UNIVERSITY OF BATH

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ES22016 Introduction to finance

Indicative answers to seminar problems

The problems provided as part of this module are designed to apply the theories learned to practical and realistic scenarios, allowing students to apply their knowledge and practice their ability to explain real-world events using economic theories in plain English. Seminars are dedicated to discussing these problems, but we are not able to discuss all problems due to time constraints. Having additional problems allows students to practice their knowledge in preparation of the assessment; they can compare their solutions with the indicative answers provided and for any additional clarifications attend the office hours.

The below table gives an indication about the problems to be discussed in class for each topic. At times it will be not be possible to discuss each problem and at other times it might be possible to discuss an additional problem if time permits. It is expected that students are prepared to discuss all problems in each seminar.

	Seminar problems
Topic 1	1, 2
Topic 2	3, 5
Topic 3	7, 10
Topic 4	11, 14
Topic 5	15, 17
Topic 6	20, 22
Topic 7	23, 26
Topic 8	28, 29
Topic 9	33, 36
Topic 10	39, 40
Topic 11	9, 22
Topic 12	72, 74
Topic 13	90, 96
Topic 14	128, 130
Topic 15	142, 179
Topic 16	2, 3
Topic 17	10, 36
Topic 18	16, 20
Topic 19	26, 38
Topic 20	43, 48

The problems for topics 11 to 20 are taken from the respective textbooks and the problem numbers refer to the numbers of the problems therein. These solutions to these problems are not included in this booklet, but are provided separately.

The difficulty of problems will vary, as the difficulty of questions in the exam will vary, to allow for an assessment of the degree to which the learning outcomes have been met and the final mark to reflect the standards achieved. The questions discussed in the seminar will therefore be a mix of more easy and more difficult questions. Furthermore, some problems will require the application of more than one model for a

complete answer, but these are not necessarily more difficult than problems requiring the use of only a single model.

Topic 1

Problem 1

An investor who is risk-averse would always choose the option that gives the lowest risk.

Is this statement true?

Indicative answer: *This statement is not correct. Being risk averse does not mean that an investor would not take on any risk, it merely implies that investors need to obtain compensation for taking on any risk, the risk premium; the more risk averse an investor is the higher the risk premium. The risk premium comes in the form of a higher expected return than a risk-free choice could provide. Hence if the risk premium, the expected return, is sufficiently high, a risk averse investor would choose a risky option.*

Problem 2

Carefully explain the difference between moral hazard and adverse selection.

Indicative answer: *A situation that is characterised by the decision-maker having a choice of actions has the potential for moral hazard; such choices might include investments with different characteristics or the exertion of effort. Moral hazard emerges if the chosen action is in the interests of the decision-maker , but other market participants who are directly involved would prefer another action. As an example, a bank might prefer the borrower to make a low-risk investment, while the company prefers this risk to be high. In contrast, if one party of the parties agreeing an exchange are better informed than the other party, this has the potential for adverse selection. The better informed party might be able to extract more surplus from the less informed party, which is referred to as adverse selection. A better informed investor might be able to sell an asset to a less informed investor at a price he knows to be too high. In adverse selection, the losses to one party do not arise out of choices of individuals (effort, investment), but are the consequence of having different information and there are being different types of assets or economic environments. Both, moral hazard and*

adverse selection can lead to a breakdown of the market; if individuals are aware its presence, they know they will make losses from interacting with other market participants and will therefore not enage in a transaction.

Topic 2

Problem 3

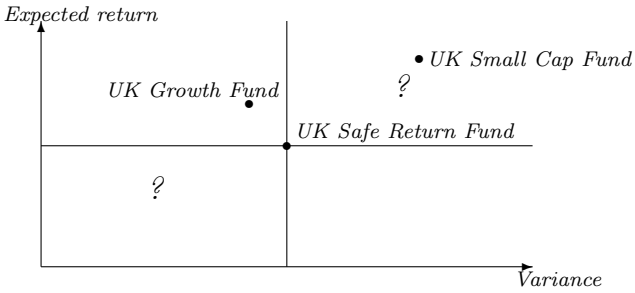
Michael Tippet is an independent financial advisor and while discussing investment options with a client, he points out three possible investment funds that might be suitable for the needs of his client. The first fund he has picked out, UK Small Cap Fund, invests into small listed companies and in the past has shown an annual return of 12% and a volatility in returns of 27%. Another fund, called UK Safe Return Fund, invests mainly in utility stocks and other defensive stocks, generating an annual return of 7% with an annual volatility of 13%. The final fund under consideration, UK Growth Fund, has a different investment strategy in that it not only invests into stocks, but also bonds issued by companies. Its annual return and volatility are 8% and 11%, respectively. As financial advisor, he believes that market conditions are stable and expects similar returns to those observed in the recent past also in the future from all funds. His client tells him that he finds the investment strategy followed by UK Safe Return Fund appealing and wants to consider this fund further.

a. As his financial advisor, should Michael Tippet recommend to invest into UK Safe Return Fund?

b. Recently, Michael Tippet has advised a different client and given that client's circumstances, he proposed to invest 40% of the monies considered into UK Small Cap Fund and 60% into UK Growth Fund. His current client dislikes risk more than his previous client, how would he change the recommended portfolio for his current client?

Indicative answer:

a. UK Safe Return Fund has a lower expected return than UK Growth Fund and at the same time a lower volatility (risk). Applying the mean-variance criterion, the fund should not be considered any further as it will be inferior to UK Growth Fund. Going forward, only the UK Small Cap Fund and UK Growth Fund should be considered. The below figure illustrates this and we see that UK Safe Return Fund is inferior and can thus be disregarded as long as we restrict our investment to one fund. Were we able to invest into two or three funds, the correlation of the funds with each other would have to be considered and we might not be able to exclude UK Safe Return Fund.



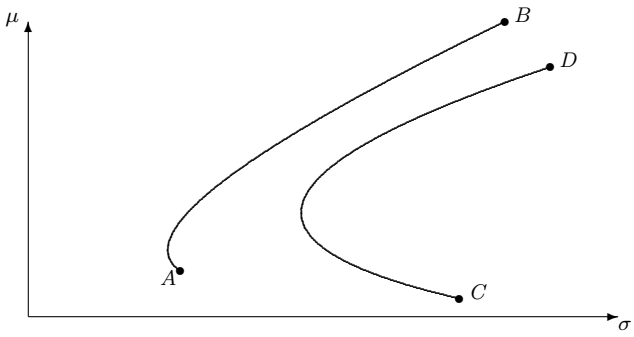
b. He would have to recommend a lower allocation to the UK Small Cap Fund, and consequently a higher allocation into UK Growth Fund. If an investor is more risk averse, like in this case, he will seek to lower the risk he takes; this can only be achieved if the allocation to the high-risk asset, UK Small Cap Fund, is reduced and that of the asset with the lower risk, UK Growth Fund, increased.

Problem 4

During a discussion with family members, your older brother asserts with his usual confidence that diversification of investments is always beneficial. His claim is that if you only have a small number of assets you can invest in, say two, then you choose the two that have the lowest correlation as that is best for diversification.

Is your brother's claim correct?

Indicative answer: *Diversification is beneficial as it reduces risks, but this has to be weighed against the risks and returns these assets generate. It is easily the case that two assets with low risks and high returns, but a high correlation are better than two assets with high risks and low returns, but a low correlation. Hence, while diversification reduces risks and is generally better, this cannot be generalised when comparing portfolios comprising of different assets with different risks and returns. In the below illustration, assets A and B have a high correlation, while assets C and D have a low correlation, but any portfolio of assets A and B are superior to a portfolio consisting of assets C and D.*



Problem 5

Georgina Warren seeks advice on her investments from her long-standing investment advisor, Janice Young, with the aim of investing into stocks included in the FTSE100. Unknown to Janice Young, she has sought a second opinion from a different investment advisor. Comparing the datasheets she has obtained from both advisors about the stocks included in the FTSE100, Georgina Warren observes that both advisors agree to a large degree on the risks and returns of each stock. The recommended investments, however, differ substantially between both advisors. Janice Young recommends to invest 85% of Georgina Warren's wealth into the stock market and 15% into government bonds, while the other advisor suggested a mix of 60% to 40%. The portfolio of stocks also differs between both advisors substantially; Janice Young recommends to invest in a small number of stocks only, while the new advisor recommends a roughly equal investment into all stocks contained in the FTSE100.

Can you explain these differences in recommendations using portfolio selection theory?

Indicative answer: *The differences in the allocation into the risky assets, the FTSE100 stocks, and the risk-free asset, government bonds, might be explained through different assessments of Georgina Warren's risk aversion. If Janice Young assess her to be less risk-averse than the new advisor, this different allocation might be justified. Both advisors agree on the risks and returns of the risky assets, and thus they should agree on the composition of the risky portfolio, the optimal risky portfolio consisting of the FTSE100 stocks. Here they do not agree on this portfolio, even though they agree on the properties of all assets, this is inconsistent with portfolio selection theory. As their respective optimal risky portfolios differ, this might drive the difference in the allocation between risky and risk-free assets, rather than different assessments of Georgina Warren's risk aversion, assuming that the optimal risky portfolio of Janice Young has a lower risk than that of the second advisor. One possible explanation for the differences in the optimal risky portfolio could be that one advisor considers short sales in the stocks, while the other does not. This difference would result in different*

optimal risky portfolios.

