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Portfolio selection

- We will discuss how investors would optimally decide how to make their investments into a range of assets they consider.
- We will see which considerations investors have and how they will select the assets that are optimal for them.

# Selecting portfolios

- ▶ Individuals do not only invest into a single asset, but several assets at the same time, called a portfolio
- ▶ The assets will be interacting with each other and the total risk and return of a portfolio will depend on all assets chosen
- ▶ Portfolio selection addresses the optimal weights of assets in the portfolio
- ▶ The optimal portfolio will depend on the asset characteristics and the preferences for risks of individual investors

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- ▶
    - While much of the discussion about investing evolves around identifying the best assets to invest in, mostly defined as those that are expected to show the highest return with occasionally risks also being considered, investors typically do not invest only into a single assets, but several assets.
    - Such a combination of assets an investor holds is called a portfolio
  - ▶ Assets are not independent of each other, but they will be moving more or less closely with each other, or sometimes tend to move in opposite directions. This will affect the total return and also the risk that an investor is exposed to. The optimal portfolio to invest in will take such interactions into account.
  - ▶ Taking all these aspect into consideration, portfolio selection theory will determine the characteristics of the best portfolio possible given the assets considered for investment, and this can then be 'reverse engineered' to obtain the combination of assets that achieve these characteristics with the assets given. The result will be weights of the assets in the portfolio.
  - ▶
    - We will see that the optimal portfolio will depend on the characteristics of the assets that are considered for investment.
    - The risk preferences of investors will also have to be considered.
- We will first develop the basic decision criteria to affect a choice between different portfolios.

## Decision criteria

- ▶ Individuals will choose a portfolio that maximizes their expected utility
- ▶ Expected utility is driven by the outcomes (expected return) and (co-)variances (risk)
- ▶ Preferences of individuals are difficult to assess and we can use the general property of risk aversion
- ▶ Using risk aversion we can exclude a wide range of portfolios

- Before addressing the specific challenge of determining the optimal portfolio, we will look at the decision criteria we can use in this context.
- ▶ The optimal portfolio will be the portfolio which shows the highest expected utility for an investor, hence we will indirectly be conducting utility maximisation.
- ▶
  - The expected utility will be determined by the outcomes the investment gives the investor.
  - In financial markets and investments we often identify outcomes with the expected return that is achieved.
  - The other determinant will be the variance of the outcome
  - This represents the risk the investor is exposed to.
- ▶ We will use the general assumption of risk averse investors as much as possible in our assessment rather than using their specific risk aversion. The reason is that risk aversion is difficult to determine. Thus we will seek to exclude the use of a specific risk aversion as long as possible.
- ▶ By only using the assumption of risk aversion, we can see that most possible portfolios can already be excluded from consideration.
- We will first look at the decision-making by investors using only the expected return (mean) and the variance (risk) of assets.



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Mean-Variance criterion

→ We will now look at making decisions only using the mean and the variance of the possible outcomes as a criterion.



# Establishing efficient frontiers


- ▶ We can reduce the possible choices to an efficient frontier that shows a positive relationship between risk and return
- ▶ The optimal choice on the efficient frontier will have to use the utility function
- ▶ The more risk averse the individual is, the less risk is taken

- Now that we have derived the main results of the model, as far as relevant for us, we will briefly discuss some implications as well as limitations of this model. This will allow us to interpret the model in its context of the initial problem and enables us to apply it appropriately in a realistic context.
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  - We have seen how using risk aversion can reduce the possible choices to a small set, called the efficient frontier.
  - Between choices that are on the efficient frontier itself cannot be chosen without further knowledge of the risky aversion. We find that on the efficient frontier there is positive relationship between risk and return.
- ▶ For the optimal choice we will still have to use the utility function, or at least know the risk aversion of the investor.
- ▶ We have seen that more risk averse investors will take less risk, given their dislike for risk they are willing to forego some return to achieve this.
- Having determined the principle about making decisions about risky outcomes, we can now apply this principle to the specific context of portfolio selection.

