Andreas Krause

Chapter 9.2.1

Identifying company types through collateral

- Companies might provide collateral in order to reduce the loan rate, but this puts their collateral at risk if the investment does not succeed.
- Although often banks insist on collateral to reduce their risks, it might be that companies voluntarily offer such collateral, while other would not make such an offer.
- We will look at the which companies would provide collateral and how this allows banks to identify the company type from this choice.

Collateral use

- Banks do not always know the risks companies are taking, but companies might be more aware of their own risks
- Setting loan rates too high to account for high-risk companies, might crowd out low-risk companies due to the lower return they obtain from their investment
- Collateral can be used to distinguish between the two types of companies, allowing for loan rates to be set according to the risk of the company

Collateral use

→ Companies might offer collateral to banks to reduce the costs of loans. We will see how such an offer can be used as information on the risks of the company by the bank.

- While banks assess the risks of companies they seek to lend to, they will not be certain about the risk and often there are many factors affecting the risk that banks will be unable to assess. For example, soft information on management skills are not easily assessed by banks.
- Companies will generally have a better idea about the risks involved given their familiarity with their business, management, and the market.

- If banks over-estimate risks or price loans with caution due to their uncertainty and charge a higher loan rate for that reason, they might charge too much for low-risk companies that generally do not achieve high returns due to the positive risk-return relationship. In this case low-risk companies might be driven out of the loan market and the bank faces only companies with higher risk, causing an adverse selection problem.

- We will see how collateral can be used to distinguish between companies of high and low risk.
- Being able to distinguish between companies of different risks, allows banks to set loan rates that suit the risks of the companies, avoiding adverse selection.

→ We will now formally develop a model which tales into account the effect of collateral on the profits of companies and banks.

Company profits

- Companies obtain their investment return and repay the loan if they are successful, if they are unsuccessful they lose their collateral
- $\Pi_C^i = \pi_i \left((1+R)\, L - \left(1 + r_L^i\right) L \right) - (1-\pi_i)\, C_i$
- The isoprofit curve of companies is given by $d\Pi_C^i = -\pi_i L dr_L^i - (1-\pi_i)\, dC_i = 0$

$\Rightarrow \frac{dr_L^i}{dC_i} = -\frac{1-\pi_i}{\pi_i L}$

- We find a negative trade-off between the loan rate and the size of the collateral

Company profits

→ We first look at the profits a company pledging collateral would generate.

- Companies use the loan to make an investment on which they will receive some return.
  - This return allows the company to repay its loans with interest, retaining the difference.
  - Investment returns are only generated if the investment is successful and only in this case the loan can be repaid.
  - If the investment is not successful, the company loses the collateral it has pledged.

- *Formula*
- We can now determine the isoprofit curve by taking the total differential. We can then determine the combinations of loan rates and collateral that give companies the same profits.

⇒ This can be solved for the relationship between the loan rate and collateral use by showing the trade-off between these two variables.

- The result is a straight line with a negative slope such that a higher collateral would require a lower loan rate to maintain the same level of profits. The lower loan rate that companies pay in case of success compensates the company for the larger losses it makes when losing the collateral in case they are not successful.

→ Having determined the relationship between the loan rate and collateral for companies, we now turn to the profits generated by banks.

Bank profits

- Banks obtain collateral, but they can only sell this with a discount λ
- If the investment of the company is successful, the loan is repaid, if the investment is not successful, the bank obtains the value of the collateral
- Loans are fully financed by deposits
- $\Pi_B^i = \pi_i\left(1 + r_L^i\right) L + \left(1 - \pi_i\right) \lambda C_i - \left(1 + r_D\right) L$
- The isoprofit curve of banks is given by $d\Pi_B^i = \pi_i L dr_L^i + \lambda \left(1 - \pi_i\right) dC_i = 0$

$\Rightarrow \frac{dr_L^i}{dC_i} = -\lambda \frac{1-\pi_i}{\pi_i L}$

- The slope is negative, but flatter than the isoprofit curve for companies

Bank profits

→ In a similar way to company profits, we now determine bank profits and the relationship between loan rates and collateral.

