Andreas Krause

Portfolio selection

- We will discuss how investors would optimally decide how to make their investments into a range of assets they consider.
- We will see which considerations investors have and how they will select the assets that are optimal for them.

Selecting portfolios

Selecting portfolios

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- While much of the discussion about investing evolves around identifying the best assets to invest in, mostly defined as those that are expected to show the highest return with occasionally risks also being considered, investors typically do not invest only into a single assets, but several assets.
  - Such a combination of assets an investor holds is called a portfolio
- Assets are not independent of each other, but they will be moving more or less closely with each other, or sometimes tend to move in opposite directions. This will affect the total return and also the risk that an investor is exposed to. The optimal portfolio to invest in will take such interactions into account.
- Taking all these aspect into consideration, portfolio selection theory will determine the characteristics of the best portfolio possible given the assets considered for investment, and this can then be 'reverse engineered' to obtain the combination of assets that achieve these characteristics with the assets given. The result will be weights of the assets in the portfolio.
- We will see that the optimal portfolio will depend on the characteristics of the assets that are considered for investment.
  - The risk preferences of investors will also have to be considered.

→ We will first develop the basic decision criteria to affect a choice between different portfolios.

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- Individuals do not only invest into a single asset, but several assets at the same time

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Decision criteria

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- The optimal portfolio will be the portfolio which shows the highest expected utility for an investor, hence we will indirectly be conducting utility maximisation.
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  - The expected utility will be determined by the outcomes the investment gives the investor.
  - In financial markets and investments we often identify outcomes with the expected return that is achieved.
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- We will use the general assumption of risk averse investors as much as possible in our assessment rather than using their specific risk aversion. The reson is that risk aversion is difficult to determine. Thus we will seek to exclude the use of a specific risk aversion as long as possible.
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- Expected utility is driven by the outcomes (expected return) and (co-)variances (risk)
- Preferences of individuals are difficult to assess and we can use the general property of risk aversion
- Using risk aversion we can exclude a wide range of portfolios

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Mean-Variance criterion

→ We will now look at making decisions only using the mean and the variance of the possible outcomes as a criterion.

Establishing efficient frontiers

Establishing efficient frontiers

→ Now that we have derived the main results of the model, as far as relevant for us, we will briefly discuss some implications as well as limitations of this model. This will allow us to interpret the model in its context of the initial problem and enables us to apply it appropriately in a realistic context.

- ▶
  - We have seen how using risk aversion can reduce the possible choices to a small set, called the efficient frontier.
  - Between choices that are on the efficient frontier itself cannot be chosen without further knowledge of the risky aversion. We find that on the efficient frontier there is positive relationship between risk and return.
- ▶ For the optimal choice we will still have to use the utility function, or at least know the risk aversion of the investor.
- ▶ We have seen that more risk averse investors will take less risk, given their dislike for risk they are willing to forego some return to achieve this.

→ Having determined the principle about making decisions about risky outcomes, we can now apply this principle to the specific context of portfolio selection.

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Application of decision-criterion

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→ We build our assessment of portfolio on the decision criteria discussed above.

- We will apply the mean-variance decision criterion using a graphical analysis rather than its mathematical derivation.
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  - In order to make the decision as objective as possible, we will seek to determine portfolios as far as possible without the use of the degree risk aversion or knowldge of the full utility function. That way, we can apply the analysis to all investors, irrespective of their specific risk preferences.
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Application of decision-criterion

- We now apply the mean-variance criterion to portfolio selection using a graphical approach
- As far as possible, we will seek to base the selection of portfolios on asset properties and not preferences
- This allows us to obtain generic results that are applicable to all investors

Application of decision-criterion

→ We build our assessment of portfolio on the decision criteria discussed above.

- We will apply the mean-variance decision criterion using a graphical analysis rather than its mathematical derivation.
-
  - In order to make the decision as objective as possible, we will seek to determine portfolios as far as possible without the use of the degree risk aversion or knowldge of the full utility function. That way, we can apply the analysis to all investors, irrespective of their specific risk preferences.
  - We will only use preferences, or the degree of risk aversion if other approaches cannot be used.
- That way we obtain insights in principles that apply to all investors, whatever their risk aversion is, just assuming they are risk averse.

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Andreas Krause
Optimal portfolios

→ We will now determine the optimal portfolio of investors. To this effect we will look at simple portfolios consisting of two and then later of three assets; the ideas developed there can easily be transferred to any number of assets.

Combining a risky portfolio and risk-free asset

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- We have also seen that diversification, the inclusion of additional assets in the consideration for inclusion in the portfolio, will increase the utility of investors, as does allowing short sales.

→ It has to be noted that while many assets may be considered for inclusion in the optimal portfolio, not all asset will necessarily be actually included. Many assets can, in practice will have, a weight of zero in the optimal portfolio and thus not investment into these will be made. Excluding such assets from the derivation of the optimal portfolio would not affect the outcome, but it will only be known at the end of the process that they could have been excluded. Assets get excluded most commonly if they are very risky and have a low expected return; unless they have low correlations with other assets, their inclusion is usually not beneficial. Even as short sales, their high risks might not make them attractive enough for inclusion in the optimal portfolio. Assets with low correlations are increasing the opportunity set the most, and this is often used as a justification for including highly risky and often not well-performing hedge funds into investment portfolios; it is their investment strategies that make them attractive due to having low correlation with most other assets.

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