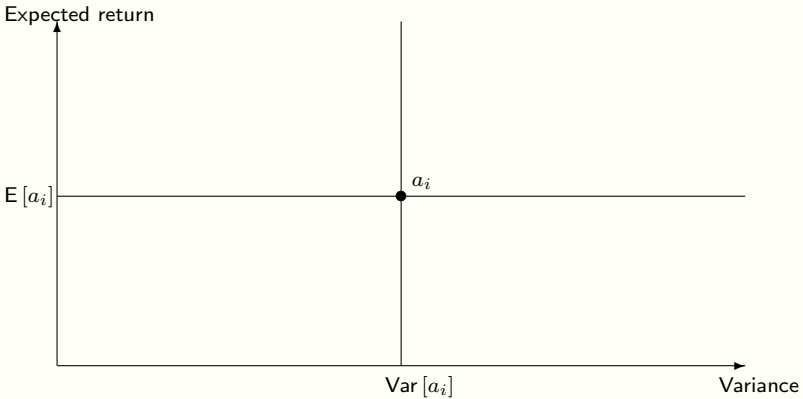

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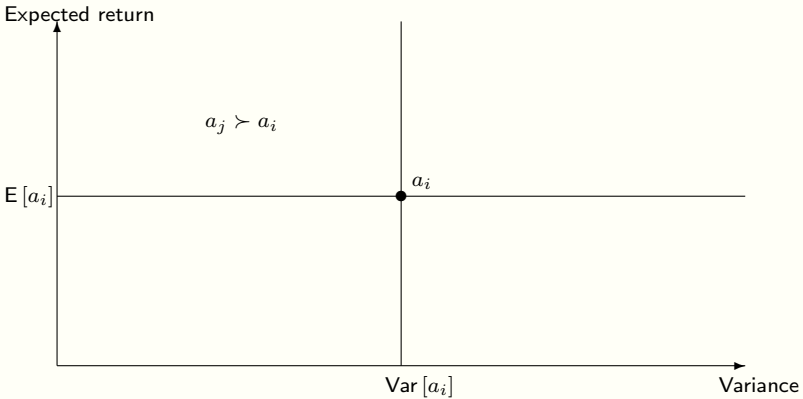

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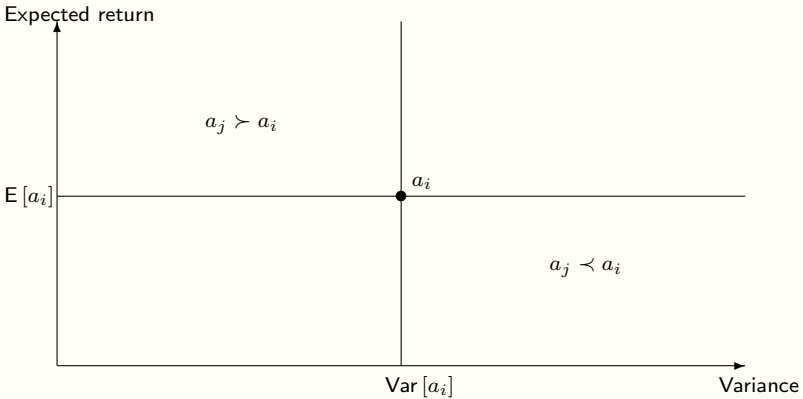

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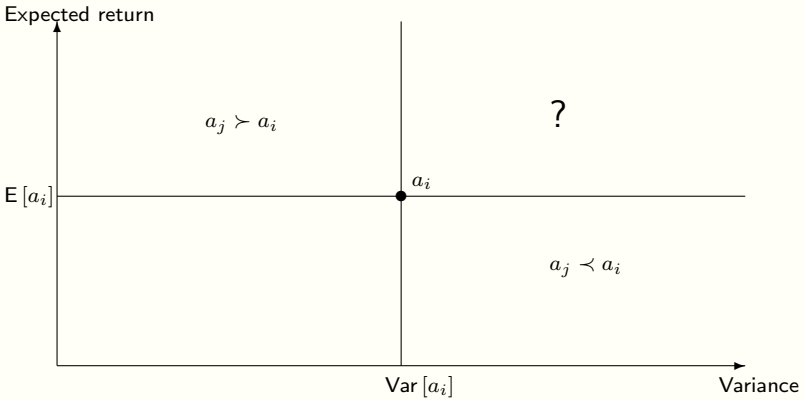

