Andreas Krause

Chapter 3.1

Maturity transformation of deposits

Outline

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- Deposits can to a large degree be withdrawn at any time, while loans are usually given for longer time periods.
- Depositors value the benefits of being able to withdraw their funds at any time.
- Companies taking loans value to security of a longer loan to be able to commit to longer investments.
- Thus deposits are short-term and loans are long-term.
- This imposes a mismatch in the maturity of deposits (liabilities) and loans (assets).
- We will see why such a mismatch is optimal.

Outline

- We will look at the preferences of depositors and how they can best be met.
- To this effect we will derive the social optimum and then compare this with various arrangements for depositors.
- We consider direct lending by depositors to companies without the presence of a bank, then extend this to include a facility to trade these loans, before finally evaluating bank loans.

- We will first look at the preferences of depositors for withdrawing funds.

Maturity mismatch

- Borrowers prefer long-term loans to meet the time horizon of their investments
- Depositors prefer the ability access their funds easily if needed

$\Rightarrow$ Banks need to be able to pay back deposits if requested, but lend out at long terms

- We show that bank lending provides the optimal solution to overcoming this maturity mismatch

Maturity mismatch

→ We will look firstly deeper at the origins of the maturity mismatch between depositors and companies.

- Investment are often taking a long time until profits are generated and this allows companies not to repay the loans before these profits are generated. Therefore companies will seek long-term loans.
- Depositors want to access their funds immediately if needed, for example to make purchases or alternative investments. Therefore depositors would want short-term deposits.
- [⇒]
  - This requirement by depositors to withdraw their funds implies that banks must be able to repay deposits at any time.
  - However, they cannot obtain these funds by calling in loans as preferably they would lend long-term.
- We will see that banks are able to implement the social optimum in this case. Banks are able to overcome the issues arising from this maturity mismatch.

→ We will first determine the details of the model used before providing a more detailed analysis.

Model specifications

- Loans are repaid after 2 time periods with probability π
- Depositors can withdraw either after 1 or 2 time periods, earning interest r_D^1 and r_D^2
- A fraction p of depositors withdraws in time period 1
- If banks have to raise cash to repay deposits, they only get a fraction λ of the loan value
- Depositor utility: $E\left[U\left(D\right)\right] = pu\left(\left(1+r_D^1\right)D\right) + \left(1-p\right)u\left(\left(1+r_D^2\right)D\right)$

Model specifications

→ We can now look at the details of the model.

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  - We simplify the analysis by requiring loans to be given for two time periods. This represents the long term loans.
  - At the end of these two time periods, the loans are repaid only if the investment companies have made has been successful, which happens with some probability. If the investment is not successful the loan will not be repaid.
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  - Depositor have the ability to withdraw after 1 time period, or they can retain their deposits until the loans are repaid. Thus deposits may have to be repaid before loans are repaid.
  - Depositors obtain interest in their deposit, which might differ depending on how long they retain their deposits.
- We assume that a known fraction of depositors will withdraw early. This might be due to them wishing to purchase goods or make alternative investments.
- Banks need cash to repay deposits and they do not hold sufficient deposits, they have to sell some of the loans they have given in the market. As the market will not be very liquid with few potential purchasers and large adverse selection, banks will only realise a fraction of the loan value.
- Depositors obtain utility from the amount withdrawn early or the amount retained with the bank, weighted by the probability of each possibility.

→ We can now assess the social optimum of this model, which will serve as a bechmark for all arrangements that we consider.

- The social optimum is the benchmark against which all arrangements will have to be judged.
- If the social optimum is achieved, we know that we have found an arrangement that cannot be improved on, even if other arrangements can exist that give the same outcome.

