Bank risk-taking and optimal bailout resources

ABSTRACT. We develop a model in which regulators determine optimally the resources they commit to bank bailouts, balancing the costs of bailouts against those of bank failures. Banks, on the other hand, balance the costs of reducing their risk with the benefits of bailouts. We find that as the costs of risk reduction increases, the actual risk taken reduces, also reducing the need for bailouts. The reason for our counter-intuitive result is that as these costs increase for banks, the regulator reacts by reducing bailouts and thus induce an actual reduction in risks taken by banks. Similarly, we find that increasing the costs of not bailing out banks, reduces bank risks and the resources committed to bailouts.
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1. Introduction

It is commonly accepted that the prospect of a bailout increases the moral hazard of banks in that they take higher risks than they would normally do, as evidenced for example in Demirgüc-Kunt and Detragiache (2002), amongst many others. A lot of the literature on bailouts focuses on assessing the impact bailouts have on moral hazard, such as the extent and how to mitigate its effects, see e.g. Farhi and Tirole (2012); Dell'Ariccia and Ratnovski (2019). What has been given much less attention is an analysis of the interaction between bank risk-taking, and hence the extent of bailouts needed, and the policy of the government or regulator financing such bailouts. Lucchetta et al. (2019) consider the liquidity requirements by banks being used by regulators to induce more prudent bank behaviour, but they also find that the prospect of being bailed out induces an incentive to increase risks.

In this paper we will focus on the interaction between a regulator committing to a certain extent of bailouts and banks responding to these commitments by choosing an optimal level of risk. In contrast to other models, we do not restrict the behaviour of banks by regulation, but solely rely on the incentives given by the regulator for banks to limit risks.

2. Optimizing banks and regulators

Let us assume that an infinite number of atomistic banks provide loans that are repaid with probability π, including interest r_L; these repayments are independent of each other across banks. Banks are fully financed by deposits, which are paid interest r_D and hence with limited liability, their expected returns from lending the amount of L are $\pi\left(r_L - r_D\right)L$. The risk banks take, measured by the success rate π, can be affected by effort and in order to obtain a success rate π, the bank faces costs of $\frac{1}{2}c\pi^2 L$, that are always borne by the bank. If the bank fails, which happens with probability $1-\pi$, they may be bailed out with probability p, in which case the income from the loans, $(1+r_L)L$ is restored to the bank at no costs to them. Hence bank profits are given by

$$\Pi_B = \pi (r_L - r_D) L + (1-\pi) p (r_L - r_D) L - \frac{1}{2} c \pi^2 L. \tag{1}$$

We now propose that a regulator bails out banks only if the fraction of banks failing, $1-\pi$ as those banks whose loans are not repaid will fail, exceeds a threshold $\underline{\lambda}$. Once this threshold is surpassed, failing banks are bailed out, but there are limited resources available and at most a fraction of $\overline{\lambda}$ can be bailed out fully. If even more banks are failing, not all will be bailed out and only a fraction $\frac{\overline{\lambda}}{1-\pi}$ can be rescued. Hence we have

$$p(\pi) = \begin{cases} 0 & \text{if} \quad 1-\pi < \underline{\lambda} \\ 1 & \text{if} \quad \underline{\lambda} \leq 1-\pi \leq \overline{\lambda} \\ \frac{\overline{\lambda}}{1-\pi} & \text{if} \quad 1-\pi > \overline{\lambda} \end{cases} \tag{2}$$

as the probability of a bank being bailed out. If the failure rate $1-\pi$ is sufficiently low, then no bail out happens as the threshold $\underline{\lambda}$ has not been reached. Once the failure rate of banks is sufficiently high, all banks are bailed out until the resources $\overline{\lambda}$ are reached. For even higher failure rates, the resources $\overline{\lambda}$ are not sufficient to bail out all banks, and hence only a fraction are bailed out, where we assume that banks are chosen randomly.

