



Chapter 14.1.2  
Coordination of deposit withdrawals

- Banks invest the deposits they obtain into loans that cannot be called in if the banks requires cash reserves.
- Banks would need to raise cash if they needed additional cash reserves, for example if more deposits than expected are withdrawn.
- Selling assets quickly to generate cash or obtaining loans will impose significant costs on banks, which might affect their ability to repay deposits.
- We will look at how depositors might want to withdraw deposits optimally, taking these losses banks make when raising cash reserves into account.

# Losses from withdrawing deposits

- ▶ Withdrawals of deposits can impose losses on banks as they need to raise additional cash reserves
- ▶ These losses can impact depositors if banks are not able to repay their deposits fully
- ▶ If such losses occur to depositors remaining with the bank, it can be rational for them to withdraw deposits, too
- ▶ This causes a bank run
- ▶ Coordinating the withdrawal of deposits can prevent a bank run and be beneficial for all depositors

# Losses from withdrawing deposits

- Banks always face the threat of deposits being withdrawn early and this can impose losses on banks and depositors alike.
  - ▶
    - If banks face the withdrawal of deposits, they face losses; these losses are not only reduced cash reserves, but actual losses.
    - These losses arise from the attempt to raise additional cash reserves to meet the demand for withdrawing deposits.
  - ▶ While losses are initially borne by the bank in the form of reduced profits, once losses become high, they will affect the ability of the bank to repay the deposits itself as the value of their assets will be depleted to such an extent that they are worth less than the deposits they have to repay.
  - ▶ If losses affect depositors remaining with the bank, but those withdrawing deposits are not affected or affected to a smaller extent, it can be rational for all depositors to withdraw.
  - ▶ We then have a bank run where depositors withdraw early only because others withdraw early; there is no fundamental reason for such a withdrawal.
  - ▶
    - If the decisions of depositors can be coordinated such that not all depositors withdraw if some withdraw early, we can prevent such a bank run to emerge.
    - This would benefit all depositors; those withdrawing early would obtain a larger repayment as the bank has to raise less cash reserves at a high cost. The same benefits would also apply to those not withdrawing early as they would obtain a repayment, including interest, at a later point of time.
- We will explore how such coordination of withdrawals can be achieved and under which conditions it will fail.

# Loan sales to finance deposit withdrawals

- ▶ Banks face withdrawals of deposits
- ▶ They repay these deposits from their existing cash reserve and the sale of loans, for which they obtain a fraction of their true value

- ▶  $R + \lambda\pi(1+r_L)\hat{L} = \gamma D$

$$\Rightarrow \hat{L} = \frac{\gamma D - R}{\lambda\pi(1+r_L)}$$

# Loan sales to finance deposit withdrawals

- We will firstly look at the amount of loans a bank needs to sell in order to be able to repay deposits that are being withdrawn.
- ▶ We consider a situation in which banks face the early withdrawals of deposits.
- ▶
  - These withdrawn deposits are repaid in the first instance using the cash reserves the bank holds.
  - As a next measure they sell some of the loans to raise additional cash reserves.
  - Given the speed with which loans have to be sold, banks are only able to obtain a fraction of their repayment value.
- ▶ The funds thus raised will match the funds needed to repay the fraction of deposits that are being withdrawn.
- ▶ [⇒] We can determine how many loans the bank needs to sell in order to repay all deposits.
- Knowing the required sales of loans, we can now determine at what point a bank would fail.

# Bank failure

- ▶ A bank fails if the remaining depositors cannot be repaid
- ▶ Banks obtain the repayments of the loans not sold, and the cash reserves, less the amount repaid for early withdrawals
- ▶ From this they need to repay the deposits of the remaining depositors
- ▶  $\pi (1 + r_L) (L - \hat{L}) + R - \gamma D < (1 - \gamma) (1 + r_D) D$

- We can now determine the situation in which a bank would fail.
- ▶ If a bank cannot meet all its obligations it is deemed to fail; here this means that the bank fails if it cannot repay all depositors, those withdrawing initially, but most importantly, those remaining with the bank.
- ▶
  - The funds available to the bank with which they can repay the remaining deposits are given by the repayment of the loans they have not sold.
  - In addition they have the cash reserved they originally had,
  - from which the early withdrawals have been repaid.
- ▶ Using these resources, they need to repay the remaining depositors, including the interest they are due.
- ▶ The bank would fail if their resources are not sufficient to repay all deposits.
- We can now see how bank failures can occur, depending on the size of deposit withdrawals.



# Insolvency with small deposit withdrawals

- ▶ If reserves are sufficient to cover the early withdrawal,  $\gamma \leq \frac{R}{D}$ , banks do not sell loans,  $\hat{L} = 0$
- ⇒  $\pi < \pi^* = \frac{(1+(1-\gamma)r_D)D-R}{(1+r_L)(D-R)}$
- ▶ If loans are very risky the bank does not generate enough profits to repay depositors and becomes insolvent

# Insolvency with small deposit withdrawals

- We initially look at the possibility of a bank failing if the deposit withdrawals are low.
- ▶ We first consider the case where deposit withdrawals are so low that the bank has sufficient cash reserves to repay them and hence does not need to sell any loans.
- ▶ [⇒] Using the bank failure condition from above in this case, we can rewrite the condition as the minimum repayment rate for loans that are required to avoid a bank failure.
- ▶
  - If loans are very risky the repayment of loans would not be sufficient to allow the bank the repayment of the remaining deposits.
  - This situation is an insolvency as the bank is not able to meet its obligations not because of the withdrawals of deposits, but because of losses arising from lending.
- For small deposit withdrawals, the bank can become insolvent if the loans are too risky.

# Illiquidity with large deposit withdrawals

- ▶ If withdrawal rates are high, banks may have to sell all loans,  $\hat{L} = L$
- ▶ Banks cannot pay all depositors withdrawing early if the money raised, plus cash reserves, is below the deposit withdrawals
- ▶  $R + \lambda\pi(1 + r_L)L < \gamma D$
- ⇒  $\pi < \pi^{**} = \frac{\gamma D - R}{\lambda(1 + r_L)(D - R)}$
- ▶ Bank are failing due to illiquidity

# Illiquidity with large deposit withdrawals

- We now turn to the opposite situation in that deposit withdrawals are very large.
- ▶ We assume that the deposit withdrawals are so high that in order to be able to meet this demand, the bank has to sell all loans.
- ▶
  - The total cash reserves would consist of the money raised from selling all loans,
  - in addition to the initial cash holdings.
  - If this amount is below the deposit withdrawals, the bank cannot repay all depositors withdrawing early.
- ▶ *Formula*
- ▶ [⇒] We can again solve this equation for the minimum repayment rate that a bank requires to avoid failing in this case.
- ▶ In this case, the bank does not fail because the risks they have taken are too high. The reason is that they cannot raise sufficient cash reserves to repay all depositors withdrawing early; thus the bank is illiquid.
- Hence for large deposit withdrawals, banks fail due to illiquidity.

# Inability to repay remaining depositors

- ▶ For intermediate deposit withdrawals, banks sell some of their loans,  $0 < \hat{L} < L$
- ⇒  $\pi < \pi^{***} = \frac{(\lambda(1+(1-\gamma)r_D)+\gamma)D-(1+\lambda)R}{\lambda(1+r_L)(D-R)}$
- ▶ The bank can repay depositors withdrawing early, but the losses from selling loans are too large to allow them to repay the remaining depositors in full

# Inability to repay remaining depositors

- We can now consider the intermediate case of deposit withdrawals being such that some of the loans need to be sold by the bank, but not all.
- ▶ We assume that banks sell some loans in order to raise cash reserves for the early withdrawal of deposits.
- ▶ [⇒] The amount of loans that needs to be raised can be determined from the previous slide by setting the amount raised equal to the deposits withdrawn; this is then inserted into the condition for the bank failing and we obtain the minimum repayment rate of loans to avoid bank failure as indicated in the *formula*.
- ▶
  - The bank sells loans and this raises sufficient cash reserves to allow the bank repaying all depositors withdrawing early.
  - The losses the bank incurs from selling the loans below their value will prevent the bank from having sufficient funds to repay all the remaining depositors. Hence the bank will fail once the remaining depositors are to be repaid; the bank is insolvent not because the loans in itself are too risky, but because losses from selling loans have been incurred.
- We can now analyse the behaviour of the remaining depositors in more detail. If they know they are not going to be repaid in full, we will analyse whether it is optimal for them to withdraw early as well.

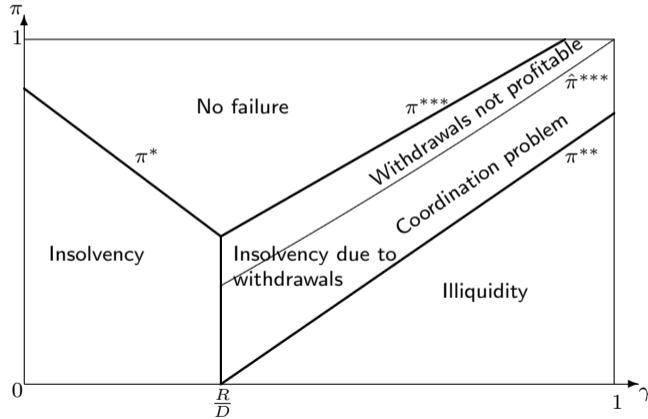
# Profitable to withdraw deposits

- ▶ The remaining depositors obtain from the bank repayments of the loans not sold, and the cash reserves, less the amount repaid for early withdrawals
  - ▶ As early withdrawal does not pay any interest, they would be worse off than those depositors if the repayments shared are below the face value of the deposit
  - ▶  $\pi (1 + r_L) (L - \hat{L}) + R - \gamma D < (1 - \gamma) D$
- $\Rightarrow \pi < \hat{\pi}^{***} = \frac{(\lambda + \gamma)D - (1 + \lambda)R}{\lambda(1 + r_L)(D - R)}$
- ▶ Depositors are better off withdrawing early than remaining with the bank, causing a bank run
  - ▶ Coordinating to not withdraw deposits would benefit all depositors

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- ▶
    - The remaining depositors will be repaid as much as the bank can repay, which is the repayment obtained from the loans the bank has not sold.
    - They also make use of their cash reserves.
    - but had to repay the deposits withdrawn early.
  - ▶ If this amount is less than the remaining deposits, then it would be better for these depositors to withdraw immediately.
  - ▶ This can again be transformed into the minimum repayment rate that is required to avoid the remaining depositors to also withdraw.
  - ▶
    - If the risk of the bank is higher than this threshold, the remaining depositors would withdraw early as well.
    - As in this case all depositors withdraw, we observe a bank run.
  - ▶ Avoiding a bank run is beneficial for all depositors, as the bank would not need to sell more loans, which would cause more losses to the bank and reduce the payment to all depositors. The bank and their depositors would benefit from a way to coordinate the behaviour of depositors to avoid this situation of it being individually rational to withdraw deposits, but this causing all depositors to suffer additional losses.
- We can now analyse the different cases we have discussed graphically.



# Bank failures due to deposit withdrawals



# Bank failures due to deposit withdrawals

- We will summarize our results to see when banks are failing due to the early withdrawal of deposits
- ▶ We investigate the failure of banks for different combination of early deposit withdrawals and the riskiness of the loans banks give.
- ▶ We firstly look at small deposit withdrawals, where we had seen that low success rates lead to the insolvency of the bank as not all deposits remaining with the bank can be repaid.
- ▶ Large deposit withdrawals will lead to the illiquidity of the bank if not sufficient cash reserves can be raised to repay all the deposits withdrawn early.
- ▶ Finally we have for intermediate withdrawals of deposits that banks can repay deposits withdrawn early, but then have accumulated too many losses to be able to repay those deposits remaining with the bank.
- ▶ We have also seen that if the repayment rate of loans is sufficiently high, those depositors not seeking to withdraw early would find it optimal to retain their deposits, despite not being repaid in full. For lower repayment rates, a coordination problem emerges in that it would be better for all remaining depositors to not withdraw, but they would find this not optimal individually. If all could agree to not withdraw deposits, then this would be preferred.
- ▶ In all other cases, the bank would be able to repay all depositors fully and thus not fail.
- We this see that banks face failure not only if their loans are too risky, but also if depositors withdraw early. In some instances, individual rationality leads to a bank run, even though it would be socially optimal for depositors to not withdraw.

# Summary

- ▶ Banks taking high risks face insolvency or illiquidity as they cannot raise enough cash to withstand early deposit withdrawals
- ▶ Banks taking low risks and facing low deposit withdrawals will not fail
- ▶ Banks between these extremes can raise enough cash for early deposit withdrawal, but might not have the resources to repay the remaining depositors
- ▶ It can be beneficial for depositors to withdraw early and cause a bank run
- ▶ This presents a coordination problem as all depositors not withdrawing would be beneficial for all

- We can now summarize under which conditions banks are failing and when a bank run might be observed.
- ▶ We have seen that, not surprisingly, banks taking high risks face insolvency. They may also face illiquidity as the value of their loans is so small that they cannot raise sufficient funds to withstand the early withdrawal of deposits.
- ▶ Those banks that take low risks and do not face significant early deposit withdrawals will not fail. Here the bank has sufficient resources to repay all deposits at all times.
- ▶
  - The interesting case emerges for banks that might be able to withstand an initial withdrawal of deposits,
  - but the losses this imposed on the bank through selling loans below their true value, will then make it impossible to repay loans that have remained with the bank.
- ▶
  - In this case it can be beneficial for the remaining depositors to also withdraw early to limit their losses.
  - This would then be a bank run as all deposits are withdrawn from the bank.
- ▶ Such bank runs could be avoided if the decisions of the remaining depositors could be coordinated such that they all would retain their deposits; in this case they would receive a higher repayment. It is, however, individually rational to withdraw deposits.
- Bank runs are individually rational, but socially not optimal. The range in which such bank runs can occur is small and hence, bank runs might be rare phenomenon.



This presentation is based on  
Andreas Krause: Theoretical Foundations of Banking, 2025

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