Repaying deposits

- The cash held will be paid out for deposits withdrawn in time period 1
- $p\left(1+r_D^1\right)D = D - L$
- The loan repayments are used to repay the deposits left in time period 2
- $(1-p)\left(1+r_D^2\right)D = \pi\left(1+r_L\right)L$
- Combined: $p\left(1+r_D^1\right) + (1-p)\frac{\left(1+r_D^2\right)}{\pi\left(1+r_L\right)} = 1$

Repaying deposits

- → We first look at the ability to repay deposits to those depositors withdrawing early and those retaining their deposits.
- ▶ We assume that cash is retained such that this can be used to repay all deposits that are withdrawn early.
- ▶ The amount of cash is the amount of deposits not lent out, $D - L$, and the amount withdrawn is the amount due to depositors after time period 1, of which a fraction p is actually withdrawn. These two amounts have to be equal.
- ▶ Those deposits that are retained are then repaid by the repayments from the loans provided.
- ▶ The amount retained is the amount due to depositors after time period 2, of which a fraction $1 - p$ is actually retained and this amount is equal to the funds received from repaid loans. Note that any intermediary makes no profits here.
- ▶ We can combine these two equalities into this *formula*.
- → We can now determine the social optimum using this combined constraint.

Optimal deposit rates

- First order condition of maximizing the utility of depositors gives
$$\frac{\partial u\left(\left(1+r_D^1\right)D\right)}{\partial\left(1+r_D^1\right)D} = \pi\left(1+r_L\right)\frac{\partial u\left(\left(1+r_D^2\right)D\right)}{\partial\left(1+r_D^2\right)D}$$
- With the combined constraint this can be solved for the optimal deposit rates to be paid
- This social optimum is the benchmark with which we compare other lending arrangements

Optimal deposit rates

→ We choose deposits rates r_D^1 and r_D^2 that maximize the expected utility of depositors here, which represents social welfare as we only consider depositors.

- We can insert for one of the deposit rates from the constraint and then maximize the utility of depositors. The first order condition gives us the *formula* here.
- If we know the utility function and using the constraint, this could be solved for the actual deposit rates.
- This first order condition and the constraint we use as the benchmark against which we will compare various ways of arranging deposits and loans.

→ We will now continue with the evaluation of such arrangements and see how they compare to the social optimum.

- We will firstly consider the case where depositors provide loans directly to companies. If they want to withdraw their funds, they will have to sell the loan at a loss.

Wealth after early withdrawal

- If depositors lend directly, they will have the liquidated loan and cash if they want to consume in time period 1
- $\left(1+r_D^1\right) D = D - L + \lambda L$
- Depositors not liquidating their loan will in period 2 obtain their cash and the loan repayments
- $\left(1+r_D^2\right) D = D - L + \pi \left(1 + r_L\right) L$

Wealth after early withdrawal

- → We will look at the wealth the depositor has after an early withdrawal if he provides a loan directly to a company.
- ▶ Assume the depositors holds the same amount of cash as in the social optimum, $D - L$ and when selling the loan to withdraw fund obtains an additional λL. This gives the total amount this depositor will be able to achieve when withdrawing early.
- ▶ The amount repaid from raising cash is such that it gives an implied deposit rate r_D^1 if withdrawing early.
- ▶ A depositor not withdrawing early will not have to sell loans and in addition to the cash held will obtain the repayments of the loan.
- ▶ The amount repaid from cash and the repaid loans is such that it gives an implied deposit rate r_D^2 if not withdrawing early.
- → We can use these repayments now to assess the constraint of the utility optimisation for depositors.

Stricter constraint

- Combining these two constraints we get $p\left(1+r_D^1\right)+(1-p)\frac{\left(1+r_D^2\right)}{\pi\left(1+r_L\right)}\leq 1$
- If $\lambda < 1$ this constraint is stricter than the social optimum, where it was an equality

$\Rightarrow$ Direct lending is not optimal for depositors

Stricter constraint

→ We use the repayments to recover the constraint imposed on depositors in the social optimum.