A regulator faces total costs of $\Psi_F L$ if banks fail and are not bailed out, consisting of covering the costs to depositors and wider macroeconomic costs. Bailing out a bank will cost $\Psi_B L$, being the costs of funding repayments of the loans $(1+r_L)L$ and the associated financing costs. The bailout here consists of the bank being given funds to cover the losses of the failed loans, but we could easily limit this to a smaller amount, such as the repayment of the initial loan L without changing the essence of the argument. We assume that $\Psi_F > \Psi_B > 1$. Then, if banks are not bailed out, the costs are $(1-\pi)\Psi_F L$ and if they are fully bailed out, it is $(1-\pi)\Psi_B L$. If not all banks can be bailed out, but only a fraction $\overline{\lambda}$ of all banks, we have $\overline{\lambda}\Psi_B L + \left(1-\pi-\overline{\lambda}\right)\Psi_F L$, i.e. the costs of bailing out a fraction $\overline{\lambda}$ of banks and letting the remaining banks fail. Thus the payments by the regulator are

$$\Pi_R = \begin{cases} -(1-\pi)\Psi_F L & \text{if} \quad 1-\pi < \underline{\lambda} \\ -(1-\pi)\Psi_B L & \text{if} \quad \underline{\lambda} \leq 1-\pi \leq \overline{\lambda} \\ -\overline{\lambda}\Psi_B L - \left(1-\pi-\overline{\lambda}\right)\Psi_F L & \text{if} \quad 1-\pi > \overline{\lambda} \end{cases}. \tag{3}$$

Let us now consider the optimal risk taking by banks, π, as well as the optimal amount of resources the regulator puts in, $\overline{\lambda}$. As banks are atomistic, the decision of one bank, does not affect the fraction of banks failing overall. Thus the likelihood of a bailout will not be affected by the decision of a single bank, but merely by the aggregate decision of the banks overall. Therefore we can take $p(\widehat{\pi})$ as given for a bank optimising their risk-taking. Then the first order condition for banks maximizing their profits by choosing the optimal risk level is given as

$$\frac{\partial \Pi_B}{\partial \pi} = ((r_L - r_D) - c\pi - p(\widehat{\pi})(r_L - r_D))L = 0, \tag{4}$$

which we can re-write as

$$p(\widehat{\pi}) = 1 - \eta\pi, \tag{5}$$

with $\eta = \frac{c}{r_L - r_D}$. This can be interpreted as the ratio of the costs increasing the success rate (c) and the benefits from doing so, $r_L - r_D$. As all banks are alike, in equilibrium we require that the equilibrium rate $\pi^*_{\overline{\lambda}} = \widehat{\pi}$. Using equation (5) to insert into (1), we get that in equilibrium

$$\Pi^*_B = \pi^*(r_L - r_D)L - \frac{1}{2}c\pi^{*2}L + (1 - \pi^*)(1 - \eta\pi^*)(r_L - r_D)L. \tag{6}$$

This gives us $\frac{\partial \Pi^*_B}{\partial \pi^*} = -c(1 - \pi^*)L < 0$ and hence if there are multiple equilibria, banks would prefer the one with the lower success rate.

Looking at possible solutions to the first order condition (5), we can consult figure 1 which shows the right-hand side of equation (5) as the straight descending line indicated by η_i and its left-hand side as the increasing line p_i. We see immediately that any equilibrium must be such that $\pi < 1 - \overline{\lambda}$. The potential second equilibrium at $\pi > \underline{\lambda}$ would be inferior as equilibria with lower success rates are preferred, which was shown above. Hence the equilibrium is given by $\frac{\overline{\lambda}}{1-\pi} = 1 - \eta\pi$, which solves for

$$\pi^* = \frac{1+\eta}{2\eta} - \sqrt{\frac{(1+\eta)^2}{4\eta^2} - \frac{1-\overline{\lambda}}{\eta}}. \tag{7}$$

Given that in this case with $\pi < 1 - \overline{\lambda}$ we have $\Pi_R = -\overline{\lambda}\Psi_B L - \left(1 - \pi - \overline{\lambda}\right)\Psi_F L$, we find that $\underline{\lambda}$ is irrelevant in equilibrium and can take any value $\underline{\lambda} \in \left[0; \overline{\lambda}^*\right]$. The optimal $\overline{\lambda}$ we obtain from maximizing Π_R, which gives us as the first order condition that

$$\frac{\partial \Pi_R}{\partial \overline{\lambda}} = \Psi_B L - \Psi_F L - \Psi_F \frac{\partial \pi^*}{\partial \overline{\lambda}} L = 0. \tag{8}$$