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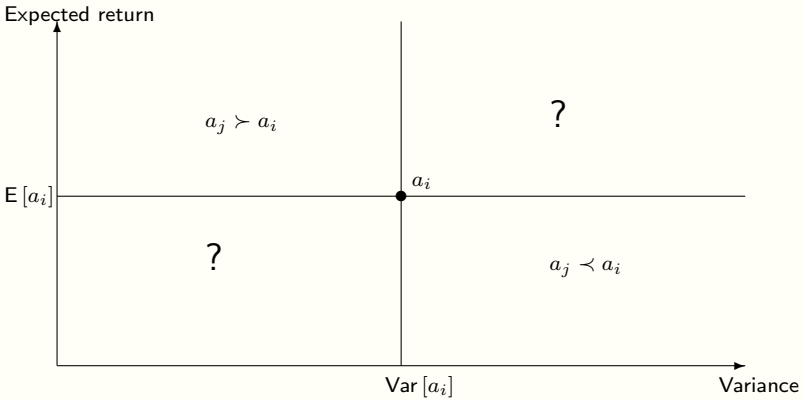

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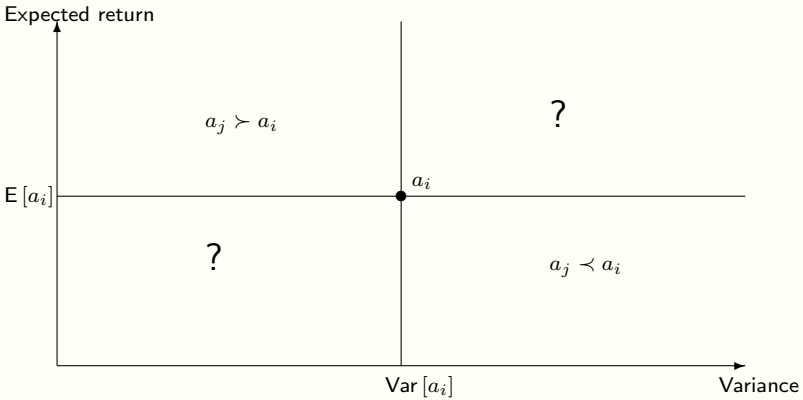

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Determining possible choices

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- By comparing any two alternatives, we can eliminate alternatives that have higher risk and lower return

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The efficient frontier

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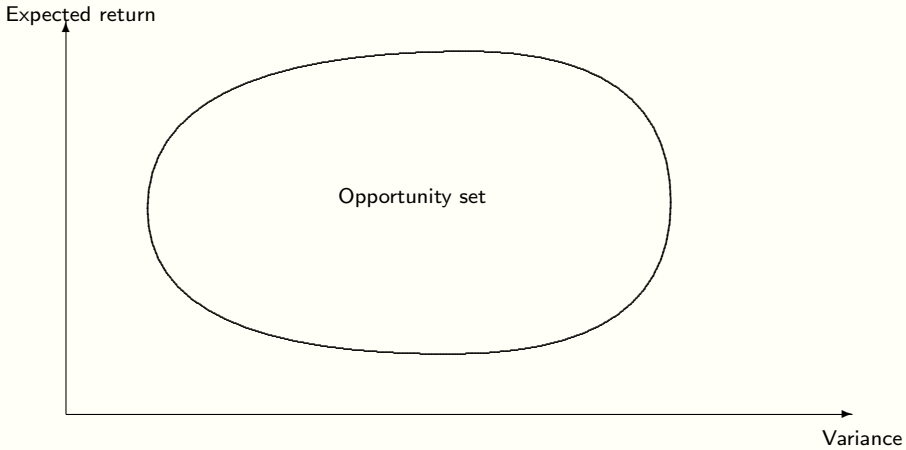