Problem 6

At a reception in the aftermath of a meeting of financial advisors, Thomas Hartford complains to colleagues that his clients are generally not interested in his complete advice. He points out that he always recommends a selection of stocks, or occasionally other risky assets, to invest in, in addition how much their client should put into safe assets such as government bonds. In most cases, however, his clients only take up his recommendation on the stocks and other assets, ignoring his recommendation to invest into government bonds, even though this is the part he probably puts most effort in as this is very individual advice. His colleagues say that they recognises this and always wonder why clients ignore an important part of the advice.

a. Why is it important to consider the advice on investing into government bonds?

b. Why is the advice on investment into government bonds individualised, but the advice on investing into other assets not?

Indicative answer:

a. The optimal portfolio consists of the optimal risky portfolio, the stocks and other assets clients accept advice for, and a risk-free asset, the government bank. Investing into a combination of risky and risk-free assets increases the utility of investors and by failing to invest into risk-free assets they do not maximize the risk-return relationship that is available to them.

b. As long as the assets considered for investment are identical across investors, the optimal risky portfolio will be identical for all investors as it is independent of their preferences; therefore financial advisors do not need to re-calculate the optimal risky portfolio for each investor. The optimal portfolio is a combination of the optimal risky portfolio and a risk-free asset. This combination will depend on the preferences of the investor, most notably the risk aversion. Financial advisors need to assess the risk aversion of each investor to obtain the optimal portfolio and hence the weight of the risk-free asset has to be determined individually for each investor.

Topic 3

Problem 7

Estimating the Capital Asset Pricing Model for a range of large stocks listed on the London Stock Exchange, you have chosen the FTSE 100 index as your market portfolio. A friend advises that it is better to use the FTSE 350 index or even the FTSE All-Share index as your market portfolio because they cover a wider range of stocks. You argue that as you only consider stocks included in the FTSE 100 index, this is the best match for your purposes.

a. Is your friend's suggestion an improvement on your approach?

b. Is either of the mentioned indices the true market portfolio?

c. Is the true market portfolio identical for all investors?

Indicative answer:

a. *The market index should encompass all risky assets that are available to investors. As the proposed indices, the FTSE 350 and FTSE All-share index, include more companies, they would be a better benchmark from which to estimate the Capital Asset Pricing Model. The market portfolio is used as all assets must be held and hence all assets are included in the assessment of the systematic risk, which forms the basis of the Capital Asset Pricing Model. Which specific assets are considered for the purposes of estimating β_i from the model, is irrelevant in this context.*

b. *The true market portfolio consists of more than stocks, it will include a wide variety of other assets, such as corporate bonds, real estate, commodities, cryptoassets, investments into human capital, and much more; all these constituents are not included into any stock market index. The FTSE All-Share index does even only cover those stocks listed on the London Stock Exchange, which makes a small fraction of stocks globally. Therefore, the FTSE All-Share or FTSE 350 indices are not the true market portfolio.*

c. *Investors will have different investment opportunities, even in the absence of any constraints on investments, such as capital controls by some countries or other investment restrictions. The most obvious difference would be the investment into human capital, it would realistically only be possible to invest into your own education and knowledge. It would take extraordinary markets that would allow*

to invest into the education and knowledge of other people and obtain the rewards of this investment exclusively without the person educated reaping any rewards. It is thus that even in very well developed markets, a single market portfolio cannot exist.

Problem 8

You seek to determine the expected return of assets you consider for investment, but want to establish which risk factors are relevant to them and what their influence is. You have identified a number of firm-specific factors that affect the returns of assets, but also various macro-economic and political factors you deem relevant. Using Arbitrage Pricing Theory, you have then estimated the influence of each of these factors for a variety of assets, giving you then the expected return for each asset. A friend tells you that you make your life too difficult, he always just uses the Capital Asset Pricing Model and he is sure that it makes hardly any difference, because, after all, the influence of the market portfolio simply aggregates all the other factors that you considered.

Is you friend correct in his assessment?

Indicative answer: *Your friend is partially correct in that the market portfolio will be influenced by the factors you have identified. However, the influence on the market portfolio will be the (weighted) average influence these factors have on individual assets. Therefore, the β_i of the Capital Asset Pricing Model will reflect how much each individual asset deviates from this average influence. This will, however, be the an imperfect indicator of the influence as it only allows for a deviation from the average influence. Suppose a stock is influenced by several factors in the same way as the market, but in two factors it deviates. Consider the case where the influence of these two factors is positive for the market as a whole, but for the asset positive for one factor and negative for the other. If the two factors is highly positive, the return of the market is increased and the return of the asset should increased. However, in reality, the influence of the two factors cancel each other partially out and the influence is much less pronounced. The Capital Asset Pricing Model is not able to capture such subtleties and these deviations in influences compared to the market portfolio would only be captures in the idiosyncratic risk. Therefore the Arbitrage Pricing Theory provides a more precise framework.*

Problem 9

Your financial advisor suggests you invest in 16 stocks only, which he has handpicked for their future prospects. He points out that his chosen stocks are representative of the market as a whole, covering all the major industries that are represented. Being very conscious about the risk of stock investments, you worry about the lack of diversification given that well over 1,000 stocks are listed on the London Stock Exchange alone.

Are you right to question the approach taken by your financial advisor?

Indicative answer: *As the stocks selected are constituting a cross-section of the stock market with all major industries represented, it is reasonable to assume that the correlations between these stocks will be relatively low and be close to the average correlation within the stock market. This should allow for a large degree of diversification, which limits the level of idiosyncratic risk. Such risk will be present and not completely diversified and hence the level of risk of this portfolio could be reduced through including more stocks. However, even if concerned about risk, there are costs associated with trading large number of different stocks, fixed trading costs on the one hand, but also the costs of remaining informed about the future prospects of these stocks. These costs have to be balanced against the benefits of holding more stocks.*

Problem 10

Consider a market in which 50 stocks are traded. You have established that the average volatility of stocks is 12% p.a. and that the average correlation between stocks is 0.7. Your aim is to hold an equally weighted portfolio that consists of as few 'average' stocks as possible, but has a total risk that is no more than 10% above the systematic risk of the market.

How many stocks do you need to hold?

Indicative answer:
Identification of parameters: *Average variance:* $\overline{\sigma}_i = 0.12$, *Average covariance:* $\overline{\sigma}_{ij} = \overline{\sigma}_i^2 \times 0.7 = 0.01008$
Portfolio variance: $\sigma_P^2 = \frac{1}{N}\left(\overline{\sigma}_i^2 - \overline{\sigma}_{ij}\right) + \overline{\sigma}_{ij} = \frac{0.12^2 - 0.01008}{N} + 0.01008 = \frac{0.00432}{N} + 0.01008$
Maximum portfolio variance: $\sigma_P^2 \leq 1.1 \times \overline{\sigma}_{ij} \Rightarrow \frac{0.00432}{N} + 0.01008 \leq 1.1 \times 0.01008 \Rightarrow N \geq 4.2857$
Final result: *The investor needs to hold at least 5 stocks to achieve a total risk that is no higher than 110% of the systematic risk.*

The number of stocks required to achieve a significant reduction in total risk is usually very small, giving credence to the approach of carefully selecting wisely only a small number of stocks to achieve good diversification.

Topic 4

Problem 11

The shares of Trafalgar plc. trade at 431p and you are seeking information whether it is advisable to invest into this company. When collecting information about the company you establish that it has paid a dividend of 13p during the last year, having grown its dividend by 4% each year in the past and you believe that the company will able to maintain this growth for the foreseeable future. You have further determined the volatility of the shares as 22% p.a. and its correlation with a broad market index is 0.82. This market index is expected to increase by 8% p.a. and has shown a variance of 0.04 and treasury bills yield 3.5% p.a.

a. Would you recommend to purchase shares of Trafalgar plc.?

b. Would your answer change if the dividend were to grow at 5% each year?

c. How do you explain that such a small difference between the growth rates affects your results significantly?

Indicative answer:

a. *Identification of parameters: Dividend:* $D_t = 13$, *growth rate:* $g = 0.04$, *risk-free rate:* $r = 0.035$, *Stock variance:* $\sigma^2 = 0.22^2 = 0.0484$, *market variance:* $\sigma_M^2 = 0.04$, *market return:* $\mu_M = 0.08$, *Correlation with market:* $\rho = 0.82$
Determining expected stock return: Use the CAPM: $\mu_i = r + \beta_i (\mu_M - r)$, $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\sqrt{\sigma^2}\sqrt{\sigma_M^2}\rho}{\sigma_M^2} = \frac{0.03608}{0.04} = 0.902 \Rightarrow \mu_i = 0.035 + 0.902\,(0.08 - 0.035) = 0.07559$
Stock value: $P_t = \frac{1+g}{\mu_i - g} D_t = \frac{1+0.04}{0.07559-0.04} 13 = 380$
Final result: As the current stock price exceeds its value, the stock is not a good investment and buying it is not recommended.

b. *Change required: All results from part a. can be reused as only the growth rate of dividends changes to* $g = 0.05$.
Stock value: $P_t = \frac{1+g}{\mu_i - g} D_t = \frac{1+0.05}{0.07559-0.05} 13 = 528$
Final result: As the current stock price is below its value, the stock is a good investment and buying it is recommended.

c. *The value of the stock increases by nearly 40% when increasing the expected growth rate of dividends from 4% to 5%. The reason is that while the growth rate of dividends is only slightly higher, the cumulative effect is substantial. This is*

especially true if the growth rate g is close to the discount rate μ_i as in this case future dividends are growing at nearly the same rate as the discount rate, making their present value remain nearly constant year after year, adding substantial value to the stock.