- We can use the expressions from the repayment and multiplying it with the respective probabilities, we easily see that this expression is less than 1.
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  - We can easily show that if selling loans is causing a loss, the inequality is strict.
  - In the social optimum above this constraint was equal to 1.
- [⇒] If the constraint is the more strict, the social optimum cannot be achieved as the optimisation in general will not result in the same outcome.

→ We have seen that direct lending does not achieve the social optimum.

- We now extend the framework of direct lending by allowing loans to be traded amongst depositors rather than having to be liquidated by selling to outside investors at a loss.

Wealth after trading

- Loans can be sold at price P, rather than be liquidated at a loss
- If needing to liquidate the loan early to withdraw funds, it is sold: $\left(1 + r_D^1\right) D = D - L + \pi \left(1 + r_L\right) LP$
- If keeping the loan, the cash can be used to buy additional loans: $\left(1 + r_D^2\right) D = \frac{D-L}{P} + \pi \left(1 + r_L\right) L$

Wealth after trading

- → We focus first on the wealth of depositors after a trade of loans has occurred.
- ▶
  - Loans can be sold at some price relative to the expected repayment from this loan, which we will determine endogenously later and this revenue is added to its cash position for withdrawal.
  - This sale of the loan is an alternative to the liquidation of the loan at a loss.
- ▶ A depositor seeking to withdraw funds will sell the loan and the implied deposit rate is given by r_D^1.
- ▶ A depositor not withdrawing early has excess cash and can use this cash, $D - L$, to purchase these loans at a price P, giving it a total value of $\frac{D-L}{P}$, in addition to the loan repayment of the loans the depositor has retained. A depositor not seeking to withdraw funds will buy the loan and the implied deposit rate is given by r_D^2.
- → We can now determine the price at which the loan should be sold.

Loan price

- $P = \frac{1}{\pi(1+r_L)}$
- If $P > \frac{1}{\pi(1+r_L)}$ all deposits are invested into loans, as $\left(1+r_D^1\right) D = D - L + \pi\left(1+r_L\right) LP$ would increase in L

$\Rightarrow$ No cash remains to buy loans that are sold to raise cash for withdrawals

- If $P < \frac{1}{\pi(1+r_L)}$ all deposits are kept in cash, as $\left(1+r_D^1\right) D = D - L + \pi\left(1+r_L\right) LP$ would decrease in L

$\Rightarrow$ No loans are given

→ The loan price must be determined such that no arbitrage is possible by depositors.

- We propose that the price at which the loan is sold as given in the *formula*.
-
  - Assume the price were higher, then all deposits would be invested into loans, no cash would be held.
  - This is because the more loans are held the higher the payment when withdrawing. Depositors can sell loans at more than what they would obtain if they held cash.
- [⇒] This implies that no depositor would hold cash, but then depositors not withdrawing funds would have no cash to purchase the bonds those depositors withdrawing early want to sell. Thus the sale of bonds would not be possible and the price of the bond cannot exceed $\frac{1}{\pi(1+r_L)}$.
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  - Assume the loan price is lower, then no depositor would invest into loans but retain cash.
  - The reason is that the less loans are held the higher the payment when withdrawing. Investors would get so little for their loans, that they would make a loss, hence do not invest into them.
- [⇒] In this case no loans are given in the first instance and hence no selling and buying of loans could occur.

→ We have this established the price of the loans and can use this to determine the market equilibrium.

Market clearing

- Loans from those selling have to equal the cash kept by those not selling
- $pP\pi\left(1+r_L\right)L=\left(1-p\right)\left(D-L\right)$

$\Rightarrow$ $L=\left(1-p\right)D$

Market clearing

- → The market is in equilibrium if the demand for loans equals the supply for loans.
- The supply of loans consists of the fraction of depositors selling their loans at that price, while the demand is the cash holding of those not withdrawing early. These two expressions have to be equal in equilibrium.
- *Formula*
- Inserting for the price we can now determine how many loans can be given. The difference between loans and deposits is the amount of cash held.
- → Having established the market equilibrium in the loans, we can now continue to assess the implications for the optimal deposit rates and hence the utility of depositors.

