Some tips for Math 263: ODEs for Engineers Matt Roberts

$y^{(n)}$ means "the nth derivative of y", i.e. $y^{(n)} = \frac{d^n y}{dx^n}$. On the other hand, y^n means "$y \times y \times y \times \ldots \times y$" ($n$ times). So $y^{(n)} \neq y^n$.

Check equations for trivial solutions, like $y = 0$ or $y = c$, first. Often the methods we have learnt for solving equations fail in trivial cases, usually because they involve dividing by 0.

The object of the course is to be able to solve equations that involve derivatives of an unknown function (y) of one variable (usually x). Ideally we could always write our solution in the form $y = f(x)$, but sometimes that's not possible. We call something of the form $g(x, y) = 0$ an implicit solution — sometimes that's the best we can do. The important thing is that our solution doesn't involve derivatives. For example, the equation

$$y' = \frac{e^{-y}}{1 + y}$$

has solutions

$$ye^y = x + C$$

which we can't write $y = f(x)$ (at least not using common functions).

If $W(y_1, y_2)(x) \neq 0$, then y_1 and y_2 are linearly independent. However, this does NOT mean that if $W(y_1, y_2)(x) = 0$ then y_1 and y_2 are linearly dependent. For general functions y_1 and y_2, it is sometimes possible to choose a bad point x such that $W(y_1, y_2)(x) = 0$ yet y_1 and y_2 are still linearly independent. However, if y_1 and y_2 are solutions to a linear homogeneous ODE, then things are much nicer. In this case $W(y_1, y_2)(x)$ is either zero for ALL x, in which case y_1 and y_2 are LD, or $W(y_1, y_2)(x) \neq 0$ for ALL x, in which case y_1 and y_2 are LI.

Complex numbers can be written in two forms: $a + ib$, where a and b are real numbers, or $re^{i\theta}$, where r and θ are real numbers. We can move between the two as follows:

$$a + ib = (a^2 + b^2)e^{i \tan^{-1}(b/a)};$$

$$re^{i\theta} = r\cos\theta + ir\sin\theta.$$

More tips will appear here as the course progresses. If you have any suggestions, please let me know: `matthew.roberts@mcgill.ca`