Problem 12

Jones & Co. seek to expand their business and have drawn up three possible avenues for investment:

- Expanding their current business in the domestic market would require an investment of £100m and they expect to generate profits giving a return on investment of 11% each year for the foreseeable future. Having discussed this investment with their bank, the bank has agreed to grant a loan of £60m at a loan rate of 8% p.a.
- They could expand in an overseas market, investing £70m, which would generate an initial return on investment of 7%, but their profits are expected to grow by 4% every year. Their bank would provide a loan of £35m at a loan rate of 9% p.a.
- The final investment opportunity would be to diversify their business by expanding into new markets, which would require an investment of £110m, to which the bank would lend £65m at a loan rate of 9.5% p.a. The investment is expected to yield a return of 11.5% p.a. for the foreseeable future.

For these investments they are able to raise up to £100m in equity.

If Jones & Co. face equity costs of 12% p.a., what would be your advice on their investments?

Indicative answer: *The investments need to be evaluated using Net Present Value. and then the optimal combination of investments chosen.*
Weighted Average Cost of Capital: Cost of equity: $\mu = 0.12$
Investment 1: Investment: $I_1 = 100$*, debt:* $D_1 = 60$*, equity:* $E_1 = I_1 - D_1 = 100 - 60 = 40$*, loan rate:* $r_L^1 = 0.08 \Rightarrow R_1 = \mu\frac{E_1}{D_1+E_1} + r_L^1\frac{D_1}{D_1+E_1} = 0.12\frac{40}{60+40} + 0.08\frac{60}{60+40} = 0.096$
Investment 2: Investment: $I_2 = 170$*, debt:* $D_2 = 35$*, equity:* $E_2 = I_2 - D_2 = 70 - 35 = 35$*, loan rate:* $r_L^2 = 0.09 \Rightarrow R_2 = \mu\frac{E_2}{D_2+E_2} + r_L^2\frac{D_2}{D_2+E_2} = 0.12\frac{35}{35+35} + 0.09\frac{35}{35+35} = 0.105$
Investment 3: Investment: $I_3 = 110$*, debt:* $D_3 = 65$*, equity:* $E_3 = I_3 - D_3 = 110 - 65 = 45$*, loan rate:* $r_L^3 = 0.095 \Rightarrow R_3 = \mu\frac{E_3}{D_3+E_3} + r_L^3\frac{D_3}{D_3+E_3} = 0.12\frac{65}{65+45} + 0.095\frac{45}{65+45} = 0.1098$
Net Present Values: Profits: $\Pi_1 = 0.11 \times 100 = 11 \Rightarrow NPV_1 = \frac{\Pi_1}{R_1} - I_1 = \frac{11}{0.096} - 100 =$

14.58
Profits: Growth rate of profits: $g = 0.04$, *Initial profits:* $\Pi_2 = 0.07 \times 70 = 4.9 \Rightarrow NPV_2 = \frac{1+g}{R_1-g}\Pi_2 - I_2 = \frac{1+0.04}{0.105-0.04}4.9 - 70 = 8.40$
Profits: $\Pi_3 = 0.115 \times 110 = 12.65 \Rightarrow NPV_3 = \frac{\Pi_3}{R_3} - I_3 = \frac{12.65}{0.1098} - 110 = 5.21$
Assessment of individual investments: All investments have positive Net Present Value and all investments should be made.
Choice of investments: The company cannot finance all investments as it cannot raise sufficient equity. The maximum that can be raised is £100m and the investments require equity of £40m, ££35m, and £45m, respectively. We can therefore at most make two of these investments and should choose those two that give the highest Net Present Value when combined. In this case this is investment 1 and investment 2.
Final result: Jones & Co. should expand their current business in their domestic market and expand overseas.

Problem 13

You observe that after an increase in the stock price the next price change of the stock is more likely to be positive than negative. After a decrease in the stock price, you find that the next price change is more likely to be positive than negative. You conclude that the market cannot be efficient as whatever the last observation of the price change, the price is more likely to increase than decrease.

a. Is your conclusion correct, assuming you have performed appropriate statistical test supporting your observation?

b. If instead, you observed that after a decrease in the stock price the next price change is more likely to be negative than positive, would your conclusion hold?

Indicative answer:

a. You cannot make this conclusion. Market efficiency implies that the returns of stocks are uncorrelated over time, it makes no claims about the signs of any stock movements. The average return of a stock is positive, hence it will be more likely to observe positive price changes than negative price changes, as in this case. What is relevant is the correlation of returns, thus any variations around this trend of the stock price. Thus from your observations you cannot conclude that the market is inefficient, but you can also not conclude that it is efficient.

b. Your observation suggests serial correlation as after a positive return, positive returns are more likely and after a negative return another negative return is

more likely. Therefore we have a positive serial correlation, which contradicts the implications of efficient markets and, provided statistical tests show a sufficiently high level of significance, you can conclude that the market in this stock is not efficient.

Problem 14

You have two companies that compete in the same market, facing comparable competitive forces. Despite them being so similar, the profits of the listed company Pillmeyer AG has grown at a rate of 4% p.a., while Hertig KG, which is privately owned, has only grown at 3% p.a. With the current owners of Hertig KG seeking to sell their company, they need to determine its value to be able to set an appropriate price. As Pillmeyer AG is listed on the stock exchange, they know that its value is €117m and according to their latest annual report they generated profits of €4.5m. Internal numbers show that Hertig KG showed profits of €1.7m.

What is the value of Hertig KG?

Indicative answer:
Identifying the problem: To value Hertig KG we need to have information on the current profits $D_2 = 1.7$, the growth rate of these profits, $g_2 = 0.03$, and the discount rate of future profits, the expected return on the equity of the company. This information s not given. As both companies are identical and subject to the same market forces, it is reasonable to assume that the expected returns on both companies are identical. We would thus turn to Pillmeyer AG and use their expected return, but this is also not given. However, we can derive this expected return.
Obtaining the expected return: For Pillmeyer AG we have $P_1 = \frac{1+g_1}{\mu_1 - g_1} D_1 \Rightarrow 117 = \frac{1+0.04}{\mu_1 - 0.04} 4.5 \Rightarrow \mu_1 = 0.08$, assuming that the expected returns for both companies are identical, we have $\mu_1 = \mu_2 = 0.08$.
Value of Hertig KG: $P_2 = \frac{1+g_2}{\mu_2 - g_2} D_2 = \frac{1+0.03}{0.08-0.03} 1.7 = 35.02$
Final result: The value of Hertig KG is €35.02m.

Topic 5

Problem 15

You observe the following term structure on government bonds:

Maturity (years)	1	2	3	4	5	6	7	8	9	10
Yield (% p.a.)	3.24	3.31	3.57	3.82	4.13	4.41	4.93	5.02	5.52	5.97
Maturity (years)	11	12	13	14	15	16	17	18	19	20
Yield (% p.a.)	6.08	6.12	5.86	5.68	5.52	5.35	5.27	5.22	5.20	5.19

a. What is the expected yield of a 3-year bond in 4 years' time?

b. How do you explain the change of the yield the 3-year bond experiences, given your answer to part a.?

c. In how many years' time do you expect short-term yields to decline?

d. In how many years' time do you expected the yield 5-year bonds to decline?

Indicative answer:

a. *Determining the relevant bonds: The bond in question matures in 7 years time, hence we need to choose the return on the 7-year bond:* $(1+0.0493)^7$. *This return needs to be equal to that of a 4-year bond bought now,* $(1+0.0382)^4$, *followed by a 3-year bond, whose yield is unknown:* $(1+r)^3$
Solution: $(1+0.0493)^7 = (1+0.0382)^4 (1+r)^3 \Rightarrow r = 0.0643$
Final result: The bond in question is expected to have a yield of 6.43%.

b. *The yield of 3-year bonds is expected to increase from 3.57% to 6.43%, this is due to the yield curve being increasing, implying that investors expected yields to increase over time.*

c. *An increasing yield curve implies that yields are expected to rise, while a decreasing yield curve implies that they are expected to decrease. The yield curve obtains a negative slope after 12 years, hence we would expect short term yield to decrease again in 12 years time.*

d. *The slope of the yield curve over a time period of 5 years becomes negative after 10 years. Hence yields on 5-year bonds are expected to fall in 10 years time.*

Problem 16

A bank has obtained deposits to the amount of \$50m from a wealthy individual and agreed to pay interest of 5.5% p.a. for a period of 2 years, while the current level of interest rates for all maturities is 5.25%. It can grant loans at the current level of interest to companies with repayments to be scheduled in 5 years. In order to completely eliminate any interest rate risk, how much loans do they need to provide to companies?

