



Chapter 3.1  
Maturity transformation of deposits

# Outline

- Problem and model assumptions
- Social optimum
- Direct lending
- Direct lending with trading
- Bank lending
- Summary

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