Andreas Krause

Risk aversion

Dislike of risk

- Individuals prefer, *ceteris paribus*, higher outcomes to lower outcomes
- Individuals prefer, *ceteris paribus*, lower risks to higher risks
- The attitude towards risk is commonly referred to as risk aversion
- More formally, we define
  Individuals are risk averse if they always prefer to receive a fixed payment to a random payment of equal expected value.

  (B. Dumas and B. Allaz, Financial Securities: Market Equilibrium and Pricing Methods, Cengage Learning, London 1996)

Measuring risk and risk aversion

- The most common risk measure in finance is the variance of outcomes
- Variance is a measure of how much outcomes deviate from the expected value
- $\mathsf{Var}\left[V\right] = \mathsf{E}\left[\left(V - \mathsf{E}\left[V\right]\right)^2\right]$
- Risk aversion is more difficult to measure as it will depend on the utility function of individuals

Risk premium

- Using the definition of risk aversion, we see can compare the expected utility of risky outcomes and the utility non-risky outcomes
- The utility of the risky outcomes, should be less than that of a non-risky-outcome with the same expected value
- The amount to be deducted from the safe outcome to obtain the same utility as the risky outcome is the risk premium
- $E\left[U\left(V\right)\right] \quad \leq = U\left(E\left[V\right] - \Pi\right)$

Approximating the risky outcome

- We use a quadratic approximation of the expected utility of the individual facing risky outcomes
- We use the expected value as a starting point, then make a linear adjustment, and a quadratic adjustment
- $$\mathsf{E}\left[U\left(V\right)\right] = \mathsf{E}\left[U\left(\mathsf{E}\left[V\right]\right) + \frac{\partial U(\mathsf{E}[V])}{\partial V}(V - \mathsf{E}[V]) + \frac{1}{2}\frac{\partial^2 U(\mathsf{E}[V])}{\partial V^2}\left(V - \mathsf{E}[V]\right)^2\right]$$
$$= U(\mathsf{E}[V]) + \frac{1}{2}\frac{\partial^2 U(\mathsf{E}[V])}{\partial V^2}\mathsf{Var}\left[V\right]$$

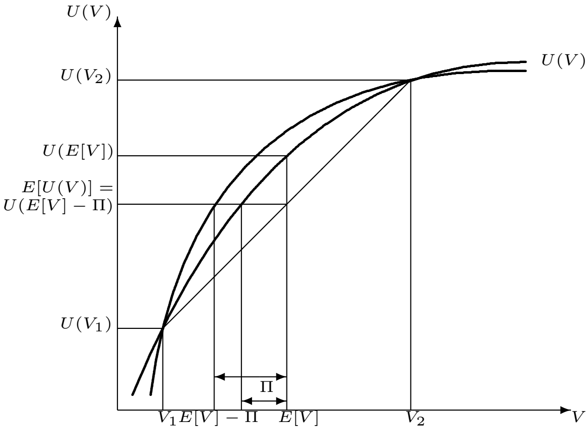
Approximating the safe outcome

- We use a linear approximation of the expected utility of the individual facing a safe outcome
- We use the expected value as a starting point and then make a linear adjustment
- $U(\mathsf{E}[V]-\Pi) = U(\mathsf{E}[V]) + \frac{\partial U(\mathsf{E}[V])}{\partial V}\Pi$
- Setting this equal to the approximation of the risky outcome gives $\Pi = \frac{1}{2}\left(-\frac{\frac{\partial^2 U(\mathsf{E}[V])}{\partial V^2}}{\frac{\partial U(\mathsf{E}[V])}{\partial V}}\right)\mathsf{Var}\left[V\right]$
- The risk premium is increasing in the risk and the risk aversion

Arrow-Pratt measure of risk aversion

- We define $z = -\frac{\frac{\partial^2 U(\mathsf{E}[V])}{\partial V^2}}{\frac{\partial U(\mathsf{E}[V])}{\partial V}}$ as the Arrow-Pratt measure of absolute risk aversion
- The more risk averse an individual is, the higher the risk premium: $\Pi = \frac{1}{2} z \mathsf{Var}\left[V\right]$
- The second derivative of the utility function reflects the curvature of the utility function

Risk premium with two outcomes



Approximating expected utility

- We have the expected utility given as $E\left[U\left(V\right)\right] = U\left(E\left[V\right] - \Pi\right)$
- The risk premium was $\Pi = \frac{1}{2} z \text{Var}\left[V\right]$

$\Rightarrow$ $E\left[U\left(V\right)\right] = U\left(E\left[V\right] - \frac{1}{2} z \text{Var}\left[V\right]\right)$

- As marginal utility is positive, we can often maximize expected utility by maximizing only the argument $E\left[V\right] - \frac{1}{2} z \text{Var}\left[V\right]$
- We can maximize expected utility without knowing the utility function, we only require the risk aversion

Summary

- Risk aversion measures the attitude towards risk
- Risk aversion will be dependent on the preferences of individuals
- Risk itself can be measured objectively through the variance

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Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk