Previously on MA40189:

- Gibbs sampler: sample from a high-dimensional distribution by sampling from low-dimensional conditionals
- distribution of interest is $\pi(\theta)$ where $\theta = (\theta_1, \ldots, \theta_d)$
- for each $p = 1, \ldots, d$ obtain $\theta_p^{(t)}$ from conditional distribution

$$\pi(\theta_p \,|\, \theta_1^{(t)}, \ldots, \theta_{p-1}^{(t)}, \theta_{p+1}^{(t-1)}, \ldots, \theta_d^{(t-1)})$$

Today on MA40189:

- sampling from the joint distribution of $\theta = (\theta_1, \theta_2)$ where

$$f(\theta_1, \theta_2) \;\propto\; \binom{n}{\theta_1} \theta_2^{\theta_1+\alpha-1}(1-\theta_2)^{n-\theta_1+\beta-1}$$

  for $\theta_1 \in \{0, 1, \ldots, n\}$ and $0 \leq \theta_2 \leq 1$

  - $f(\theta_1 \,|\, \theta_2) \propto f(\theta_1, \theta_2)$ as a function of θ_1
  - $f(\theta_2 \,|\, \theta_1) \propto f(\theta_1, \theta_2)$ as a function of θ_2
- Illustrate the example
- Gibbs sampler can be viewed as a special case of the Metropolis-Hastings algorithm:
  - each iteration t consists of d Metropolis-Hastings steps
  - each with an acceptance probability of one
- Explore why the Metropolis-Hastings algorithm works