Indicative answer: *We need to employ a duration-based hedge. For this, we need to obtain the duration of bonds, which in turn requires the 'bond' value. We derive these first for a nominal value of 100. The coupon payment are then the interest rate agreed multiplied by the nominal value.* Identification of parameters: $C_1 = 0.055 \times 100 = 5.5$, $r_1 = 0.0525$, $T_1 = 2$, $C_2 = 0.0525 \times 100 = 5.25$, $r_2 = 0.0525$, $T_2 = 5$
'Bond' values: Deposits: $B_1 = \frac{5.5}{1+0.0525} + \frac{5.5}{(1+0.0525)^2} + \frac{100}{(1+0.0525)^2} = 100.46$
Loans: $B_2 = \frac{5.25}{1+0.0525} + \frac{5.25}{(1+0.0525)^2} + \frac{5.25}{(1+0.025)^3} + \frac{5.25}{(1+0.0525)^4} + \frac{5.25}{(1+0.0525)^5} + \frac{100}{(1+0.0525)^5} = 100.00$
Duration: $D_1 = \frac{\sum_{\tau=1}^{T_1} \tau \frac{C_1}{(1+r_1)^\tau} + T_1 \frac{100}{(1+r_1)^{T_1}}}{B_1} = \frac{\frac{5.5}{1+0.0525} + 2\frac{5.5}{(1+0.0525)^2} + 2\frac{100}{(1+0.0525)^2}}{100.45} = 1.9481$
$D_2 = \frac{\sum_{\tau=1}^{T_2} \tau \frac{C_2}{(1+r_2)^\tau} + T_2 \frac{100}{(1+r_2)^{T_2}}}{B_2} = \frac{\frac{5.25}{1+0.0525} + 2\frac{5.25}{(1+0.0525)^2} + 3\frac{5.25}{(1+0.0525)^3} + 4\frac{5.25}{(1+0.0525)^4} + 5\frac{5.25}{(1+0.0525)^5} + 5\frac{100}{(1+0.0525)^5}}{100} = 4.5470$
Weight of loans: $\omega_2 = \frac{D_1}{D_1 - D_2} = -0.7496$
Final result: *As deposits are liabilities to banks loans are assets, having different signs, doe snot imply a short position in loans. The total amount of loans that need to be given is then* $-0.7496 \times 50 = 37.4793$. *The total value of loans that need to be given is \$37.4793m. The amount of loans given is less than the deposits obtains as the loans have longer durations and are therefore more sensitive to interest rate changes.*

Problem 17

As an intern at Khalili Bank in the country of Avar, you are asked to provide a brief statement for a client meeting on the likely economic conditions in the next few years. You have only recently moved to Avar and are very much unaware of the economic situation. However, you observe that the yield curve has an inverse hump shape, i. e. it initially decreases and from year 4 onwards increases again.

What would be your conclusions?

Indicative answer: *The yield curve gives an indication about the future development of interest rates. The initially decreasing yield curve suggests that short-term*

interest rates are falling and in 4 years time they will start to increase again. This indicates that over the next four years the market expects the central bank to lower interest rates before than increasing them. Applying basic macroeconomic theory, this suggests economic growth is slowing down or a recession is entered from which Avar will emerge in approximately four years.

Problem 18

A pension fund has long-term liabilities due to commitments of future pension payments. It determines the value of these commitments by discounting at the currently prevailing interest rate for 25-year government bonds. Until pension payments have to be made, the pension fund has invested a considerable part of their reserves into Treasury Bills and comparable short-term debt instruments. They valued the instant access these investments guaranteed, without being too much affected by changing interest rates. The yields on long-term bonds have been falling recently and the pension fund made considerable losses as a consequence.

a. Why does the pension fund suffer losses as long-term interest rates fall?

b. How can the pension fund reduce the risk of losses from future decreases in the long-term interest rate?

Indicative answer:

a. A lower discount rate, here the interest rate of the long-term bonds, increases the value of the 'bond'. The 'bond' in this case is a liability of the pension fund, hence the value of their liabilities increase. This is not matched by a comparable increase in the value of assets; they are short-term bonds and as such will have shorter duration, so even if short-term interest rates were falling, the increase in the value of assets would be smaller. Hence, the value of liabilities will increase more than the value of assets, akin to debt increasing without the value of assets increasing. Thus overall the net-value of the pension fund will decrease, causing it make losses.

b. If matching the duration of assets to the duration of liabilities, increases in the value of liabilities will be much better matched by an increase in the value of assets, reducing losses. Therefore, the pension fund should commence investing their reserves into long-term bonds rather than short-term bonds.

Topic 6

Problem 19

Steaucescu SRL has purchased new machinery to upgrade and expand their current production facilities. The machinery was purchased from a manufacturer in France for a price of €12m. The main currency at which Steaucescu SRL invoices their customers is the Romanian Leu and due to recent considerable volatility of their currency, they seek to hedge their payment to their French supplier, which is due in 4 months. The current exchange rate is 5 Leu/€ and the interest rates in Romania are 7.2% p.a., while in France they are 4.8% p.a.

a. What is the forward rate that Steaucescu SRL would obtain?

b. Explain why the forward rate differs from the spot rate.

Indicative answer:

a. *Identification of variables: underlying asset: is the Euro, so the current exchange rate is* $S_t = \frac{1}{5} = 0.2$*, financing rate* $r_L = 0.072$*, yield on underlying asset* $r_S = 0.048$*, time to maturity is to be given in years to make it consistent with the interest rates* $T = \frac{4}{12} = \frac{1}{3}$
Forward rate: $K = S_t + (r_L - r_S)\,TS_t = 0.2 + (0.072 - 0.048)\,\frac{1}{3}0.2 = 0.2016$
Final result: The forward rate would be 4.96 Leu/€.

b. *The forward rate is lower than the spot rate as the interest rate in the Romanian Leu is higher then in the Euro. This allows investors to gain higher profits from investing into the Romanian Leu than the Euro and this difference is compensated for by a lower forward rate, which can be interpreted as the equivalent of a depreciation of the Romanian Leu.*

Problem 20

RiskConsult LLP is a hedge fund that invests extensively in small listed companies that are mostly neglected by investors; their current portfolio encompasses holdings

in approximately 150 of such companies. They have achieved annual returns of 11% with a volatility of 18%, while the market showed an annual volatility of 27% and generated only 7%, although some investors were not satisfied with their investment strategy as it showed a correlation of 0.8 with the market, limiting their diversification benefits from investing into RiskConsult LLP. With some of their investments being financed through a loan, which is due to be repaid in three months time, RiskConsult LLP seeks to lock in any gains they have made to avoid having to sell holdings at a loss in the future. Due to agreements with their investors, they are not able to sell these stocks and retain the proceeds in cash as they normally would do. They have set aside a representative part of their portfolio, valued at \$15m, that they seek to sell for the repayment of the loan.

a. Which instrument is most suitable to hedge the exposure of RiskConsult LLP?

b. If only derivatives on the market are available, how many contracts would you need to obtain for your hedging strategy in part a.?

Indicative answer:

a. RiskConsult LLP should hedge their exposure through selling a futures. Their preferred strategy is to sell the stocks instantly, which is not possible, hence selling them in the future at a price already fixed now is the equivalent as it give RiskConsult LLP a set price agreed now and the sale can be delayed until the loan needs to be repaid.

b. We need to determine the hedge ratio of the futures identified in part a. The hedge ration requires the β_i of the portfolio, which can be determined from the information given: $h_i = \beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{0.18 \times 0.27 \times 0.8}{o.27^2} = 0.5333$. If the total position to be hedged is worth \$15m, then the value of the underlying market portfolio on which is the futures is based will have to be $0.5333 \times 15 = 8$. Thus RiskConsult LLP needs to sell futures for the equivalent of \$8m of the market portfolio.

Problem 21

Seven years ago, a company has agreed a £25m loan with a bank at a variable loan rate that is adjusted every 3 months to the SOFR + 0.25%. As the company has mainly fixed obligations, but at the time could not find a bank offering competitive loan rates on 10-year bonds, they entered a swap agreement with their bank, which had an initial time to maturity of 10 years, matching the terms of their loan. The swap rate at the time of the agreement was 7.5% p.a. and the current term structure

for government bonds is

Maturity (years)	SOFR	1	2	3	4	5	6	7	8	9	10
Yield (% p.a.)	2.96	3.24	3.31	3.57	3.82	4.13	4.41	4.93	5.02	5.52	5.97

The company is generally regarded as safe by investors due to their market position and safety of business and attracts a risk-premium of 0.25% p.a..

If the interest rate on their loan has just been reset, what is the current value of their swap?

Indicative answer: *We need to determine the value of fixed-rate bond they have obtained and the variable-rate bond they have swapped.*
Value of variable-rate bond: As the bond has only recently been reset, it will be worth its face value, which for convenience set at $B_{var} = 100$ *for now.*
Value of fixed rate bond: The bond has only 3 years left, so the relevant discount rate is the 3-year bond yield , plus the risk premium the company is charged, the bond agreed had a swap rate of 7.5% and this will be the coupon payments on this bond: $B_{fix} = \frac{7.5}{1+0.0357+0.0025} + \frac{7.5}{(1+0.0357+0.0025)^2} + \frac{7.5}{(1+0.0357+0.0025)^3} + \frac{100}{(1+0.0357+0.0025)^3} = 110.2473$
Value of the swap $V = B_{fix} - B_{var} = 110.2473 - 100 = 10.2473$
Final result: The value of the swap overall has to be based on £25m, not 100, hence the total value £2.5618m

Problem 22

As part of their investment strategy, Alliance Insurance plc has developed a small division that purchases swaps and forwards from selected banks who have agreed such contracts with their clients, but may want to divest for reasons of risk management. A junior trader is observing the negotiation between an experienced trader and her counterparty at the bank. They are discussing the terms of a USD-GBP currency swap for a nominal \$100m, which the bank has agreed for one of their leading customers. This currency swap has a remaining time to maturity of 8 years. The junior trader is astounded to hear that the insurance company has eventually bought the swap and received a payment of \$4m for purchasing this swap; he has never seen a situation where a security is bought and the seller pays the purchaser.