Constraints for optimum

- Repayments in time periods 1 and 2 become
  $\left(1+r_D^1\right) D = D$
  $\left(1+r_D^2\right) D = \pi \left(1+r_L\right) D$
- The combined constraint is then $p\left(1+r_D^1\right) + (1-p)\frac{\left(1+r_D^2\right)D}{\pi(1+r_L)} = 1$
- The deposits rates are given and do not depend on the amount of lending
- First order conditions require $\frac{\partial u(D)}{\partial\left(1+r_D^1\right)} = \pi\left(1+r_L\right)\frac{\partial u(\pi(1+r_L)D)}{\partial\left(1+r_D^2\right)}$
- This will only be fulfilled for a specific utility function

$\Rightarrow$ Direct lending with trading is not optimal

Constraints for optimum

→ We can use all the results obtained so far to assess the resulting optimum for depositors.

- We can use the price and the relationship between deposits and loans to determine the repayments in both time periods.
- [] *Formula*
- [] *Formula*
- We can again combine these two expressions to form the constraint from the social optimum and we see that it is identical to that in the social optimum.
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  - From the expressions above, the deposit rates can easily be obtained
  - These deposit rates for both time periods do not depend on the amount of lending depositors allow.
- As the utility of depositors is unaffected and the constraint for this optimisation is the same as in the social optimum, the first order conditions will be the same.
- However, as the deposit rates are given this first order conditions will only be fulfilled for very specific values of the loan rate or success probability, or a specific utility function.
- [⇒] In general the first order condition is not fulfilled and hence the deposit rates obtained here do not reflect the social optimum.

→ Direct lending when allowing for loans to be traded between depositors will also not achieve the social optimum.

- Having so far seen that direct lending is not socially optimal, we now turn to assess bank lending.

Obtaining the social optimum

- All deposits are made with banks and the bank retains $p\left(1+r_D^1\right)D$ as cash to pay withdrawals, the remainder given as loans
- This recovers the social optimum as the arrangement is identical

$\Rightarrow$ Banks are optimal

Obtaining the social optimum

→ We propose a way banks can achieve the social optimum and then argue that it actually achieves this social optimum.

- - We propose that all deposits are made with banks, there will therefore be no direct lending and all lending will be made by banks.
  - The bank will retain some of the deposits as cash and this amount of cash will be exactly the same as the cash holding in the social optimum.
  - The remainder of the deposits will be given as loans.
- The arrangements are identical to the social optimal and we will use the first order condition and the constraint to determine the deposit rates. All depositors wanting to withdraw early will be able to do so using the cash reserves of the bank, who will not be required to sell and loans and incur losses.
- Hence banks can achieve the social optimum through pooling the deposits and lending them out jointly.

→ It is the focus on individual depositors that makes direct lending socially suboptimal. There depositors have to sell or trade loans if they seek to withdraw early, causing an inefficiency, which does not exist as banks pool the deposits and can hold back sufficient cash so as to never be required to sell loans. This is the benefit of banks.

Banking equilibrium

- We need $r_D^1 < r_D^2$ to prevent depositors withdrawing funds early
- The first order condition is $\frac{\partial u\left(\left(1+r_D^1\right)D\right)}{\partial\left(1+r_D^1\right)D} = \pi\left(1+r_L\right)\frac{\partial u\left(\left(1+r_D^2\right)D\right)}{\partial\left(1+r_D^2\right)D}$
- We need $\frac{\partial u\left(\left(1+r_D^1\right)D\right)}{\partial\left(1+r_D^1\right)D} > \frac{\partial u\left(\left(1+r_D^2\right)D\right)}{\partial\left(1+r_D^2\right)D}$ if $\pi\left(1+r_L\right) \geq 1$
- This is fulfilled if $r_D^1 < r_D^2$
- It is also not optimal for depositors to withdraw funds and force the bank to sell loans to raise cash, which depositors then buy
- Banks are an equilibrium outcome

→ While banks are optimal, they do not necessarily have to an equilibrium outcome. We will here show that banks are indeed an equilibrium outcome.

