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Chapter 3.1

Maturity transformation of deposits

Outline

- Problem and model assumptions
- Social optimum
- Direct lending
- Direct lending with trading
- Bank lending
- Summary

Maturity mismatch

- Borrowers prefer long-term loans to meet the time horizon of their investments
- Depositors prefer the ability access their funds easily if needed

$\Rightarrow$ Banks need to be able to pay back deposits if requested, but lend out at long terms

- We show that bank lending provides the optimal solution to overcoming this maturity mismatch

Model specifications

- Loans are repaid after 2 time periods with probability π
- Depositors can withdraw either after 1 or 2 time periods, earning interest r_D^1 and r_D^2
- A fraction p of depositors withdraws in time period 1
- If banks have to raise cash to repay deposits, they only get a fraction λ of the loan value
- Depositor utility: $E\left[U\left(D\right)\right] = pu\left(\left(1+r_D^1\right)D\right) + \left(1-p\right)u\left(\left(1+r_D^2\right)D\right)$

Repaying deposits

- The cash held will be paid out for deposits withdrawn in time period 1
- $p\left(1+r_D^1\right)D = D - L$
- The loan repayments are used to repay the deposits left in time period 2
- $(1-p)\left(1+r_D^2\right)D = \pi\left(1+r_L\right)L$
- Combined: $p\left(1+r_D^1\right) + (1-p)\frac{\left(1+r_D^2\right)}{\pi\left(1+r_L\right)} = 1$

Optimal deposit rates

- First order condition of maximizing the utility of depositors gives $\frac{\partial u\left(\left(1+r_D^1\right)D\right)}{\partial\left(1+r_D^1\right)D} = \pi\left(1+r_L\right)\frac{\partial u\left(\left(1+r_D^2\right)D\right)}{\partial\left(1+r_D^2\right)D}$
- With the combined constraint this can be solved for the optimal deposit rates to be paid
- This social optimum is the benchmark with which we compare other lending arrangements

Wealth after early withdrawal

- If depositors lend directly, they will have the liquidated loan and cash if they want to consume in time period 1
- $\left(1+r_D^1\right)D = D - L + \lambda L$
- Depositors not liquidating their loan will in period 2 obtain their cash and the loan repayments
- $\left(1+r_D^2\right)D = D - L + \pi\left(1+r_L\right)L$

Stricter constraint

- Combining these two constraints we get $p\left(1+r_D^1\right)+(1-p)\frac{\left(1+r_D^2\right)}{\pi\left(1+r_L\right)}\leq 1$
- If $\lambda<1$ this constraint is stricter than the social optimum, where it was an equality

$\Rightarrow$ Direct lending is not optimal for depositors

Wealth after trading

- Loans can be sold at price P, rather than be liquidated at a loss
- If needing to liquidate the loan early to withdraw funds, it is sold: $\left(1+r_D^1\right) D = D - L + \pi \left(1 + r_L\right) LP$
- If keeping the loan, the cash can be used to buy additional loans: $\left(1+r_D^2\right) D = \frac{D-L}{P} + \pi \left(1 + r_L\right) L$

Loan price

- $P = \frac{1}{\pi(1+r_L)}$
- If $P > \frac{1}{\pi(1+r_L)}$ all deposits are invested into loans, as $\left(1 + r_D^1\right) D = D - L + \pi \left(1 + r_L\right) LP$ would increase in L

$\Rightarrow$ No cash remains to buy loans that are sold to raise cash for withdrawals

- If $P < \frac{1}{\pi(1+r_L)}$ all deposits are kept in cash, as $\left(1 + r_D^1\right) D = D - L + \pi \left(1 + r_L\right) LP$ would decrease in L

$\Rightarrow$ No loans are given

Market clearing

- Loans from those selling have to equal the cash kept by those not selling
- $pP\pi\left(1+r_L\right)L=\left(1-p\right)\left(D-L\right)$

$\Rightarrow$ $L=\left(1-p\right)D$

Constraints for optimum

- Repayments in time periods 1 and 2 become
  $\left(1 + r_D^1\right) D = D$
  $\left(1 + r_D^2\right) D = \pi \left(1 + r_L\right) D$
- The combined constraint is then $p \left(1 + r_D^1\right) + (1 - p) \frac{\left(1+r_D^2\right) D}{\pi(1+r_L)} = 1$
- The deposits rates are given and do not depend on the amount of lending
- First order conditions require $\frac{\partial u(D)}{\partial\left(1+r_D^1\right)} = \pi \left(1 + r_L\right) \frac{\partial u(\pi(1+r_L)D)}{\partial\left(1+r_D^2\right)}$
- This will only be fulfilled for a specific utility function

$\Rightarrow$ Direct lending with trading is not optimal

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- **Bank lending**
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Obtaining the social optimum

- All deposits are made with banks and the bank retains $p\left(1+r_D^1\right)D$ as cash to pay withdrawals, the remainder given as loans
- This recovers the social optimum as the arrangement is identical

$\Rightarrow$ Banks are optimal

Banking equilibrium

- We need $r_D^1 < r_D^2$ to prevent depositors withdrawing funds early
- The first order condition is $\frac{\partial u\left(\left(1+r_D^1\right)D\right)}{\partial\left(1+r_D^1\right)D} = \pi\left(1+r_L\right)\frac{\partial u\left(\left(1+r_D^2\right)D\right)}{\partial\left(1+r_D^2\right)D}$
- We need $\frac{\partial u\left(\left(1+r_D^1\right)D\right)}{\partial\left(1+r_D^1\right)D} > \frac{\partial u\left(\left(1+r_D^2\right)D\right)}{\partial\left(1+r_D^2\right)D}$ if $\pi\left(1+r_L\right) \geq 1$
- This is fulfilled if $r_D^1 < r_D^2$
- It is also not optimal for depositors to withdraw funds and force the bank to sell loans to raise cash, which depositors then buy
- Banks are an equilibrium outcome

Alternative equilibrium

- No depositor has an incentive to withdraw deposits if they do not need to
- But if they expect other depositors to do so, they have an incentive to withdraw to avoid losses

$\Rightarrow$ A bank run equilibrium exists

Optimality of banks

- Banks are implementing the social optimum to address the maturity mismatch
- Their existence is an equilibrium

The threat of bank runs

- An alternative equilibrium with bank runs exists
- Bank runs cause banks to fail and impose high costs on economies
- This reduces the benefits of banks

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