- Banks obtain a collateral from the company they lend to. The value of this collateral is less than the value it has for the company as the bank will have no direct use for this collateral, but will have to sell it. Such a sale is costly, may take time and the full value might not be realised; we therefore assume that the collateral value to be only a fraction of the value to the company.
-
  - The bank will be repaid the loan only if the company's investment is successful; otherwise the bank will receive no repayment.
  - If the investment of the company is not successful and therefore the loan is not repaid, the bank will seize the collateral and realise its value.
- The bank faces costs in that the loan is financed by deposits, which will have to be repaid after the repayment of the loan.
- *Formula*
- We can now determine the isoprofit curve by taking the total differential. We can then determine the combinations of loan rates and collateral that give banks the same profits.

⇒ This can be solved for the relationship between the loan rate and collateral use by showing the trade-off between these two variables.

- The result is a straight line with a negative slope such that a higher collateral would allow only for a lower loan rate to maintain the same level of profits. The lower loan rate in case the company is successful and repays the loan is necessary to compensate for the larger collateral in case the company is not successful.
- The result is a straight line with a negative slope such that a higher collateral would require a lower loan rate to maintain the same level of profits. This negative slope is smaller than that of companies due the banks not being able to realise the full value of the collateral.

→ Having determined the profits of companies and banks, as well as the relationship between loan rates and thew amount of collateral, we can now analyse the resulting equilibrium loan contracts.

Bank offering a single contract only

- Banks know the composition of low-risk and high-risk companies, the average success rate is $\pi = p\pi_H + (1-p)\,\pi_L$

$\Rightarrow \Pi_B^P = \pi\,(1+r_L)\,L + (1-\pi)\,\lambda C - (1+r_D)\,L$

$\Rightarrow \frac{dr_L}{dC} = -\lambda\frac{1-\pi}{\pi L}$

- This slope is between banks only lending to high-risk and low-risk companies
- If this bank is competitive then we require $\Pi_B^P = 0$

$\Rightarrow 1 + r_L = \frac{1+r_D}{\pi} - \frac{1-\pi}{\pi}\frac{C}{L}$

Bank offering a single contract only

→ Thus far we assumed that banks know the risk of companies. We will now assume that this is not the case and banks cannot distinguish between companies of different risks.

- We assume that there are two type of companies in the market, high-risk and low-risk companies. The bank is not able to distinguish between them, but it knows the composition of the market.
- Knowing the fraction of high-risk and low-risk companies, the bank can obtain the expected risk it would take when lending to a company without any further information.

⇒ We can now replace the success probability in the profits of banks with this average success probability and obtain the expected profits of banks.

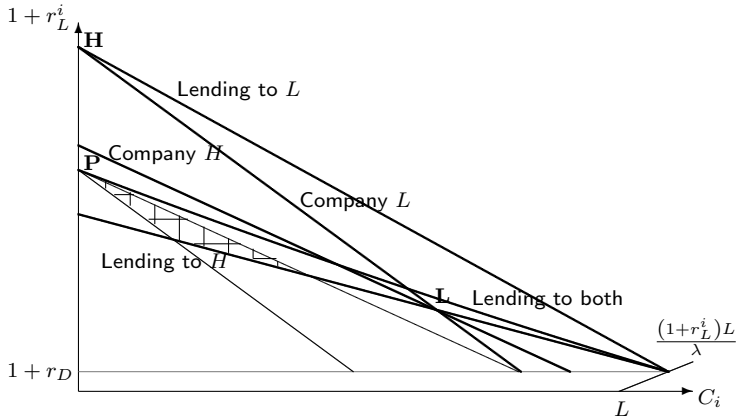
⇒ As before, we can determine the isoprofit curve and thus determine the relationship between the loan rate and collateral size.

- The slope of this relation ship is between that of lending only to a high-risk or a low-risk company as $\pi_H \geq \pi \geq \pi_L$.
- If we assume that banks are competitive, then their profits will be zero.

⇒ This can be solved for the resulting loan rate.

→ We can now analyse potential equilibria for the loan rate and collateral provision. We will do so graphically rather than algebraically.

Separating equilibrium with collateral



Separating equilibrium with collateral

→ We will now see how an equilibrium emerges in this model. We will first assume that banks know the type of company and then later show how they can distinguish them to make this assumption valid.