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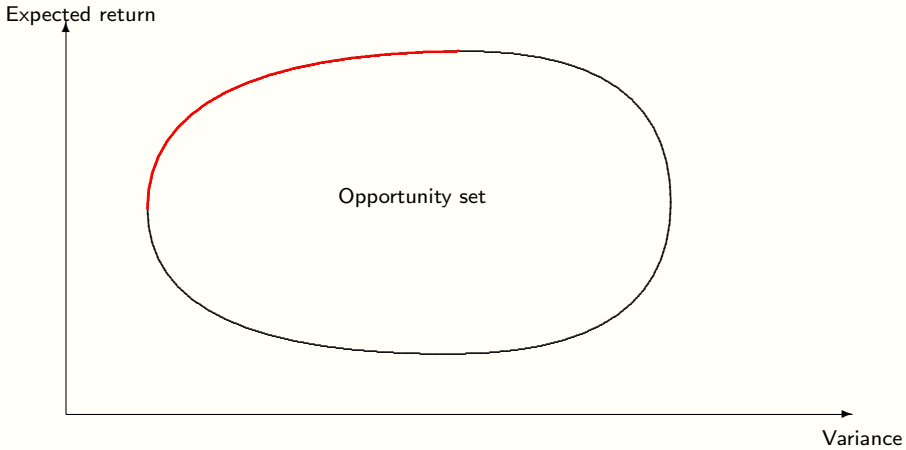

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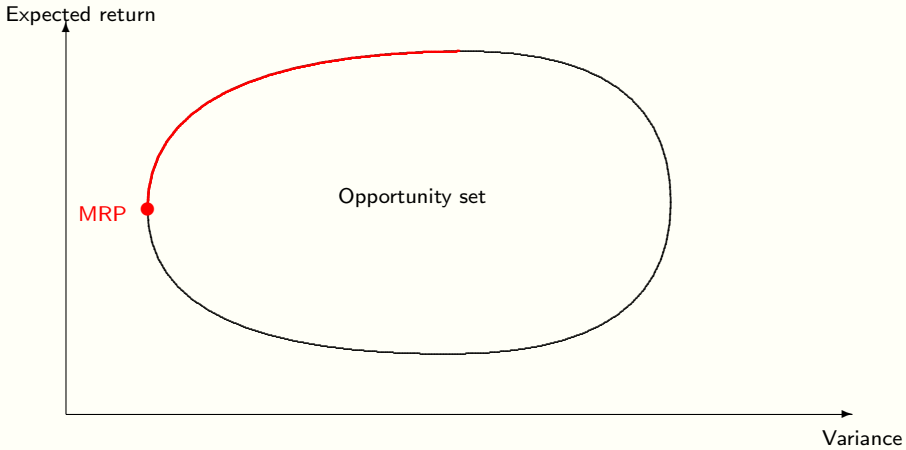

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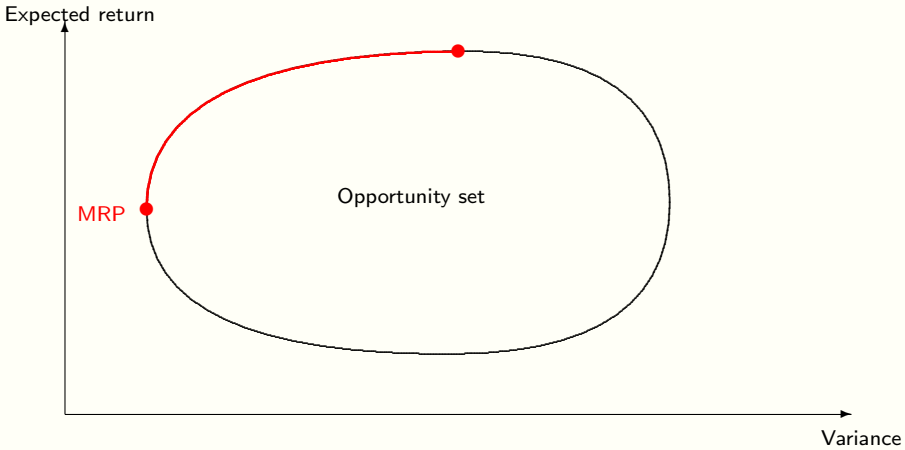

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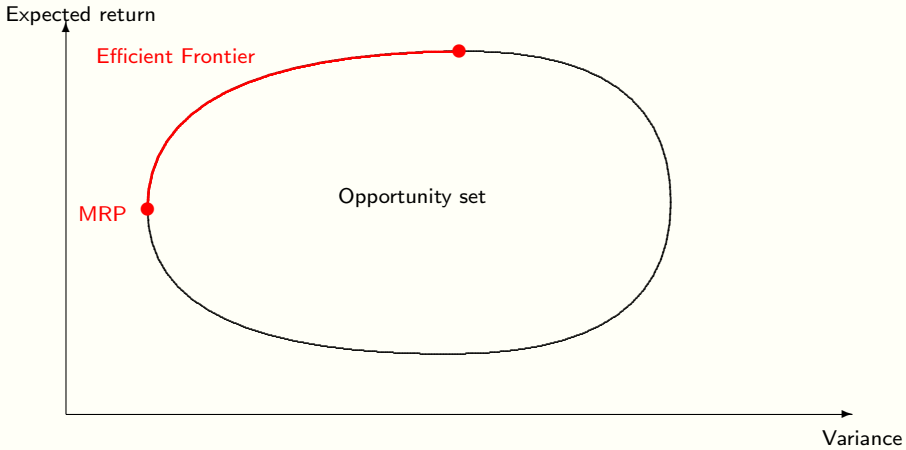

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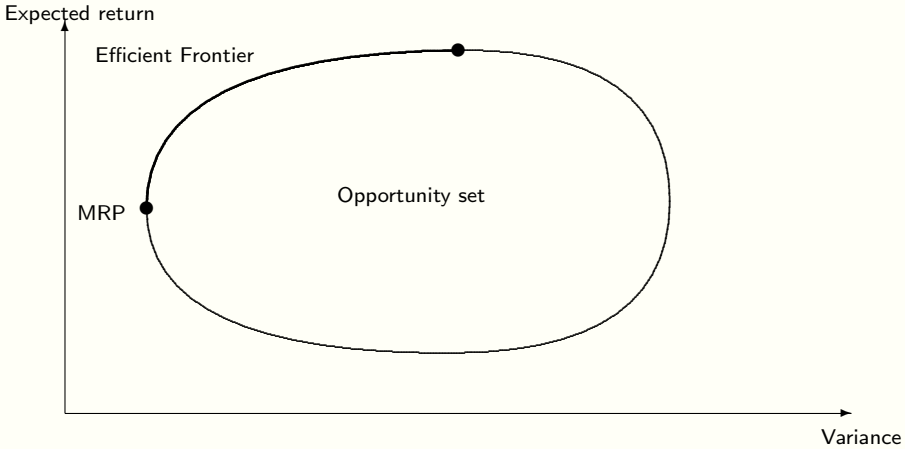

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Selecting the best choice

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Indifference curves

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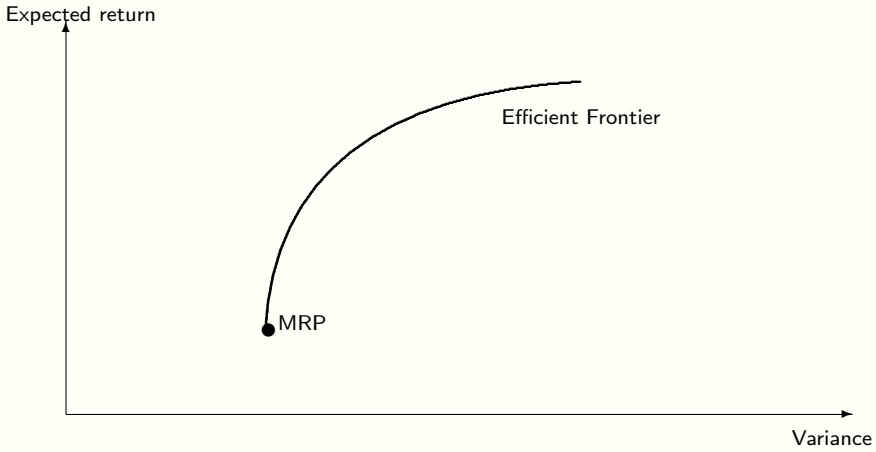

