

Chapter 8.1

The consequences of uncertain outcomes

- Banks will in general not know the out come of investment companies make, while they may form expectations, there is uncertainty surrounding these expectations.
- The uncertainty might affect the return the investment generates or the probability of success.
- We will investigate what the implication of such uncertainty is for the provision of loans.

Financing investments

- ▶ Companies can fund investment using debt and equity
- ▶ Companies choose the optimal combination of these funding sources
- ▶ With uncertain outcomes, banks cannot be sure to be repaid their loan
- ▶ Larger loans imply larger repayments, which requires higher outcomes

Financing investments

- Investments by companies can be financed through debt and equity. The choice of financing has implications for the provision of loans.
 - ▶
 - Companies need funds for investment, and we often assume that these funds are provided by banks through loans.
 - However, in addition to obtaining loans, companies will also be able to use their own funds, equity to finance investments.
 - ▶ There is generally an optimal combination of debt and equity that companies choose; this combination is commonly referred to as the capital structure and in corporate finance theories about the optimal capital structure have been developed. Here we take this decision by companies as given.
 - ▶ Given that outcomes from the investment are uncertain, banks do not know how much, if any, if their loans are repaid.
 - ▶
 - The idea is that a larger loan will obviously also result in a larger repayment.
 - In order to repay such larger loans, the outcome has to be larger, We assume here that the total investment remains constant and companies breach the gap to the loan amount provided through equity.
- We can now look at the optimal size of the loans a bank would provide.

Company profits

- ▶ Companies retain the outcome once the loan has been repaid, taking into account their own investment

- ▶ $\Pi_C = \int_{(1+\bar{r}_L)L}^{+\infty} \pi(1+R) L dF(\pi(1+R)L) - E$

- ▶ Companies seek loans if $\Pi_C \geq 0$, which gives a maximum loan rate \bar{r}_L

$$\Rightarrow \frac{\partial(1+\bar{r}_L)}{\partial L} = - \frac{\frac{\partial \Pi_C}{\partial L}}{\frac{\partial \Pi_C}{\partial(1+\bar{r}_L)}} = \frac{1-(1+\bar{r}_L)^2 L f(\pi(1+\bar{r}_L)L)}{(1+\bar{r}_L)L^2 f(\pi(1+\bar{r}_L)L)}$$

- ▶ The isoprofit curve has a negative slope

- We will first determine the demand of companies for loans and its properties.
 - ▶
 - The company retains the net outcome of the investment,
 - But only after the loan has been repaid; for lower outcomes the company obtains no return and defaults.
 - From this surplus, the investment of the company using its own funds, its equity, needs to be deduced.
 - ▶ *Formula*
 - ▶ For companies to demand loans, they need to make a profit, which implies that there will exist a maximum loan rate that can be charged by the bank.
- ⇒ We can now look at the relationship between the maximum loan rate and the loan amount. The maximum loan rate is given when the profits are zero and we can determine the isoprofit curve by totally differentiating the company profits and solving this for the slope of the isoprofit curve.
 - ▶ The denominator of this expression will be positive and the numerator negative as $1 + \bar{r} > 1$, and the loan amount will be large. If we assume that the density of the outcome just covering the loan repayment at this maximum loan rate is not too small. Hence we will have that the slope of the isoprofit curve is negative; this implies a negative slope of the demand curve for loans in the loan rate.
- Having determined the demand for loans we will now seek to determine the supply of bank loans.

Bank profits

- ▶ Banks obtain the outcome if the loan cannot be repaid and are repaid for higher outcomes, up to the maximum loan rate at which companies demand loans, and repay deposits

$$\begin{aligned}\text{▶ } \Pi_B &= \int_0^{(1+r_L)L} \pi(1+R) L dF(\pi(1+R)L) \\ &\quad + \int_{(1+r_L)L}^{(1+\bar{r})L} (1+r_L) L dF(\pi(1+R)L) - (1+r_D)L\end{aligned}$$

$$\Rightarrow \frac{\partial \Pi_B}{\partial (1+r_L)} = (F((1+\bar{r}_L)L) - F((1+r_L)L))L > 0$$

$$\frac{\partial \Pi_B}{\partial L} = (F((1+\bar{r}_L)L) - F((1+r_L)L))(1+r_L) + \frac{1+r_L}{1+\bar{r}_L} - (1+r_D)$$

- ▶ If L is small, the first term will be small, the two final terms are negative, making this negative
- ▶ If L is large, the first term will be small, the two final terms are negative, making this negative
- ▶ If L is intermediate, the first term will be positive, the two final terms are negative, allowing this to be positive

→ We derive the supply of banks loans through banks optimizing their profits

- ▶
 - If the loan cannot be repaid in full, thus the outcome of the investment is below the loan repayment, the bank seizes the investment outcome to obtain a partial loan repayment.
 - If the outcome of the investment is sufficiently high, the bank obtains the full loan repayment.
 - This can only be achieved up to the maximum loan rate as for higher loan rates, no loans are demanded.
 - Banks finance loans using deposits, which also have to be repaid.

· *Formula*

⇒ We can now take the first derivative of the profits with respect to the loan rate.

· We can now take the first derivative of the profits with respect to the loan size.

- ▶
 - For a small loan size the differences between these two expressions will be small and close to zero.
 - The fraction will be less than 1 as the loan rate will be below the maximum loan rate and the final term will exceed 1, making this negative.
 - This will make the whole expression negative.
- ▶
 - For a large loan size the differences between these two expressions will be small and close to 1.
 - The fraction will be less than 1 as the loan rate will be below the maximum loan rate and the final term will exceed 1, making this negative.
 - This will make the whole expression negative.
- ▶
 - For an intermediate loan size the differences between these two expressions will be positive .
 - The fraction will be less than 1 as the loan rate will be below the maximum loan rate and the final term will exceed 1, making this negative.
 - With the positive first term, this expression can be overall positive.

