Andreas Krause

Risk aversion

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  Individuals are risk averse if they always prefer to receive a fixed payment to a random payment of equal expected value.

  (B. Dumas and B. Allaz, Financial Securities: Market Equilibrium and Pricing Methods, Cengage Learning, London 1996)

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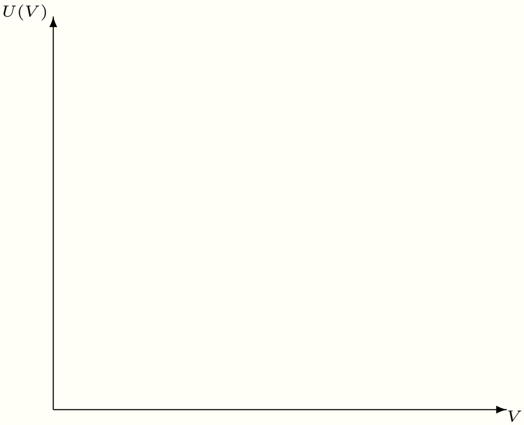

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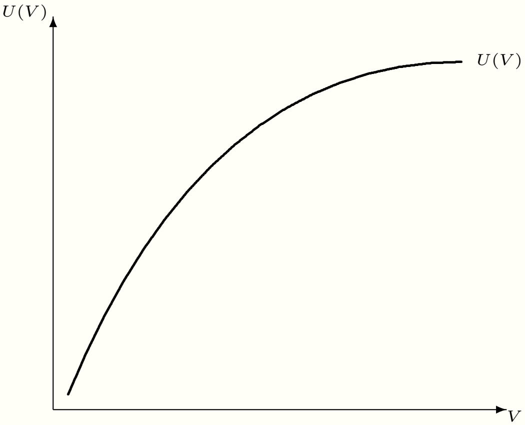

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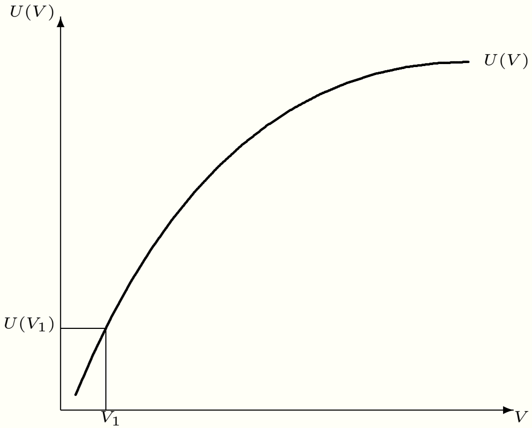

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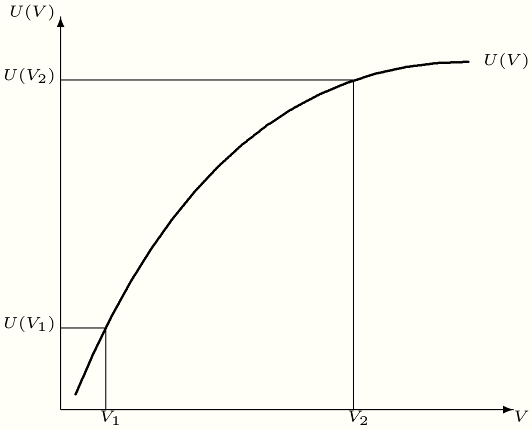

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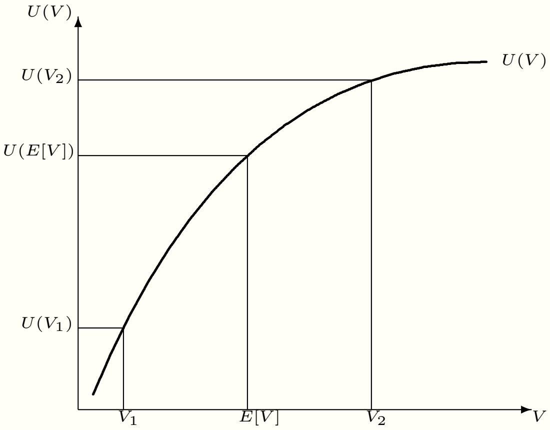

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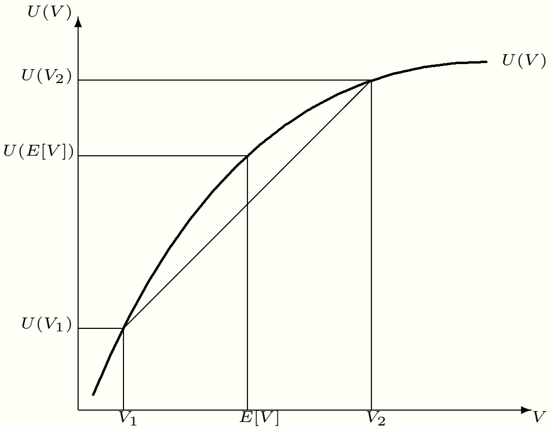