From equation (7) we easily get that $\frac{\partial \pi^*}{\partial \overline{\lambda}} = -\frac{1}{2\eta\sqrt{\frac{(1+\eta)^2}{4\eta^2} - \frac{1-\overline{\lambda}}{\eta}}}$. Defining $\widehat{\Psi} = \frac{\Psi_F - \Psi_B}{\Psi_F}$ as the relative costs to regulators of banks failing and being bailed out, we then can rewrite the first order condition (8) as $\frac{\partial \pi^*}{\partial \overline{\lambda}} = -\widehat{\Psi}$, which after inserting solves for

$$\overline{\lambda}^* = 1 - \frac{(1+\eta)^2\widehat{\Psi}^2 - 1}{4\eta\widehat{\Psi}^2}. \tag{9}$$

As we need to ensure that $0 \leq \overline{\lambda}^* \leq 1$, we require that $\widehat{\Psi}^2(1-\eta)^2 \leq 1 \leq \widehat{\Psi}^2(1+\eta)^2$. If these conditions are violated, the optimal solutions are $\overline{\lambda}^* = 1$ or $\overline{\lambda}^* = 0$, corresponding to the case of a guaranteed full bailout and no possible bailout, respectively. These cases are considered separately in the appendix.

Using from the first order condition that $\sqrt{\frac{(1+\eta)^2}{4\eta^2}-\frac{1-\lambda}{\eta}}=\frac{1}{2\eta\widehat{\Psi}}$, we can rewrite equation (7) as

$$\pi^*_{\overline{\lambda}}=\frac{(1+\eta)\,\widehat{\Psi}-1}{2\eta\widehat{\Psi}}. \tag{10}$$

Using equation (5) we easily get that

$$p^*_{\overline{\lambda}}=\frac{\widehat{\Psi}\,(1-\eta)+1}{2\widehat{\Psi}}. \tag{11}$$

We have thus established the key result of our model, which we summarize in the following proposition.

Proposition 1 (Equilibrium risk-taking and bailout resources)**.** *Let $\widehat{\Psi}$ and η be defined as in the main text. If $\widehat{\Psi}^2\,(1-\eta)^2\leq 1\leq\widehat{\Psi}^2\,(1+\eta)^2$ the equilibrium is given by*

$$\begin{aligned}
\pi^*_{\overline{\lambda}} &= \frac{(1+\eta)\,\widehat{\Psi}-1}{2\eta\widehat{\Psi}},\\
\overline{\lambda}^* &= 1-\frac{(1+\eta)^2\,\widehat{\Psi}^2-1}{4\eta\widehat{\Psi}^2},\\
\underline{\lambda}^* &\in \left[0;\overline{\lambda}^*\right].
\end{aligned}$$

In order to analyse our result, we can now easily show that

$$\begin{aligned}
\frac{\partial\overline{\lambda}^*}{\partial\widehat{\Psi}} &= -\frac{1}{2\eta\widehat{\Psi}^3}<0. \\
\frac{\partial\overline{\lambda}^*}{\partial\eta} &= \frac{1}{4\eta^2}\left(1-\frac{1}{\widehat{\Psi}^2}-\eta^2\right)<0,\\
\frac{\partial\pi^*_{\overline{\lambda}}}{\partial\widehat{\Psi}} &= \frac{1}{2\eta\widehat{\Psi}^2}>0,\\
\frac{\partial\pi^*_{\overline{\lambda}}}{\partial\eta} &= \frac{1}{2\eta^2}\left(\frac{1}{\widehat{\Psi}}-1\right)>0,\\
\frac{\partial p^*_{\overline{\lambda}}}{\partial\widehat{\Psi}} &= -\eta\frac{\partial\pi^*}{\partial\widehat{\Psi}}<0,\\
\frac{\partial p^*_{\overline{\lambda}}}{\partial\eta} &= -\pi^*-\eta\frac{\partial\pi^*}{\partial\eta}<0,
\end{aligned} \tag{12}$$

where for the second and fourth results we note that $\widehat{\Psi}<1$ and the final two results can be derived from equation (5) or directly.

