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Option pricing

The difficulty in pricing options

- Option pricing is a major field in financial economics
- Assumptions on the way asset prices evolve into the future are an essential ingredient into models
- Once such a stochastic process has been identified, solving for option vales is not trivial
- In many cases no analytical solutions can be obtained, with numerical methods or Monte-Carlo simulations being employed

Restrictive assumptions

- In order to obtain some insights into option pricing, restrictive assumptions are necessary
- These assumptions will allow to derive explicit solutions, but may not meet the reality of how the price of the underlying asset evolves
- While more realistic assumptions provide a better match with observed prices, the general insights restrictive models provide remain valid

Arbitrage pricing

- The basic concept in options pricing is that the payoff profile of an option is matched with that of an portfolio of assets whose values are known
- Analysing the composition of this portfolio will then lead to the value of the option
- The simplest form of assumptions is to assume that asset prices move discretely from one time period to another
- This is commonly known as the binomial model of option pricing


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Binomial pricing of options

Benefits and limitations of binomial option pricing

- Binomial option pricing is very flexible in that any form of option can be analysed
- The changes in the price of the underlying asset can also be modelled flexibly by making different assumptions
- Using computers, a large number of time periods can be considered, making this methodology realistic by allowing frequent price changes
- A major limitation of binomial asset pricing is that no analytical solution exists and general properties of options prices can only be analysed numerically

Providing an explicit solution for option prices

- Seeking an analytical solution to the value of an option, additional assumptions need be made to allow for an explicit solution
- The Black-Scholes model is the most widely used model for standard European options
- It can be derived using different approaches, but is in all cases involving advanced statistical and mathematical methods


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Black-Scholes model

Limitations to markets with specific properties

- Assuming that asset values are log-normally distributed allows an analytical formula of the value of an option
- This formula is restricted by the assumptions made and cannot easily be transferred to markets in which these are not fulfilled
- The formula is usually stated for call options only, but using the Put-Call parity, the value of put options can be obtained easily

? Why do investor purchase options and not routinely replicate them themselves?

! In order to ensure the value of the option is met, the Δ and loan amount needs to be maintained at all times, but it changes as parameters change - including time to maturity - and thus requires constant (costly) updating of these holdings

Knowing properties to use options

- Option pricing formulae involve a large number of parameters that affect their value
- Current price of the underlying asset, strike price, volatility, time to maturity, risk-free rate
- Knowing how these parameters affect option prices also allows investors to completely hedge their positions


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Properties of option prices

Option values and risks

- Options are increasing in the volatility of the underlying asset, making it one of the few asset whose value increases as risks increase
- The increased value can be explained with the increase in utility the insurance against these risks provides

? Why do investors not always use Δ-hedging but instead rely on futures and swaps?

! In order to ensure the value of the position is not changing, the hedge ratio needs to be maintained at all times, but it changes as variables change - including time to maturity - and thus requires constant (costly) updating of option holdings

Summary

- Option pricing depends critically on the assumptions about the future evolution of the price of the underlying asset
- Explicit solutions for option prices can only be obtained if these assumptions are very restrictive
- Option pricing suggests that they can be replicated using the underlying asset and investment into a risk-free asst (short position)

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