## Application of decision-criterion

- ▶ We now apply the mean-variance criterion to portfolio selection using a graphical approach
- ▶ As far as possible, we will seek to base the selection of portfolios on asset properties and not preferences
- ▶ This allows us to obtain generic results that are applicable to all investors

- We build our assessment of portfolio on the decision criteria discussed above.
- ▶ We will apply the mean-variance decision criterion using a graphical analysis rather than its mathematical derivation.
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  - In order to make the decision as objective as possible, we will seek to determine portfolios as far as possible without the use of the degree risk aversion or knowledge of the full utility function. That way, we can apply the analysis to all investors, irrespective of their specific risk preferences.
  - We will only use preferences, or the degree of risk aversion if other approaches cannot be used.
- ▶ That way we obtain insights in principles that apply to all investors, whatever their risk aversion is, just assuming they are risk averse.
- We can now continue with the determination of the optimal portfolio investors should invest in.



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Optimal portfolios

→ We will now determine the optimal portfolio of investors. To this effect we will look at simple portfolios consisting of two and then later of three assets; the ideas developed there can easily be transferred to any number of assets.

## Combining a risky portfolio and risk-free asset

- ▶ The optimal portfolio consists of combination of a risky portfolio and the risk-free asset
- ▶ Only the combination depends on preferences, not the composition of the optimal risky portfolio
- ? Will all individuals hold the same optimal risky portfolio?
- ! The optimal risky portfolio is dependent on the asset characteristics, if individuals disagree on these characteristics, they will have different optimal risky portfolios

# Combining a risky portfolio and risk-free asset

- Now that we have derived the main results of the model, as far as relevant for us, we will briefly discuss some implications as well as limitations of this model. This will allow us to interpret the model in its context of the initial problem and enables us to apply it appropriately in a realistic context.
- ▶ We have seen that the optimal portfolio consists of two components that are identical across investors, the optimal risky portfolio and the risk-free asset.
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    - The combination of these two assets depends on the risk preferences of investors.
    - The optimal risky portfolio will be the same across investors, regardless of their risk preferences, as long as they agree on the asset characteristics.
  - ▶ The optimal risky portfolio is the same for all investors, why do risk preferences do not matter?
  - ▶ We were able to obtain the efficient frontier by only relying on asset characteristics and the assumption that investors are risk averse; we did not need to know the degree of risk aversion. The ORP is located on the efficient frontier and it is the point where the line between the efficient frontier and the risk-free asset is tangential. This again uses only asset characteristics. Thus the ORP does not rely on risk preferences. To have the same ORP for all investors, we need to make the additional assumption that all investors agree on the asset characteristics. If they do not agree, the efficient frontier will be different and hence the ORP will be different.
- Thus the optimal portfolio can be determined by first selecting the optimal risky portfolio and then combining this with the risk-free asset.



# Summary

- ▶ Portfolio selection consists of determining a preference-free optimal risky portfolio and then combining this with a risk-free asset
- ▶ Only this final combination depends on the risk aversion of the individual
- ▶ Diversification reduces risk and increases the utility of the portfolio

- We can now summarize the key results we have obtained about decision-making under uncertainty.
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    - We have established that the process of portfolio selection in its first step requires the determination of an optimal risky portfolio, which can be determined objectively without reference to the preferences of investors, but which will be driven only by the characteristics of all assets that are considered for investment.
    - This optimal risky portfolio is then combined with a risk-free asset to determine the optimal portfolio.
  - ▶ Only the combination of the optimal risky portfolio with the risk-free asset will require knowledge of the risk aversion of investors.
  - ▶ We have also seen that diversification, the inclusion of additional assets in the consideration for inclusion in the portfolio, will increase the utility of investors, as does allowing short sales.
- It has to be noted that while many assets may be considered for inclusion in the optimal portfolio, not all assets will necessarily be actually included. Many assets can, in practice, have a weight of zero in the optimal portfolio and thus not investment into these will be made. Excluding such assets from the derivation of the optimal portfolio would not affect the outcome, but it will only be known at the end of the process that they could have been excluded. Assets get excluded most commonly if they are very risky and have a low expected return; unless they have low correlations with other assets, their inclusion is usually not beneficial. Even as short sales, their high risks might not make them attractive enough for inclusion in the optimal portfolio. Assets with low correlations are increasing the opportunity set the most, and this is often used as a justification for including highly risky and often not well-performing hedge funds into investment portfolios; it is their investment strategies that make them attractive due to having low correlation with most other assets.



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