- We will look at combinations of the loan rate and collateral that are viable for the company and the bank.
- Firstly, we consider situation where the value of the collateral to the bank fully covers the loan amount, including interest. In this case the bank is always fully repaid and a higher collateral will not be rationally provided by the company.
- If the bank always is repaid its loan in full, and markets are competitive, the loan rate will be the costs of the bank, which is the costs of its deposits used to finance the loan.
- We introduce isoprofit curve of the bank showing the relationship between the loan rate and the collateral at which the bank makes zero profits, where we consider the company with a low success rate (high risk) first.
- The same relationship is presented for this company, where we know that the slope is steeper. We have shown the isoprofits curve that shows the highest company profits; companies will seek the lowest loan rate and the lowest collateral, hence seek the isoprofit curve that is closest to the origin in this graph. The isoprofit curve of the company has to cross the isoprofit curve of the bank making no profits to be feasible. Thus the line shown here is the best possible like for companies.
- The lines cross at this point H where no collateral is required.
- We now show the isoprofit curve of the bank lending to the company with a high success rate (low probability of default). We can show that this slope is lower than that of the company with a low success rate.
- Of course the best solution for the company would again be to provide no collateral, but this would not allow the bank to distinguish between company types. Hence we choose a different approach. The bank offers the company a contract that crosses its isoprofit curve marginall to the right of the isoprofit curve of the other company. This point is the best point at which the contract of this company is not attractive to the other company, who would prefer contract H as the isoprofit curve is marginally closer to the origin there.
- Hence the low-risk company will be offered a contract at point L, requiring collateral. This company finds point H less attractive as it is further from the origin. Thus the bank can distinguish the two types of companies, the high-risk company does not provide collateral, while the low-risk company does.
- Let us now consider the case where a bank is only offering a single contract and lends to both using the same contract. The isoprofit curve will be located between the two types of companies.
- The isoprofit curves of both companies cross at the point where no collateral is provided.
- Hence we would have an equilibrium is point P without collateral. This contract P is more attractive to both types of companies. Thus a bank offering a common contract would attract all companies.
- In the shaded area, a bank could offer a contract that is more attractive to the low-risk companies, generates excess profits to the bank, but which find high-risk companies less attractive. Hence a bank could enter and offer such a contract requiring collateral and low-risk companies would switch, making high-risk companies stay at P, which then becomes loss-making as it consists of only high-risk companies. This would require the bank to move the equilibrium to H and the other bank would move to L. Hence if P is chosen, a bank offering separating contracts can enter the market.

Banks offering distinct contracts

- If banks are competitive, they make zero profits as long as they follow the same strategy: all offer two contracts (separating) or all offer one contract (pooling)
- If a bank offers a single contract, another bank can offer a marginally better contract near $\mathbf{P}$
- This bank would obtain all low-risk companies, but no high-risk companies

$\Rightarrow \Pi_B^* = \pi_H (1 + r_L) L - (1 + r_D) L = (1 - p) \frac{\pi_H - \pi_L}{\pi} (1 + r_D) L$

- The other bank would be left with high-risk companies only

$\Rightarrow \Pi_B^{**} = \pi_L (1 + r_L) L - (1 + r_D) L = -p \frac{\pi_H - \pi_L}{\pi} (1 + r_D) L$

Banks offering distinct contracts

→ Banks can now react to only a single contract being offered and we will be looking at what implications this has for their profits.

▶
- In perfect competition, banks offering the same goods (contracts) will make no profits.
- They may offer two contracts, one with and one without collateral requirements for a separating equilibrium.
- Alternatively they may offer a single contract without collateral for a pooling equilibrium.

▶ A bank can offer a contract which is marginally better than the pooling contract P in the hatched area. This contract is marginally better in that it requires a very small collateral and charges a loan rate minimally lower. We can neglect such small amounts and state that this contract still uses no collateral and the same loan rate as the contract in P.

▶
- This contract would attract all low-risk companies as it is a marginally better contract, resulting in a marginally higher profit.
- The contract would not be attractive to high-risk companies, who would remain at the contract in P

⇒ The bank profits in this case are given as in the *formula*, without collateral and the loan rate is the loan rate from the pooling equilibrium as given above, where we note that only low-risk companies are provided with loans, π_H. Inserting this loan rate gives us the expression in the *formula* and we see easily that this expression is positive.