- If the interest paid when withdrawing early is higher than when retaining funds, then depositors would always withdraw early as they would obtain a higher payment. We need to rule this scenario out.
- We have the first order condition of the social optimum given by the *formula*.
- If the investment is such that it allows the loan to be profitable to banks, thus the expected repayment to exceed the initial investment, then the marginal utility in time period 1 will exceed the marginal utility in time period 2.
- With the usual assumption of declining marginal utility, this implies that the deposit rate in time period 1 has to be smaller than in time period 2, as required.
- Depositors could force banks to sell loans by withdrawing deposits, even though they do not need the funds. They could then use that amount the bank has repaid to purchase the loans from the bank. The depositors would make a loss from this withdrawal of funds, and then make a gain from purchasing the loans below value. These two aspects cancel each other out and depositors make no profits. This is akin to trading loans among depositors and will not be socially optimal in general.
- Therefore the existence of banks is an equilibrium as there is no incentive for depositors to withdraw funds early.

→ We have thus established that banks provide the social optimum and that they are an equilibrium.

Alternative equilibrium

- No depositor has an incentive to withdraw deposits if they do not need to
- But if they expect other depositors to do so, they have an incentive to withdraw to avoid losses

$\Rightarrow$ A bank run equilibrium exists

→ While banks are an equilibrium, a second equilibrium exists in which the banking system will fail.

- We have seen that no depositor has an incentive if no one else withdraws early, apart from those that require funds and for which cash reserves have been established.
- However, if some investors were to withdraw early, they would force the bank to sell loans, making a loss, which will reduce the return of remaining depositors, which then is an incentive for them to withdraw early. However, we do not even need actual withdrawals, if depositors expect such withdrawals, they would withdraw themselves, actual withdrawals are not required.
- [⇒] This is referred to as a bank run and is another equilibrium in which the banking system will break down.

→ While banks are an equilibrium, this equilibrium is vulnerable to expectations about the behaviour of other depositors.

- We have shown that banks are able to implement the social optimal for depositors seeking access to their funds while companies seek long term loans. But we have also found that this equilibrium can be easily disturbed and bank runs occur.

Optimality of banks

- Banks are implementing the social optimum to address the maturity mismatch
- Their existence is an equilibrium

- → Banks are optimal in terms of implementing the social optimum.
  - The role of banks here is to accommodate the maturity mismatch between short-term deposits and long-term loans and they do so in a socially optimal way.
  - Furthermore, this social optimum is an equilibrium.
- → Banks are using their ability to pool deposits to circumvent the problem of having to sell loans if early withdrawals happen. Their cash holdings allow for all those seeking to withdraw early, to do so without selling loans.

The threat of bank runs

- An alternative equilibrium with bank runs exists
- Bank runs cause banks to fail and impose high costs on economies
- This reduces the benefits of banks

The threat of bank runs

→ The equilibrium with banks is fragile, however as there is the threat of bank runs.

- While banks are an equilibrium, there is a second equilibrium which consist of a bank run.
- Such bank runs will force the bank to sell loans and will impose losses on depositors as loans are only sold at a loss. It also imposes high costs on the economy itself, not only on the depositors directly affected if the trust in banks is eroded and they cannot function properly by achieving the social optimum and overcome the maturity mismatch.
- Hence banks increase welfare compared to direct lending, but this welfare increase is limited by the reduction in welfare if bank runs occur.

→ Banks runs in well-managed banking systems are rare and it can therefore reasonably be argued that banks overall increase welfare.

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Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk