Propagation in a reaction-diffusion equation

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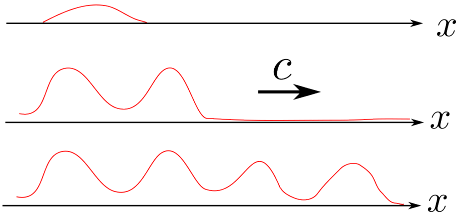
Probability meets Biology II

The Fisher-KPP equation

$u(t,x)$: density of individuals

$$\begin{array}{cccccc} \text{Evolution} & = & \text{movements} & + & \text{demography}-\text{competition} \\ \partial_t u(t,x) & = & \Delta u & + & (r(x)-u)u \end{array}$$

Heterogeneous environments :

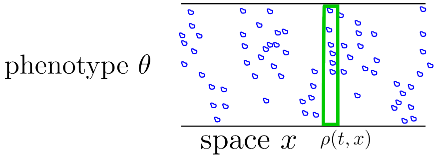


Fisher 1937, Kolmogorov, Petrovsky and Piskounov 1937

Add a phenotypic variable

$u(t,x) \to u(t,x,\theta), \qquad \theta \in [0,1]$

$$\begin{array}{ccccccc} \text{Evolution} & = & \text{movements} & + & \text{mutations} & + & \text{demography}-\text{compet.} \\ \partial_t u(t,x,\theta) & = & \Delta_x u & + & \Delta_\theta u & + & (r(x,\theta)-\rho)u \end{array}$$



Here,

$$\rho(t,x) = \int_0^1 u(t,x,\theta)\,\mathrm{d}\theta.$$

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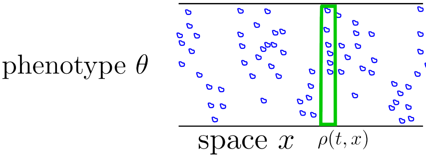

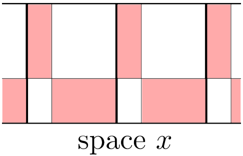

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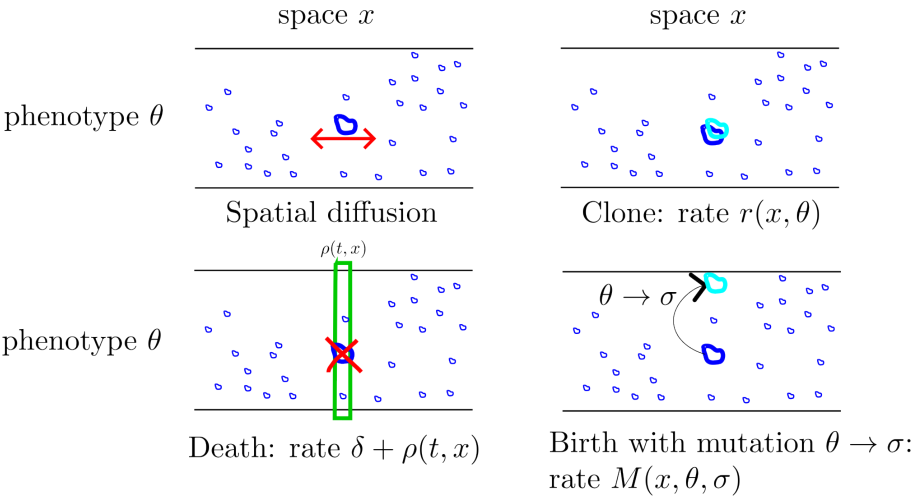
$$\rho(t,x) = \int_0^1 u(t,x,\theta)\,\mathrm{d}\theta.$$

- Periodic heterogeneous environment
- **Goal : Spreading speed.**

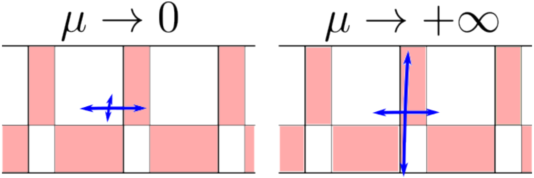
Origin : Individual-based model

Champagnat et Méléard, 2007 :



Scaling

$$\begin{array}{ccccccc} \text{Evolution} & = & \text{movements} & + & \text{mutations} & + & \text{demography}-\text{compet.} \\ \partial_t u(t,x,\theta) & = & \Delta_x u & + & \mu\Delta_\theta u & + & r(x)\,u - \rho u \end{array}$$



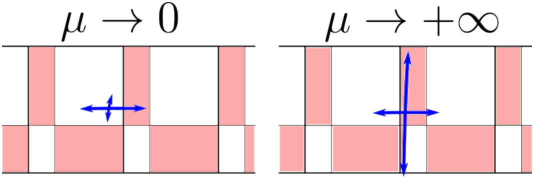
$\rightarrow$ Behaviour of the spreading speed with respect to μ.

Property : Optimal mutation rate μ^* :

$$c(\mu^*) = \max_{\mu > 0} c(\mu).$$

Tradeoff :

- μ too small : cannot enjoy the better adapted phenotypes ;
- μ too large : suffer from the ill-adapted phenotypes.



A qualitative property

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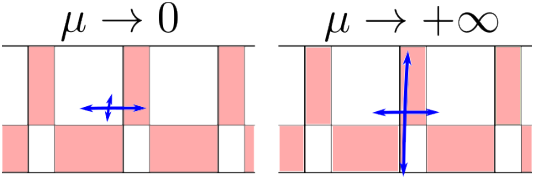

To what extent is this result realistic from a biological point of view ?
What kind of data could we use to test it ?

Thank you !

Bibliography :

Fisher, 1937.
Kolmogorov, Petrovsky, Piskounov, 1937.

Prévost, 2004.
Champagnat, Méléard, 2007.