How do you explain this agreement?

Indicative answer: *Swaps (as forwards and futures) can have a negative value. In this specific case, the value of the swap would be given by* $V - B_{domestic} - eB_{foreign}$ *and this can become negative if the value of the foreign 'bond' increases more than the value*

of the domestic 'bond', adjusted for any changes in the exchnage rate. Such a scenario is realistic if the interest rate of the foregein 'bond', here is USD bond, decreases mosre than that of the domestic 'bond', the one denominated in GBP, and any interest rate changes are not fully reflected by an adjustment of the exchange rate. The negative value arises in this case as the purchaser pays a higher interest rate on this foreign bond than is the current market rate, thus it overpays and would be better off issuing the bond at the current market rate, relative to the amount the purchaser receives on the domestic bond. This implicit overpayment in the swap needs to be compensated for and for the seller to get out of the swap agreement, where he overpays on the foreign bond, he is willing to offer any purchaser compensation. This compensation is then used by the purchaser to offset the overpayments they will have to make for the remainder of the swap.

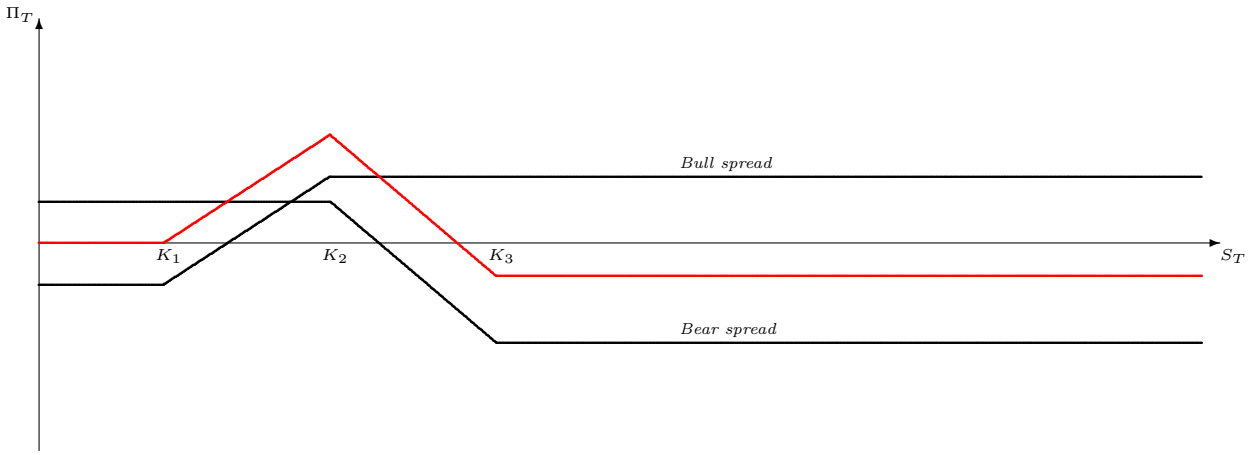
Topic 7

Problem 23

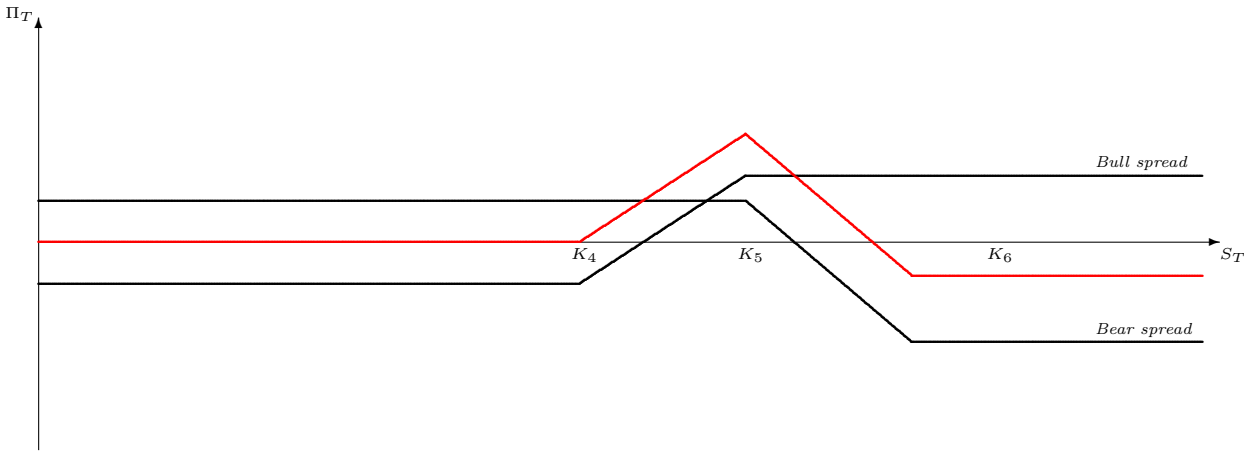
An investor has obtained information on a company, but is unsure about the impact this will have on the stock price, once it becomes publicly known. The company is seeking to expand overseas to counter increased competition in their domestic markets. This could be seen as a positive move by the company to secure future profitability and the stock price should increase. On the other hand, it can be seen as a sign that profitability in their domestic market, which makes up the majority of their current profits, is under more strain than the market has anticipated and this would reduce the stock price. Having gone through both scenarios, he believes that the stock price will either increase by about 10% if the information is seen positively, or the stock price will decrease by 20% if the information is interpreted negatively.

Develop an investment strategy using options that allows the investor to take advantage of his information.

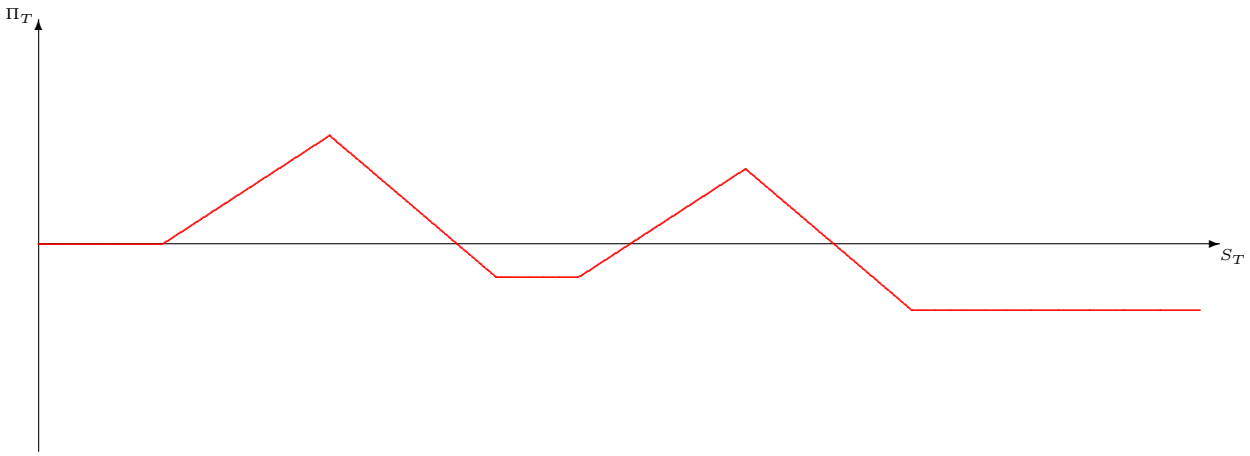
Indicative answer: *The investor should be making profits if the prices are increasing by about 10% or decreasing by about 20%. We can use bull and bear spreads with appropriate strike prices. For simplicity set the current price at 100. Then we set bull and bear spreads as follows to achieve maximum payoffs at a stock price of 80:*
Bull spread: Select a low strike price below 80, and a higher strike price at 80
Bear spread: Select a low strike price at 80 and a higher strike price below 80



This is now repeated for strike prices for the positive impact of the information:
Bull spread: Select a low strike price below 110, and a higher strike price at 110
Bear spread: Select a low strike price at 110 and a higher strike price below 110



Combining these two, we get the desired payoff profile



Problem 24

You seek to invest further into Vamelia plc as your analysis suggests that this company will outperform many competitors in the near future. However, you have already hold

24.85% of the outstanding share as a long-term investment and financial regulations require you to make a takeover offer for all outstanding shares if your shareholding reaches 25%. You are not interested in acquiring Vamelia plc completely, and would also not have the financial resources to do so.

a. How can you benefit from the prospects of Vamelia plc without purchasing their shares?

b. If no derivatives on Vamelia plc are available in the market, could you use derivatives on a broader stock index to achieve a similar, although not identical outcome?