We can now interpret these results as follows. If, for regulators, letting banks fail becomes relatively more expensive, i.e. $\widehat{\psi}$ increases, the regulator will reduce the resources available for bailouts, $\overline{\lambda}^*$. This seems counterintuitive at first as reducing the resources available for bailouts on its own would increase the number of banks failing and thus costs to the regulator. There is, however, a secondary effect in that the reduced resources for

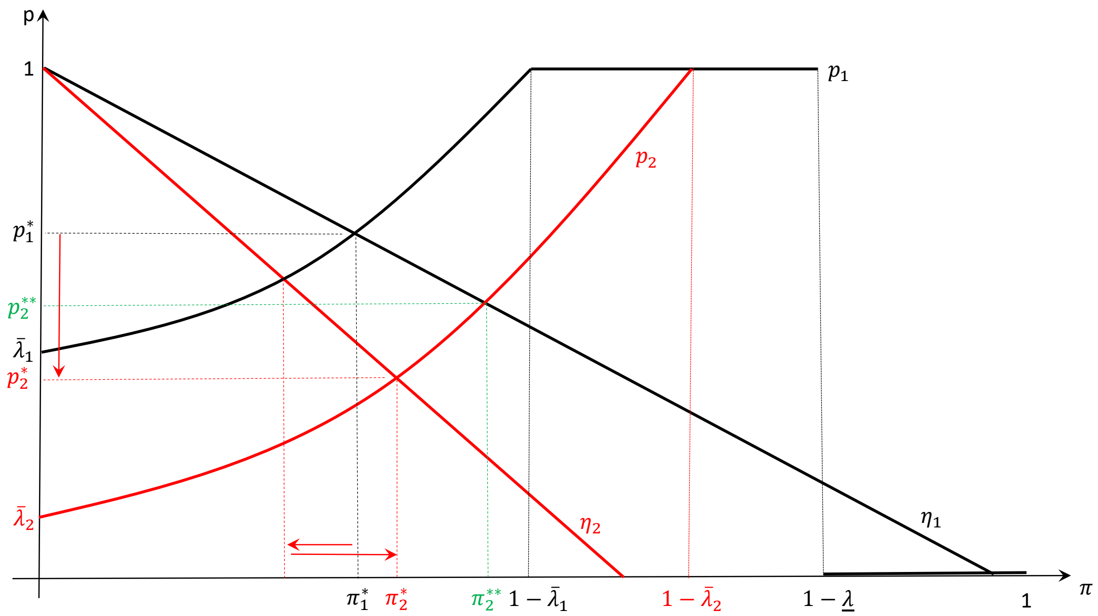

FIGURE 1. Comparative statics of the equilibrium success rate

bailouts also affects the banks' risk-taking. In the light of increased costs of not bailing out failed banks, regulators could employ two strategies to reduce these costs. Firstly, they could increase the resources available for bailouts, but this would increase the overall costs. The second strategy would be the reduce the number of failing banks, which would reduce the costs. The only mechanism to pursue the second strategy for the regulator is to reduce the resources available for bailouts. In this case as from above $\frac{\partial \pi^*_{\overline{\lambda}}}{\partial \overline{\lambda}} < 0$, the risk banks take, reduces and therefore the costs of not bailing out banks will reduce, too (*resource effect*). Of course, this reduction in costs due to lower risk-taking by banks has to be balanced against the reduced resources available to bail out banks and thus the regulator having to let banks fail, which is more costly (*cost effect*). This second effect of increasing costs is lower than the resource effect of lowering regulator costs. In figure 1 we indicate this overall effect where we assume that overall the resources available reduce to $\overline{\lambda}_2$.

If we increase the costs to banks of reducing risks (c), or reduce the benefits of such reduced risk ($r_L - r_D$), i.e. increase η, we again observe two effects. Firstly, taking risks becomes more attractive to banks, increasing the risk they are taking (*cost effect*). In figure 1 this is indicated by increasing the slope of the curve indicated η_2, which would lead to an equilibrium with lower success rate, thus higher risks. However, the regulator will also react and reduce the resources available for bailouts, $\overline{\lambda}$ as this is the only way a regulator can induce less risky behaviour and thus reduce the overall costs to the regulator. This *resource effect* reduces the risk taking by banks. The resource effect overall dominates the cost effect. Figure 1 illustrates this result where the bank costs increase to η_2, causing the regulator to reduce bailout resources to $\overline{\lambda}_2$, which reduced the risks taken overall.

We can now summarize our findings in the following corollary.

Corollary 1. *If for regulators the costs of banks not being bailed out increases relative to a bail out, regulators will reduce the bailout resources and the risks banks take reduce.*

If the costs to banks of reducing risks increase or the interest margins reduce, bailout resources are reduced and banks take less risks.

The predictions of our model are therefore that bailout resources and bank risks are decreasing if it is more expensive to not bail out banks and if the cost-benefit ratio of banks in reducing risks are higher.

