

Andreas Krause

Option pricing

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Binomial pricing of options

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Black-Scholes model

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Properties of option prices

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