▶ The bank offering the contract at P would be left with the high-risk companies only.

⇒ The bank profits in this case are given as in the *formula*, without collateral and the loan rate is the loan rate from the pooling equilibrium as given above, where we not that only high-risk companies are provided with loans, π_L. Inserting this loan rate gives us the expression in the *formula* and we see easily that this expression is negative.

→ We can now determine the equilibrium of the decisions by the two banks whether to offer a single contract without collateral or two contracts, one with and the other without collateral requirements.

Strategic choice of loan contracts

Bank 2 \ Bank 1	pooling	separating
pooling	$0, 0$	Π_B^*, Π_B^{**}
separating	Π_B^{**}, Π_B^*	$0, 0$

$\Pi_B^* > 0 > \Pi_B^{**}$ $\quad$ $\Pi_B^* > 0 > \Pi_B^{**}$

Strategic choice of loan contracts

→ We now consider the strategic interaction between two banks regarding their choice of contract they offer.
- Each bank can either offer a single contract (pooling) or two contracts (separating).
- Due to perfect competition, if both banks offer the pooling contract, they will make zero profits.
- The same is true if both companies offer separating contracts.
- If bank 1 offers a separating contract while bank 2 offers a pooling contract, we can use the previous results to determine the respective profits of the banks.
- A symmetric outcome emerges if the roles of banks 1 and 2 are reversed.
- We can now analyse the resulting choices by banks. We first look at the decision of bank 1 if bank 2 chooses the pooling equilibrium. We now that in this case offering a separating contract will generate the bank a positive profit.
- Therefore bank 1 would choose the separating equilibrium.
- If bank 2 were to choose the pooling equilibrium, we know that the bank would make a loss.
- Therefore bank 1 would choose the separating equilibrium.
- We now consider the decision of bank 2 in the same way. If bank 1 were to choose the pooling contract, it would make a profit from offering the separating contract.
- Therefore bank 2 would choose the separating equilibrium.
- If bank 1 were to choose the separating contract, it would make a loss from offering the pooling contract.
- Therefore bank 2 would choose the separating equilibrium.
- It is therefore that the only equilibrium contract is for both banks to offer separating contracts.

→ We have thus established that in equilibrium, banks will offer separating contracts. Low-risk companies will choose the contract requiring collateral, while high-risk companies will choose the contract without collateral requirements. Even though the bank cannot distinguish the two companies using its information, their choice of contract allows the banks to distinguish between them from their choice of contract. Thus collateral can be used to identify the riskiness of companies as they themselves perceive it.

Summary

- Collateral can be used to separate companies taking different risks
- Low-risk companies will offer collateral in exchange for a lower loan rate
- High-risk companies will not offer collateral, but pay a higher loan rate
- Banks will offer contracts with and without collateral for companies to choose

Summary

→ Collateral is more than a means to reduce the risks to banks by providing them with a (partial) repayment of the loans in case the investment of the company is not successful; it may be used to provide information to the bank.

- We can use the decisions by companies on the provision of collateral to distinguish companies of different risks, even if we cannot differentiate between them otherwise.
- We have seen that low risk companies, those with high success rates for their investments, will offer a collateral and obtain a lower loan rate. This arrangement is beneficial for them as the reduction in the loan rate is worth more to them than the potential loss of the collateral due to the low likelihood of the collateral actually being lost.
- For high-risk companies, those with low success rates, the provision of collateral is not attractive as the benefits of the lower loan rate are not sufficient; the high likelihood of losing the collateral combined with the low likelihood of actually repaying the loan at the higher loan rate, makes the contract without collateral more attractive.
- We have also seen that it is an equilibrium for banks to offer two such contracts. Providing only a single contract to all companies exposes them to adverse selection if another bank enters to offer separate contracts.

→ Collateral can play a role in reducing adverse selection between banks and the companies they lend to. The decision of companies whether to provide collateral will reveal information about the risks they are taking with their investments that might otherwise not be available to banks.

This presentation is based on
Andreas Krause: Theoretical Foundations of Banking, 2025

Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk