Previously on MA40189:

- Beta-Binomial updating
- prior $\theta \sim Beta(\alpha, \beta)$

$$f(\theta) = \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1}$$

- likelihood $X \mid \theta \sim Bin(n, \theta)$

$$f(x \mid \theta) = P(X = x \mid \theta) = \binom{n}{x} \theta^{x}(1-\theta)^{n-x}$$

Today on MA40189:

- Posterior $\propto$ Prior $\times$ Likelihood

$$f(\theta \mid x) = c\theta^{\alpha+x-1}(1-\theta)^{\beta+n-x-1}$$

  for some constant c not involving θ

- posterior $\theta \mid x \sim Beta(\alpha + x, \beta + n - x)$
- the prior and posterior are from the same family: this is a conjugate update
- formally define conjugacy
- with respect to the Binomial likelihood, the Beta distribution is a conjugate family
- revisit example of tossing coins and drawing pins
  - role of the prior in the posterior
  - strong and weak priors