Indicative answer:

a. Creating a synthetic asset, combining a short put and a long call with the same strike price on Vamelia plc would generate the same payoff profile as the stock itself.

b. This would not be possible as techniques like β-hedging only account for systematic price changes and not the idiosyncratic price changes that arise from specific information on a stock. Using options or ither derivatives on a stock index would not generate the performance that is the result of the specific information related to Vamelia plc.

Problem 25

Can option values be negative, like those of futures and swaps?

Indicative answer: *Option values cannot be negative. The negative values in futures and swap arise from the requirement to purchase or sell the asset in futures, or to swap the payments from assets in swaps; such an exchange has to happen even if it is loss-making for the purchaser of that derivative. With options, no such transaction is required. The purchaser of an option can let the option expire without exercising his right to buy or sell the underlying asset. He will exercise the option if it is profitable to do so, but will not exercise the option it this would cause him a loss. It is thus that the payoff of options to the purchaser can never be negative and hence the value of the option cannot be negative. The most the purchaser of an option can lose is the options premium (price) paid when purchasing the option, but this is a sunk cost as the purchase price cannot be recovered in any case and therefore does not affect the value of the option.*

Problem 26

If two portfolios have the same value at some point in the future, why will the prices of those two portfolios always be identical, even before this point in time?

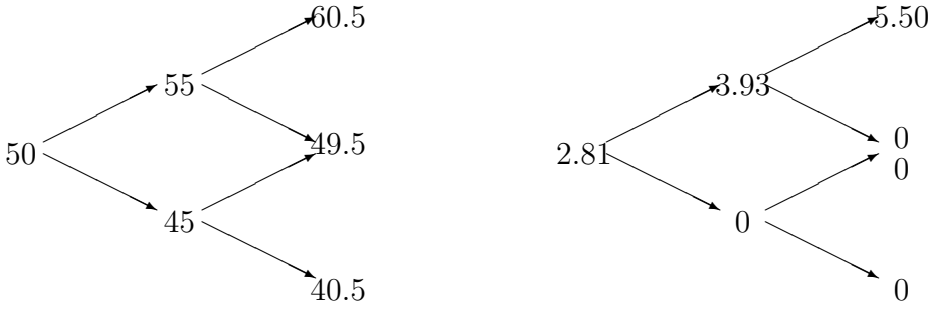
Indicative answer: *The argument is that if the prices were not identical, arbitrage could generate profits without investment, giving investors profits for free. Assume at some time T the portfolios have the same value V_T. At some time prior to this, their values are V_1 and V_2, respectively, where $V_1 \neq V_2$. I know form a portfolio of these two portfolios by choosing 1 unit of portfolio 1 and ω units of portfolio 2. I set these values such that $V_1 + \omega V_2 = 0$, and I make no investment. We need $\omega = -\frac{V_1}{V_2}$. At time T this portfolio will be worth $\Pi_T = V_T + \omega V_T = \left(1 - \frac{V_1}{V_2}\right) V_T$. If $V_2 > V_1$, we see that $\Pi_T > 0$. If $V_2 < V_1$, we swap the portfolios 1 and 2. Only if $V_1 = V_2$ do we have $\Pi_T = 0$. If $V_1 \neq V_2$, we can generate profits without making an initial investment. Such 'free money' cannot exist as every investor could instantly generate an infinite amount of profits.*

Topic 8

Problem 27

In each time period, a stock, currently trading at £50, increases or decreases by 10% with equal probability. If you buy a European call option with a strike price of £55 maturing in 2 time periods, how much do you expect to pay if the risk-free rate is 5% per time period and the return on the stock is expected to be zero?

Indicative answer:



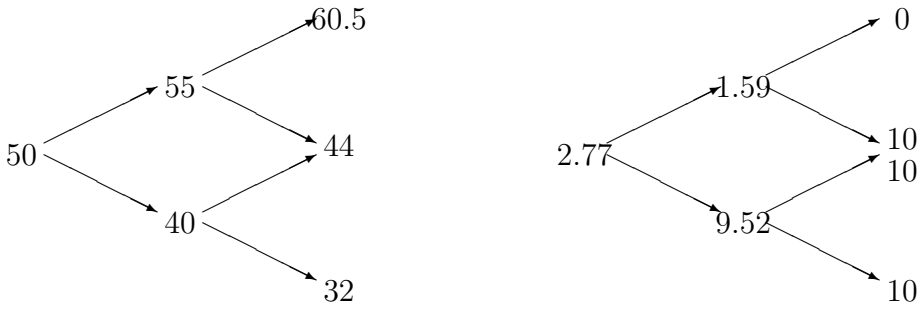
We solve for the option price backwards. As a Call option, the payments at maturity are
$C_{uu} = \max\{0; u^2S - K\} = \max\{0; 60.5 - 55\} = 5.5$
$C_{ud} = C_{du} = \max\{0; udS - K\} = \max\{0; 49.5 - 55\} = 0$
$C_{dd} = \max\{0; d^2S - K\} = \max\{0; 40.5 - 55\} = 0$
We now work backwards to the first time period:
$\Delta_u = \frac{C_{uu}-C_{ud}}{S_u(u-d)} = \frac{5.5-0}{55(1.1-0.9)} = 0.5$, $B_u = \frac{1}{1+r}\frac{uC_{ud}-dC_{uu}}{u-d} = \frac{1}{1+0.05}\frac{1.1\times0-0.9\times5.5}{1.1-0.9} = -23.57 \Rightarrow$
$C_u = \Delta_u S_u + B_u = 0.5 \times 55 - 23.57 = 3.93$
$\Delta_d = \frac{C_{du}-C_{dd}}{S_d(u-d)} = \frac{0-0}{45(1.1-0.9)} = 0$, $B_d = \frac{1}{1+r}\frac{uC_{dd}-dC_{du}}{u-d} = \frac{1}{1+0.05}\frac{1.1\times0-0.9\times0}{1.1-0.9} = 0 \Rightarrow$
$C_d = \Delta_d S_d + B_d = 0 \times 45 + 0 = 0$
For the first time period we get
$\Delta = \frac{C_u-C_d}{S(u-d)} = \frac{3.93-0}{50(1.1-0.9)} = 0.393$, $B = \frac{1}{1+r}\frac{uC_d-dC_u}{u-d} = \frac{1}{1+0.05}\frac{1.1\times0-0.9\times3.93}{1.1-0.9} = -16.8429 \Rightarrow$
$C = \Delta S + B = 0.393 \times 50 - 16.8429 = 2.8071$
The option is worth £2.81.

Problem 28

A Digital Option is an option that pays a fixed amount if it is exercised and is otherwise similar to a standard option. A Digital Call option pays the agreed amount if at maturity the price of the underlying asset is above the strike price and a Digital Put option pays the agreed amount if at maturity the price of the underlying asset is below the strike price.

Consider a stock, currently trading at £50, which increases by 10% or decreases by 20% with equal probability. If you buy a Digital put option with a strike price of £45 maturing in 2 time periods which pays you £10, how much do you expect to pay if the risk-free rate is 5% per time period?

Indicative answer:



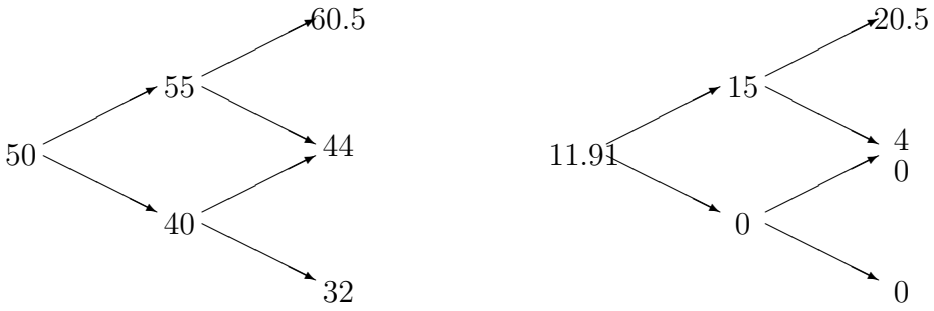
We solve for the option price backwards. As a Call option, the payments at maturity are
$C_{uu} = 0$ *as the price is above the strike price*
$C_{ud} = C_{du} = 10$ *as the price is below the strike price*
$C_{dd} = 10$ *as the price is below the strike price*
We now work backwards to the first time period:
$\Delta_u = \frac{C_{uu}-C_{ud}}{S_u(u-d)} = \frac{0-10}{55(1.1-0.8)} = -0.6061$, $B_u = \frac{1}{1+r}\frac{uC_{ud}-dC_{uu}}{u-d} = \frac{1}{1+0.05}\frac{1.1\times 10-0.8\times 0}{1.1-0.8} = 34.9206 \Rightarrow C_u = \Delta_u S_u + B_u = -0.6061 \times 55 + 34.9206 = 1.5851$
$\Delta_d = \frac{C_{du}-C_{dd}}{S_d(u-d)} = \frac{10-10}{40(1.1-0.8)} = 0$, $B_d = \frac{1}{1+r}\frac{uC_{dd}-dC_{du}}{u-d} = \frac{1}{1+0.05}\frac{1.1\times 10-0.8\times 10}{1.1-0.8} = 9.5238 \Rightarrow C_d = \Delta_d S_d + B_d = 0 \times 40 + 9.5238 = 9.5238$
For the first time period we get
$\Delta = \frac{C_u-C_d}{S(u-d)} = \frac{1.5851-9.5238}{50(1.1-0.8)} = -0.5292$, $B = \frac{1}{1+r}\frac{uC_d-dC_u}{u-d} = \frac{1}{1+0.05}\frac{1.1\times 9.5238-0.8\times 1.5851}{1.1-0.8} = 29.2321 \Rightarrow C = \Delta S + B = -0.5292 \times 50 + 29.2321 = 2.7721$
The option is worth £2.77.

Problem 29

A Barrier Option is an option that pays the normal option payoff, but only if during the life time of the option certain price limits (barriers) either have been breached or have not been breached. An 'Up-and-in' Call option allows the buyer to obtain the usual payments of a call option, provided during the life of the option, the stock price has exceeded the barrier price.

Consider a stock, currently trading at £50, which increases by 10% or decreases by 20% with equal probability. If you buy an 'Up-and-in' Call option with a strike price of £40 and a barrier at £53 maturing in 2 time periods, how much do you expect to pay if the risk-free rate is 5% per time period?

Indicative answer:



We solve for the option price backwards. As a Call option, the payments at maturity are
$C_{uu} = \max\{0; u^2S - K\} = \max\{0; 60.5 - 40\} = 20.5$ *as the barrier has been crossed*
$C_{ud} = \max\{0; u^2S - K\} = \max\{0; 44 - 40\} = 4$ *as the barrier has been crossed*
$C_{du} = 0$ *as the barrier has not been crossed*
$C_{dd} = 0$ *as the barrier has been crossed and the strike price is above the stock price at maturity*
We now work backwards to the first time period:
$\Delta_u = \frac{C_{uu}-C_{ud}}{S_u(u-d)} = \frac{20.5-4}{55(1.1-0.8)} = 1$, $B_u = \frac{1}{1+r}\frac{uC_{ud}-dC_{uu}}{u-d} = \frac{1}{1+0.05}\frac{1.1\times4-0.8\times20.5}{1.1-0.8} = -40 \Rightarrow$
$C_u = \Delta_u S_u + B_u = 1 \times 55 - 40 = 15$
$\Delta_d = \frac{C_{du}-C_{dd}}{S_d(u-d)} = \frac{0-0}{40(1.1-0.8)} = 0$, $B_d = \frac{1}{1+r}\frac{uC_{dd}-dC_{du}}{u-d} = \frac{1}{1+0.05}\frac{1.1\times0-0.8\times0}{1.1-0.8} = 0 \Rightarrow$
$C_d = \Delta_d S_d + B_d = 0 \times 40 + 0 = 0$
For the first time period we get
$\Delta = \frac{C_u-C_d}{S(u-d)} = \frac{15-0}{50(1.1-0.8)} = 1$, $B = \frac{1}{1+r}\frac{uC_d-dC_u}{u-d} = \frac{1}{1+0.05}\frac{1.1\times0-0.8\times15}{1.1-0.8} = -38.0952 \Rightarrow$
$C = \Delta S + B = 1 \times 50 - 38.0952 = 11.9048$
The option is worth £11.91.

Problem 30

A stock, currently trading at £50 and has a volatility of 25%. You buy a European call option with a strike price of £55 maturing in 2 months, the risk-free rate is 5% p.a. and the return on the stock is expected to be 12% p.a.

a. Using the Black-Scholes formula, how much do you expect to pay for this Call option?

b. How much would be value of a Put option with the same characteristics?

Indicative answer:

a. With all variables given on an annual basis, the time to maturity must also be on an annual basis, $T = \frac{2}{12} = \frac{1}{6}$.
$d_1 = \frac{\ln \frac{S}{K} + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\ln \frac{50}{55} + \left(0.05 + \frac{1}{2}0.25^2\right)\frac{1}{6}}{0.25\sqrt{\frac{1}{6}}} = -0.8012$
$d_2 = d_1 - \sigma\sqrt{T} = -0.8012 - 0.25\sqrt{\frac{1}{6}} = -0.9032$
$C = SN(d_1) - Ke^{-rT}N(d_2) = 50N(-0.8012) - 55e^{-0.05\frac{1}{6}}N(-0.9032) = 50 \times (1 - 0.7881) - 54.5436 \times (1 - 0.8159) = 0.5535$
The option is worth £0.55.

b. We can use the Put-Call parity: $P = C - S + Ke^{-rT} = 0.5535 - 50 + 55e^{-0.05\frac{1}{6}} = 5.0971$
The Put option is worth £5.10.

Topic 9

Problem 31

You are holding 2000 stocks whose characteristics have been discussed in problem 30.

a. If you wanted to ensure to hedge the value of your position perfectly during the coming 2 months at a price of £55 using put options, how many put option would you have to purchase?

b. Once you have purchased these put options, what further actions do you have to take in order to ensure your perfect hedge?

Indicative answer:

a. *We use the Δ-hedging. From problem 30 we know that $\Delta_C = N(d_1) = 1 - 0.7881 = 0.2119$. We then get $\Delta_P = \Delta_C - 1 = -0.7881$ The hedge ratio is then given by $fh = -\frac{1}{\Delta_P} = 1.2689$. For 2000 stocks, this implies that the you have to purchase 2538 put options.*

b. *The hedge ratio is constantly changing as the option approaches maturity (T changes) and the stock price S changes. Hence you will have to constantly adjust the number of put options held, necessitating frequent buying and selling of put options.*

Problem 32

You believe to have spotted the mispricing of a call option on a leading company. The company has released new information, that has lead to widespread confusion in the market about its future prospects and has reduced the price of its stocks by just over 5% during the day. At the same time, a call option has increased in value. You now believe the call option to be overpriced as its value should have gone down with the stock price.

Is it correct that you have identified the mispricing of this call option?

Indicative answer: *The option is not necessarily mispriced. Its price should indeed fall if the price of the underlying asset decreases. However, the value of the underlying asset is not the only determinant of the option price. A further reduction should be observed because the option is closer to maturity, seemingly confirming your assessment that the option is mispriced. However, option values are most sensitive to the volatility of the underlying asset. In this case the uncertainty surrounding the company's future prospects will have increased the uncertainty and thus the anticipated volatility of the stock. If the underlying asset becomes more risky, its volatility increases, the option becomes more valuable. This effect can easily outweigh the effect the change in the value of the underlying asset had on the option price. Thus the increase in the option price may well be explained by the increased volatility. Of course, specificcally, this needs to be investigated using appropriate option pricing models, but the development of the option price is consistent with such models.*

Problem 33

Constantin Theopoleus is the CEO of CT Engineering E.P.E. As part of its investment programme to update their ageing machinery, the board considers the investment options and how these investments can be financed. Constantin Theopoleus intervenes at that point and states that obviously they will take a loan to finance the investment as they should be able to secure a loan at 8-9% p.a., while using equity currently costs 12-15% p.a., depending on how it is calculated. Therefore, loans are cheaper and it would increase the value of the firm to use loans when financing these investments. Is this assertion correct?

Indicative answer: *It is true that loans carry a lower interest rate than the required rate of return on equity. However, as loans are increased, leverage increases, and thereby the risks for equity. Any losses on the now enlarged assets of the company would reduce the given equity more than is the case if equity was raised. In the latter case the same losses would be distributed across more equity, leading to smaller losses per unit of equity. Of course, the same holds for profits. Therefore, overall, the risks of equity holders will increase when financing the investment by loans, which will necessitate that the required return on equity increases. These two effect offset each other exactly and the overall cost of capital for the company, the weighted average cost of capital, remains constant. As this is what determines the value of the company as whole, not only of the equity component of the company, the value of the company is not affected. It is therefore Constantin Theopoleus is wrong and the way the investments are financed will not affect the value of the company as a whole.*

Problem 34

James Copeland is perplexed by the reaction of markets to an announcement his company has made. As the Chief Financial Officer he has made a disclosure that the company, Engineering Solutions plc, will expand its reach by opening another five offices across the country and finance the acquisition of suitable office space and the recruitment of staff by drawing on a £50m loan facility their bank has provided. Engineering Solutions plc provides engineering consultancy mainly for infrastructure projects and their main clients include Network Rail, National Highways, and major utility companies working on the establishment of wind farms as well as nuclear power plants, making their income stream highly predictable. While such a move to expand its geographical coverage was expected by the market, their share price did not react. In contrast to that, on the same day, another company, AIChip plc, made a long-anticipated announcement of expanding its business by investing into the production of high-end computer chips for use in applications involving artificial intelligence; their market is characterised by a large degree of uncertainty about future developments. Similar to Engineering Solutions plc, they announced that the expansion of their business would be financed though a loan facility provided by their bank. In this case the stock price increased by 8% after the announcement.

How can you explain the differences in the stock market reaction?