→ We can now use this result to determine the supply function for loans.

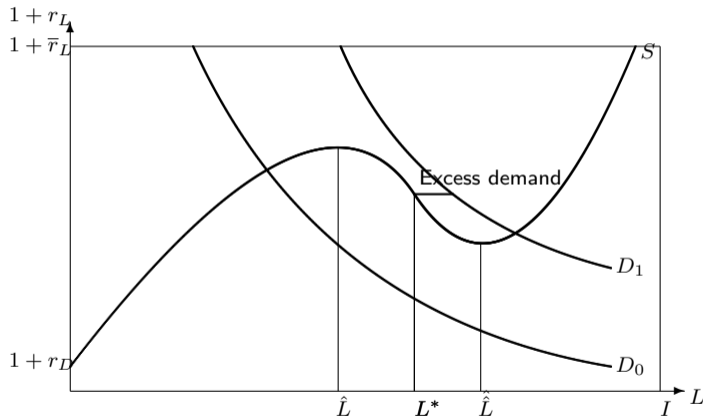
Non-monotonous supply curve

- ▶ Isoprofit curve of the bank: $\frac{\partial(1+r_L)}{\partial L} = -\frac{\frac{\partial \Pi_B}{\partial L}}{\frac{\partial \Pi_B}{\partial(1+r_L)}}$
 - ▶ Positive for small loans, negative for intermediate loans, positive for large loans
 - ▶ Optimal profits are at $\frac{\partial \Pi_B}{\partial(1+r_L)L} = 0$
- $\Rightarrow 1 + r_D = (F((1 + \bar{r}_L)L) - F((1 + r_L)L))(1 + r_L)$
- $\Rightarrow \frac{\partial \Pi_B}{\partial L} = \frac{1+r_L}{1+\bar{r}_L} > 0$
- \Rightarrow Maximal profits where the supply curve is decreasing

Non-monotonous supply curve

- We can now proceed to determine the profits of banks and from that obtain the supply curve.
- ▶ The isoprofit curve of the bank is given from taking the total differential at a given loan rate. This can be solved for the slope of the isoprofit curve, which is also the slope of the supply curve.
- ▶
 - Using the slopes of the denominator as discussed above, the slope of the supply curve is positive for small loans.
 - Using the slopes of the denominator as discussed above, the slope of the supply curve is negative for intermediate loans.
 - Using the slopes of the denominator as discussed above, the slope of the supply curve is positive for large loans.
- ▶ Banks seek to maximize their profits over the optimal amount that is repaid, and this requires the first order condition to be fulfilled.
- ⇒ Conducting this maximization, we can solve this first order condition for the deposit rate.
- ⇒ Inserting this deposit rate into the marginal profits for the loan size, we see that the expression will be positive.
- ⇒ This implies that the slope of the supply curve at the maximal profits for banks is negative.
- We can now proceed to determine the equilibrium between demand and supply.

Demand and supply



Demand and supply

- We can now determine the properties of the equilibrium and apply some comparative statics using a graphical analysis.
- ▶ We will look at the relationship between the loan size and the loan rate, taking the maximum loan rate and deposit rate as given.
- ▶ The supply curve is increasing for small and large loan and decreasing for an intermediate range as depicted here.
- ▶ The maximum profits for banks were shown to be in the area having a negative slope of the supply curve. Any loans larger than L^* would not be optimal and the bank would not supply them, thus the supply in green area is not relevant.
- ▶ If demand is low, the bank cannot obtain its optimal loan amount as the demand is not sufficient and instead the equilibrium is where demand and supply meet.
- ▶ If the demand is high, the demand and supply would intersect in the green area, but the bank would not meet that high demand as it can generate higher profits at L^* . Hence demand and supply are not equal.
- ▶ Applying the optimal loan rate and loan size, excess demand will emerge. We thus have an equilibrium which exhibits excess demand.
- As not the full loan as demanded is given, loans are rationed and this is referred to as credit rationing.

Summary

- ▶ If loan demand is high, credit rationing can occur
- ▶ Banks will reduce loan repayments for large loans as otherwise they will not be repaid, increasing losses from default
- ▶ Increasing loan rates does not compensate for this risk if loans are not repaid
- ▶ Uncertainty about loan repayments induces banks to limit the size of loans and maximize repayments
- ▶ Companies seeking a large loan might be rationed and only obtain a smaller loan than they seek

- We can now summarize the key results of this model.
- ▶ We have seen that for low loan demands demand and supply are equal, but for high loan demands, credit rationing might occur.
- ▶
 - By restricting the loan size, the loan repayments overall are limited.
 - This is because large loans would not be repaid, or repaid with a low likelihood.
 - This would increase losses from defaulting companies.
- ▶ Increasing the loan rate does not compensate for this risk, as there is no benefit if the loan itself is not repaid; the losses remain.
- ▶ The central element is the uncertainty about the loan repayments which makes it optimal for banks to not provide too large loans and limit their exposure to this risk. With companies financing the remainder of the investment with equity, an additional buffer for smaller investment returns is given.
- ▶ The consequence is that if loan demand is high, companies might only get a fraction of the loan they apply for. Increasing the loan demand strategically to obtain a larger allocation is not effective, as the loan size the bank gives, is fixed at L^* .
- Hence the uncertainty of investment outcomes can induce credit rationing as it is optimal for banks to not provide too large loans.



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