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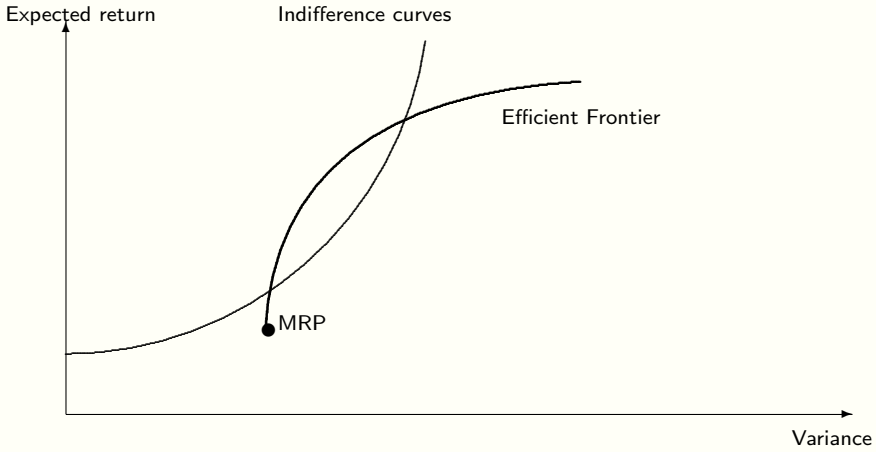

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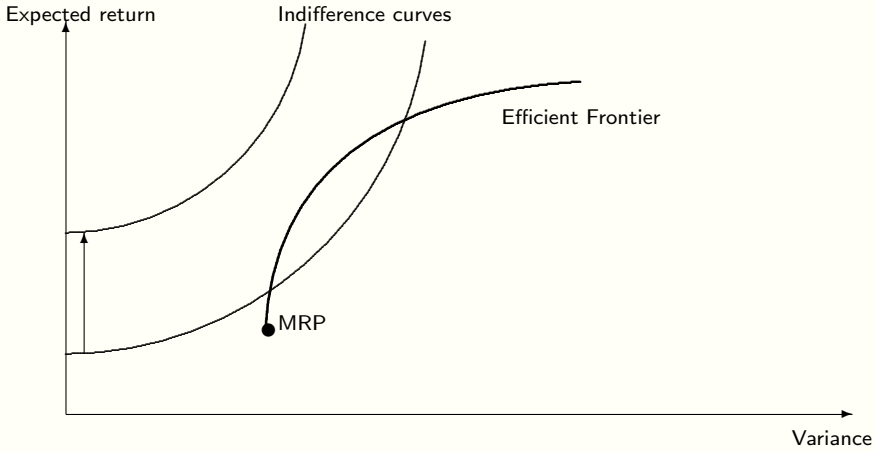

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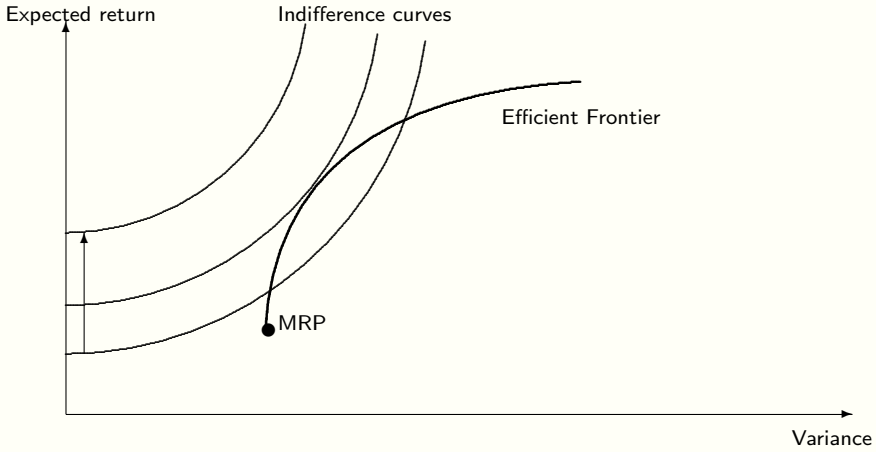