Indicative answer: *Engineering Solutions plc is a very transparent company whose business prospects are well known due to their involvement with the public sector. There is unlikely to be much asymmetric information between the company and their investors. In contrast, the business of AIChip plc is much more opaque and it is likely that the management of the company is much better informed about their future prospects than investors. The use of equity to finance their expansion can now be interpreted as a signal by management to the market that they are confident about the prospects of their investment, which then leads to the risk in the stock price. The absence of this degree of asymmetric information between managers and investors at Engineering Solutions plc leads to a situation where the use of loans is not interpreted as a signal in the management's confidence in the investment; investors have nearly the same information as managers and hence management has nothing to signal to investors. Given that the investment was anticipated, this information would already be included into the stock price (efficient markets) and the form of financing adds no further information, hence the stock price does not react.*

Problem 35

Vijay Foods plc owns a chain of supermarkets and Vijay Singh is its main shareholder and Chief Executive Officer. Recently Vijay Foods announced that as an alternative to modernising its existing supermarkets, it could expand its business by introducing budget supermarkets in the region of Marab, where Vijay Singh is a prominent local politician with ambitions to break into national politics. With their existing supermarkets, Vijay Foods plc target mainly middle class families and the business is performing well. The budget segment is highly contested by a large number of well established operators and financial analysts have been sceptical whether Vijay Foods plc can make a successful entry into this market. Undeterred, Vihjay Singh has approached his bank for a loan to finance both the modernisation of its existing supermarkets and the introduction of the new chain of budget supermarkets. His bank has declined to provide the loan and he has decided to use his considerable personal wealth to provide a private loan to the company. While the stock market had not reacted significantly when the expansion plans were announced, the stock price dropped instantly by 6% as the private loan was disclosed.

Why did the stock price drop after this announcement?

Indicative answer: *The expansion into budget supermarkets is a risky investment, which banks have seen as too risky to finance. It suggests that providing the loan would lead to risk shifting, where the use of loans to finance an investment gives an incentive to take on higher risks; the bank anticipated that rather than modernising their existing supermarkets, Vijay Foods plc would choose the riskier alternative of expanding into the budget sector of the market. I seems that investors share this view and as higher risks will lead to higher discount rates of future profits, the value of the company will reduce, resulting into the fall of the stock price. The fact that a private loan was given by Vijay Singh does not alter this assessment as that loan might be given to reap private benefits in form of increased popularity with poorer families supporting his political ambitions.*

Problem 36

Hiromara Co. is a local producer of energy drinks, but suffers from expanding international competitors taking an increasing market share. Previous launches of new products have been either a failure or lead to moderate success. They have now developed and already announced a new drink they seek to bring to the market and plan a large marketing campaign for which they need to raise ¥10bn. In many previous cases the main shareholder has agreed to finance such investments by the company

issuing new shares, but he has made it clear that his financial resources are limited and he would not be able to provide funding at the scale sought. Approaching their bank, Hiromara Co. is able to secure a loan for the required amount. Taking the willingness of the bank to provide the loan, which was only forthcoming after close scrutiny of its plans, as an endorsement of their plans, the agreement is announced publicly. Expecting a positive response from the stock market, especially as a loan has been raised, Hiromara Co. is disappointed that the stock price remains virtually unchanged.

How do you explain the lack of stock market response to the announcement of the funding of the marketing campaign through a bank loan?

Indicative answer: *Normally we would expect a company to choose a bank loan if they are confident about the prospects of the investment and this would therefore act as a positive signal to the market; here the market doe snot interpret the loan in this way. The reason is that Hiromara Co. was not free which form of financing to use. They seemed to have favoured equity financing, which would not signal confidence in the prospects of the new product , and only obtained a bank loan as equity finance was not available. Thus, even though a loan was taken, it would not act as a signalling device in this case.*

Topic 10

Problem 37

Fritelli SpA and Mariagno SpA are two stationary companies focussing on high-end writing utensils, such as fountain pens, and accessories. They are both targeting the same market and are fierce competitors, where neither of these two companies seems to have a meaningful advantage over the other. The companies are located within 10 minutes walk of each other in the small town of Limosine and are the result of two brothers falling out over the running of their company, resulting in the company being split. Fritelli SpA is owned to 74% by Giorgio Mariagno, who also manages the company, while Mariagno SpA is owned to 78% by Antonio Mariagno, but due to health concerns has delegated the running of the company to an outside manager and is not involved in any but the most important decisions by the company. By coincidence, both companies announce within days that they will seek to raise equity from outside investors to strengthen their balance sheets and fund further expansion of their respective business lines; the amounts sought would in both cases reduce the holdings of the current owners to below 50%. The reaction of the stock market to this announcement is very different; in the case of Fritelli SpA the stock price drops by 7 while there is no reaction in the stock price of Mariogno SpA.

How can you explain the difference in the reaction of the stock market to the announcement of raising equity?

Indicative answer: *Fritelli SpA is managed by the majority shareholder and raising equity would significantly dilute his stake in the company. This would negatively affect his incentives to exert effort when managing the company; sharing the benefits of his effort with other shareholders will result in less benefits accruing to him, while still facing the full costs of effort. The market therefore anticipates that Giorgio Mariagno would reduce the effort in running the company, resulting in a lower value, which causes the stock price to fall. On the other hand, Fritelli SpA is managed by an outside manager, which presumably has no or a much lower stake in the company. Increasing equity will not affect his incentives to exert effort as this has no effect on his benefits. Thus no changes in his effort is predicted and there is no reaction by the stock market.*

Problem 38

Alcatraz SA produces pipes and pumping stations for water and sewage companies. With many competitors in the field and utility companies employing strict tender rules for any large projects, it has become difficult for companies in this sector to remain profitable, Alcatraz SA being no exemption. Future business plans are often kept secret in this sector as to not give away information to competitors unnecessarily, however, Alcatraz SA has taken the unusual step to use an analyst conference and quite openly discuss their plans for the future development of the company. Financial analysts left the conference impressed by the openness of the company and were grateful for the insights not only into Alcatraz SA itself, but also the industry as a whole. A few weeks after this conference, Alcatraz SA announces that in order to increase their investments into the future development of the company, it will cut its dividend by 50% for the foreseeable future. This announcement took the market, as well as the financial analysts attending the conference, by surprise. However, the stock market did not react much to this announcement with the stock closing down 0.7% compared to the market overall closing down 0.6%.

How can you explain that there was no impact on the stock price after the dividend cut was announced?

Indicative answer: *Dividends signal information if companies hold better information than investors. In this case a reduction in dividends could be seen as a lack of confidence in their future prospect. Here, however, asymmetric information has been reduced significantly with help of the analyst conference only weeks prior. It is thus that asymmetric information about the future prospects are minimal and dividend policy does not convey information, dividends become irrelevant.*

Problem 39

Röthlisberger AG has lost significant market share in recent years and performed poorly overall, resulting in the dismissal of the senior management. The new management team is seen as highly competent and convinced that the company can be turned around quickly. Having taking up their new roles, the new management team is facing an immediate need to make investments for which additional funds are required. At the board meeting, it is discussed whether the dividend should be reduced, at least temporarily, to preserve cash and reduce the amount that needs to be borrowed for the necessary investments.

Do you think a decision to cut dividends is appropriate?

Indicative answer: *It is reasonable to assume that in the difficult circumstances the company is in, there is considerable asymmetric information between the management and investors and thus dividends could be used to convey the confidence of the management in their ability to turn around the company. By cutting dividends, Röthlisberger AG does the opposite and signals that it does not have this level of confidence. Maintaining the dividend could signal that managers are confidents about their restructuring plans as paying such dividends are only sustainable if sufficient profits are generated. This positive signal could be supported by obtaining a larger loan, to cover the dividend payments, sending another positive signal about the prospects of the company. In conclusion, the decision to cut dividends is ill-advised, provided there are no other constraints that drive such a decision.*

Problem 40

OfficeSolutions plc is a real estate company that owns large office spaces in many large cities to rent out as temporary locations for companies whose office undergo renovations or who have a sudden demand for additional office space. Their business has been performing very well recently and due to the lack of available office space to purchase, they have accumulated significant cash reserves. They have now approached RetailSpace plc with the aim of acquiring this company, who operate a similar business as OfficeSolutions plc, but focus on providing retail spaces. With the growth of online shopping, demand for retail space has been low and they have not performed well recently. Its acquisition by OfficeSolutions plc would however fulfill a long-term vision by the founder and CEO if OfficeSolutions plc to provide spaces for all commercial activities in town centres. In discussion with Lambrecht& Co, the investment bank advising them, they have been discussing how to pay for the acquisition. Lambrecht& Co have advised that the acquisition will not be well received by the market, but as OfficeSolutions plc seems insistent on completing the acquisition, they strongly recommend to make a cash offer rather than a stock offer. They advise that using their cash reserves to pay for the acquisitions would be seen as neutral by the market, while issuing shares in OfficeSolutions plc to the owners of RetailSpace plc would be seen as negative and will lead to a fall in the stock price.

What is the rationale for the advice given by Lambrecht& Co?

Indicative answer: *We can interpret this situation as equivalent to paying a dividend and the acquisition of RetailSpace plc as a vanity project by the founder and CEO to fulfill his ambition to be active in more markets, it is thus a moral hazard problem. Paying for the acquisition using cash, will reduce the cash reserves OfficeSolutions plc has available and therefore reduce the moral hazard in the future as the company has less resources which it can use unproductively. This would be seen as positive and should increase the stock price, but this will be balanced against acquiring a company*

that is not performing well. The advice by the investment bank seems to suggest that these two effects are roughly balanced. Using stocks to pay for the acquisition of RetailSpace plc on the other hand would see them acquire the same poorly performing company, but retain their cash reserves with the prospect of moral hazard in the future through the inefficient use of the cash reserves. This leads to the anticipated negative reaction of the stock market. It is therefore that using the cash reserves is preferable.

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