3. Conclusions

We have developed a model in which bailouts are the result of a regulator balancing the costs of bank failures against those of bailouts, with banks anticipating these bailouts and adjusting their risk-taking accordingly. We found that as the ratio of costs to reduce bank failures and the benefits of a bank surviving increases, the bank will choose a lower default rate. While the primary effect is that this change increases the default risk of the bank, given that the costs of reducing risks are increasing and/or the benefits of doing so decrease, it also triggers a response by the regulator who reduces the resources available for a bailout and hence the likelihood of a bailout. This second effect will incentivise the bank to reduce the risks they are taking. It is this second effect that dominates and causes the risk-taking to reduce overall.

A question, of course, remains on the credibility of the regulator to only bail out a certain fraction of banks and letting others fail. It can easily be seen that when facing a banking crisis the regulator might be tempted to increase the resources available and bail out more or even all banks. If this were to be anticipated by banks, they would choose to take higher risks. We leave these considerations on the viability of the commitment by the regulator to not bail out some banks for future research. Similarly, resources for bailouts might not be readily available if needed, resulting in the opposite effect in that banks anticipate a lower likelihood of a bailout and this reduces the risks they are taking. Again this avenue is left for future research.

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Appendix A. Alternative bailout arrangements

Apart from the finite resources, as assumed in the main model, that are deployed by a regulator minimizing its payouts and banks reacting to these bailout resources by maximizing profits, we can consider a range of other arrangements for bailouts. In this appendix we also consider the social optimum of our model as well as the cases of no bailouts, bailouts always happening and a scenario in which banks are bailed out with a probability that is determined independently of any resources committed.

A.1. **The social optimum of the main model.** In the social optimum we would maximize $\Pi_W = \Pi_B + \Pi_R$ over π, $\overline{\lambda}$, and $\underline{\lambda}$. Inserting for the different cases, we get

$$\Pi_W = \begin{cases} \pi\left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \left(1-\pi\right)\Psi_F L & \text{if} \quad 1-\pi < \underline{\lambda} \\ \left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \left(1-\pi\right)\Psi_B L & \text{if} \quad \underline{\lambda} \leq 1-\pi \leq \overline{\lambda} \\ \left(\pi + \overline{\lambda}\right)\left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \overline{\lambda}\Psi_B L - \left(1-\pi-\overline{\lambda}\right)\Psi_F L & \text{if} \quad 1-\pi > \overline{\lambda} \end{cases}.$$

We now get that $\pi\left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \left(1-\pi\right)\Psi_F L < \left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \left(1-\pi\right)\Psi_B L$ as $\Psi_B - \Psi_F < 0 < r_L - r_D$, ruling out any optimum with $1-\pi < \underline{\lambda}$. Similarly, $\left(\pi + \overline{\lambda}\right)\left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \overline{\lambda}\Psi_B L - \left(1-\pi-\overline{\lambda}\right)\Psi_F L < \left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \left(1-\pi\right)\Psi_B L$, giving the same result, thus also ruling out $1-\pi > \overline{\lambda}$. Therefore any equilibrium must fulfill $\underline{\lambda} \leq 1-\pi \leq \overline{\lambda}$ and $\Pi_W = \left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \left(1-\pi\right)\Psi_B L$, giving us the first order condition $\frac{\partial \Pi_W}{\partial \pi} = -c\pi + \Psi_B = 0$, solving for $\widehat{\pi}^*_{\overline{\lambda}} = \frac{\Psi_B}{c}$ and $\widehat{p}^*_{\overline{\lambda}} = 1$. The values of $\overline{\lambda}$ and $\underline{\lambda}$ are indeterminate as long as $\underline{\lambda} \leq 1-\pi^{**} \leq \overline{\lambda}$.

We can now show that $\widehat{\pi}^*_{\overline{\lambda}} > \pi^*_{\overline{\lambda}}$, which after inserting for all variables solves for $-\left(r_L - r_D\right)\Psi_B < 0 < \left(\Psi_F - \Psi_B\right)\left(2\Psi_B + c\right)$. Thus the social optimum has a higher success rate and due to the higher failure costs, banks are always bailed out. The bailout happens here as there is no incentive problem with the bank lowering their success rate in order to benefit from the bailouts. We thus find that bailouts with independent banks and regulators are less likely to occur and risks taken by banks are higher than in the social optimum, while bailouts happen less often than is optimal. This result is common in the literature in that the social optimum is not reached and banks take excessive risks.