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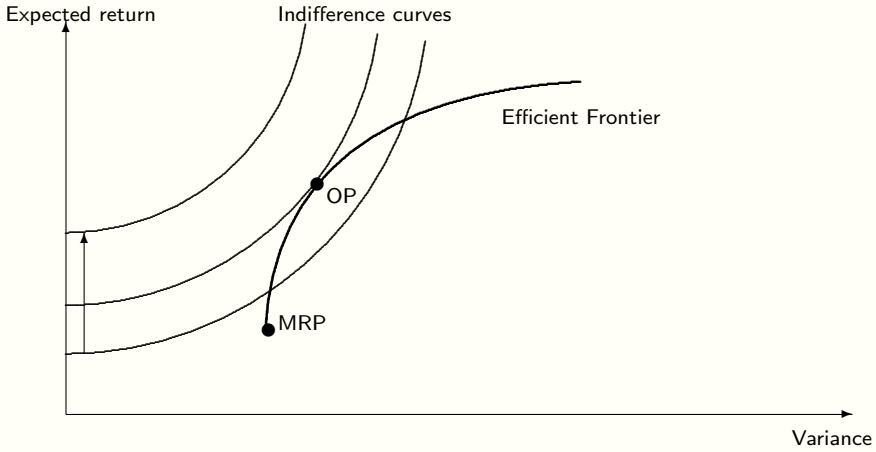

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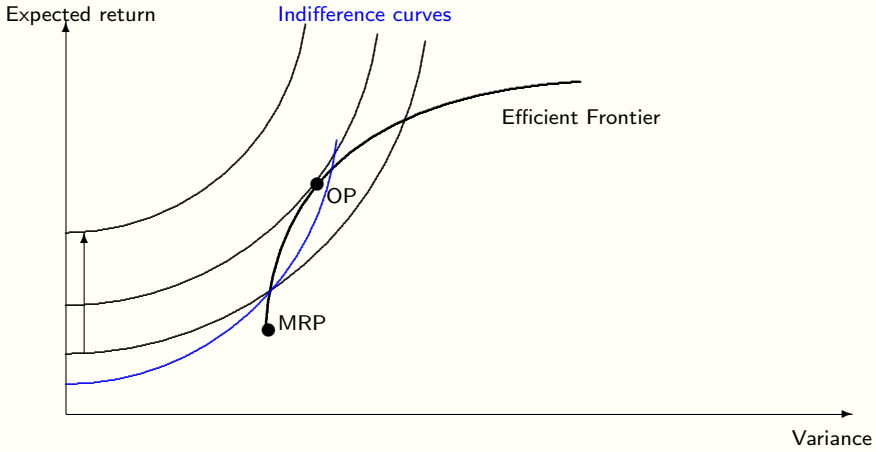

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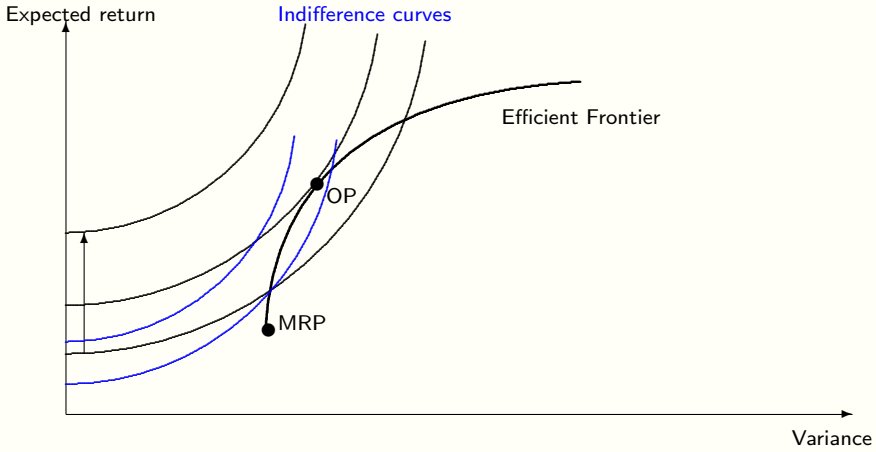

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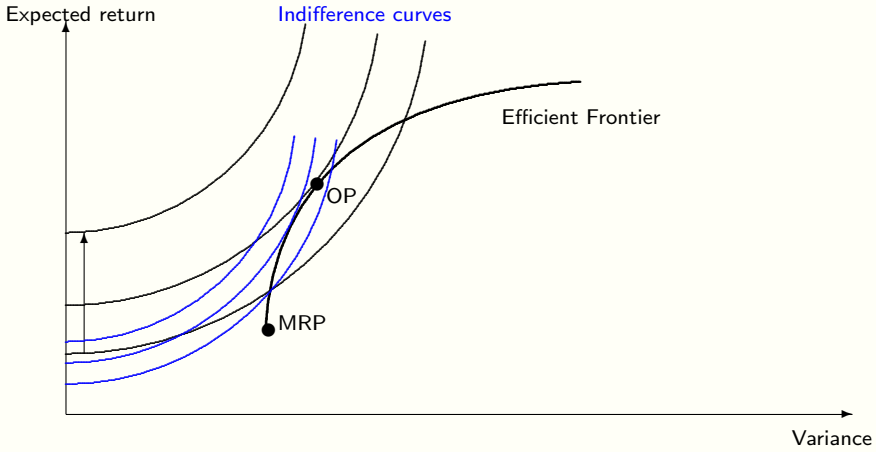

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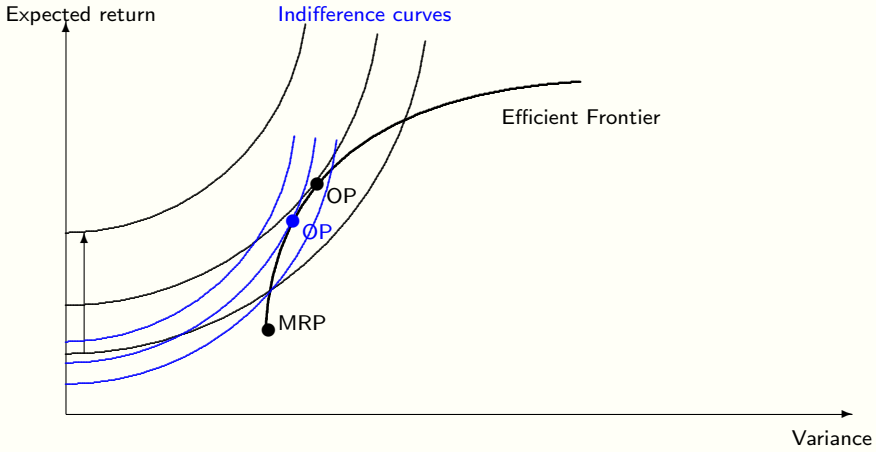

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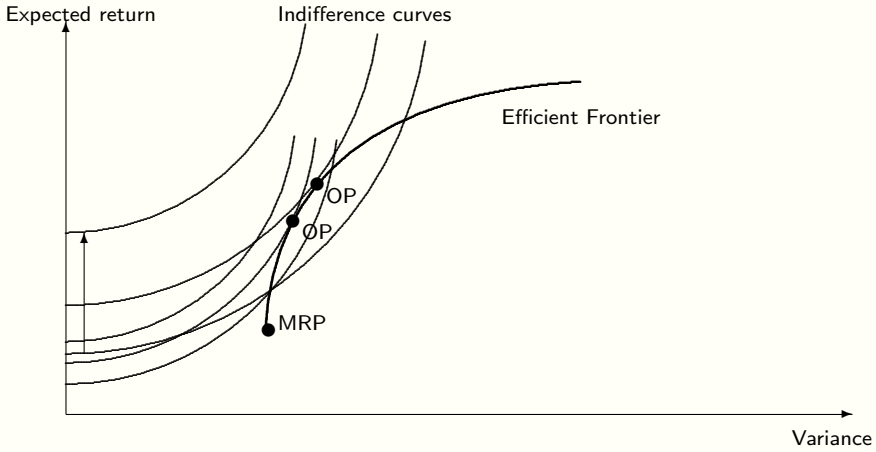

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Properties of the optimal portfolio

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