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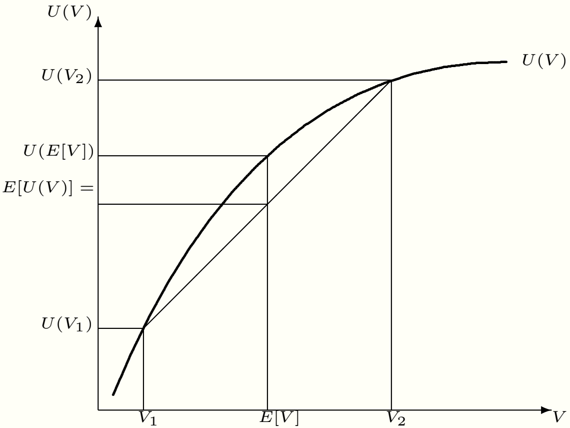

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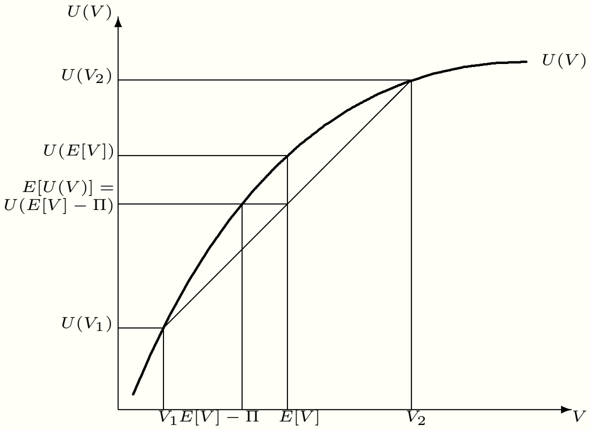

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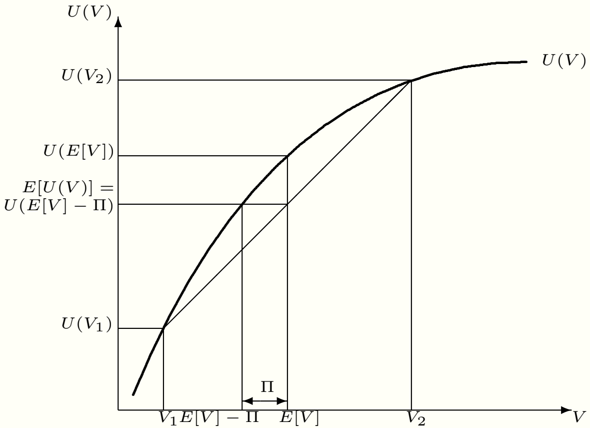

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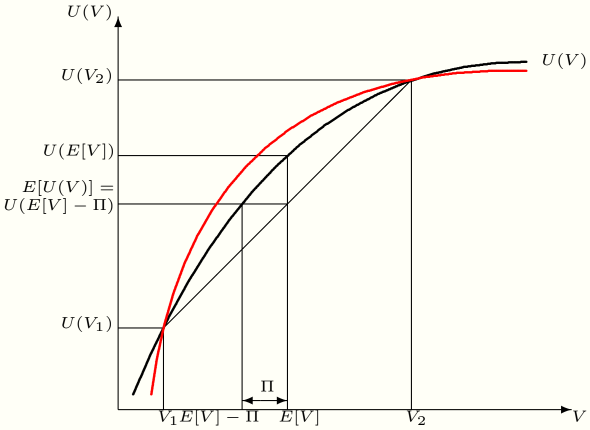

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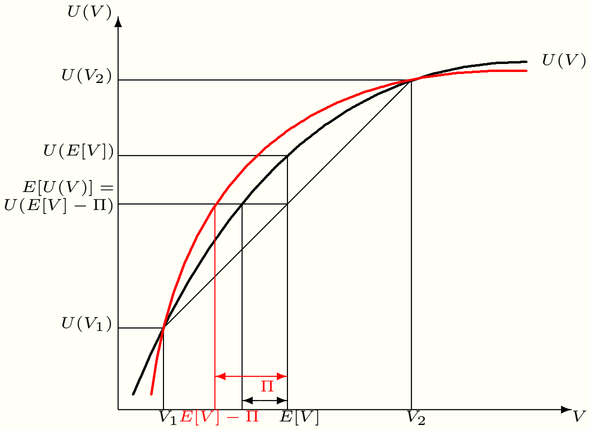

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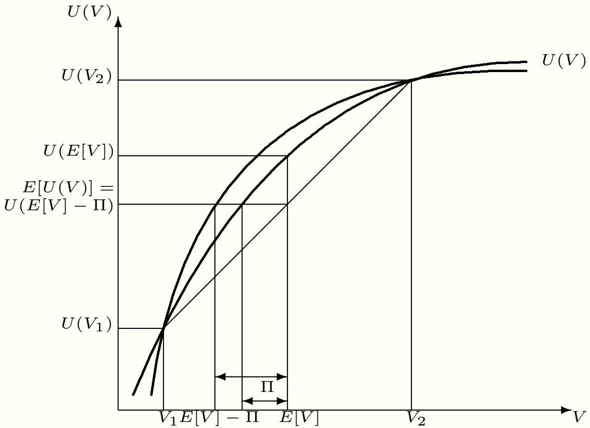

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