A.2. **No bail-outs.** Assume we do not allow for any bailouts to happen, then we can set $p = 0$, such that $\Pi_B = \pi\left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L$ and the first order condition $\frac{\partial \Pi_B}{\partial \pi} = \left(r_l - r_D\right)L - c\pi L = 0$ solves for $\pi^*_0 = \frac{1}{\eta}$.

The welfare to obtain the social optimum in this case is given by $\Pi_W = \pi\left(r_L - r_D\right)L - \frac{1}{2}c\pi^2 L - \left(1-\pi\right)\Psi_F$, which gives us $\frac{\partial \Pi_W}{\partial \pi} = \left(r_L - r_D\right)L - c\pi L + \Psi_F L = 0$, which solves for $\widehat{\pi}^*_0 = \frac{1}{\eta} + \frac{\Psi_F}{c}$. The success rate here is higher than in the case where banks and regulators maximize their respective profits as the costs of banks failing and not being bailed out is taken into account.

A.3. **Full bailout.** If we assume that banks are always bailed out, we set $p = 1$, and then $\Pi_B = (r_L - r_D) L - \frac{1}{2} c\pi^2 L$. Thus from the first order condition $\frac{\partial \Pi_B}{\partial \pi} = -c\pi L = 0$ we get that $\pi_1^* = 0$. We here get the common result that a bailout induces moral hazard by incentivising banks to take high risks.

The welfare for deriving the social optimum is $\Pi_W = (r_L - r_D) L - \frac{1}{2} c\pi^2 L - (1-\pi) \Psi_B$ and hence $\frac{\partial \Pi_B}{\partial \pi} = -c\pi L + \Psi_B = 0$, solving for $\widehat{\pi}_1^* = \frac{\Psi_B}{c}$. Taking into account the bailout costs will here limit the amount of risk banks are exposed to.

A.4. **Fixed bail-out probability.** We now assume that unlike in our model, resources are not fixed, but in all situations a fixed bailout probability p is applied. In this case $\Pi_B = (\pi + (1-\pi) p) (r_L - r_D) L - \frac{1}{2} c\pi^2 L$ and hence $\frac{\partial \Pi_B}{\partial \pi} = (1-p)(r_L - r_D) - c\pi = 0$, solving for $\pi = \frac{1-p}{\eta}$ and $\Pi_R = -(1-\pi)(p\Psi_B + (1-p)\Psi_F)$, such that after inserting for π we have from the first order condition that $p_p^* = \frac{1+(1-\eta)\widehat{\Psi}}{2\widehat{\Psi}}$. This then gives us $\pi_p^* = \frac{(1+\eta)\widehat{\Psi}-1}{2\eta\widehat{\Psi}}$.

The social welfare is given by $\Pi_W = (\pi + (1-\pi) p)(r_L - r_D) L - \frac{1}{2} c\pi^2 L - (1-\pi)(p\Psi_B + (1-p)\Psi_F)$, such that $\frac{\partial \Pi_W}{\partial p} = (1-\pi)((r_L - r_D) - (\Psi_B - \Psi_F)) L > 0$ as $\Pi_B < \Psi_F$ and thus $\widehat{p}_p^* = 1$. We also have $\frac{\partial \Pi_W}{\partial \pi} = (1-p)(r_L - r_D) L - c\pi L + (p\Psi_B + (1-p)\Psi_F) L = 0$, which after inserting $p = 1$, we solve for $\widehat{\pi}_p^* = \frac{\Psi_B}{c}$.

A.5. **Comparison of bank risks.** As we have $\widehat{\Psi}(\eta - 1) < 1$, we find $\pi_0^* > \pi_p^* = \pi_{\overline{\lambda}} > \pi_1^* = 0$. We also have that $p_p^* = p_{\overline{\lambda}}^*$. Hence committing fixed resources to a bailout as in the model of the main section of this paper or committing to bailing out a fraction of banks in any situation is equivalent. This observation arises from the construction of our model where the amount of resources the regulator commits would be such that the optimal bailout probability is achieved. Hence the two models are equivalent.

We can now easily verify that for the social optimum we have $\widehat{\pi}_0^* > \widehat{\pi}_1^* = \widehat{\pi}_p^* = \widehat{\pi}_{\overline{\lambda}}^*$ and $\widehat{p}_p^* = \widehat{p}_{\overline{\lambda}}^* = 1$. We see that unless bailouts are prohibited, the risk-taking in the social optimum is identical in all cases.