Previously on MA40189:

- pdf of exchangeable $X_1, X_2, \ldots$ can be expressed as

$$f(x_1, \ldots, x_n) \;=\; \int_\theta \left\{ \prod_{i=1}^{n} f(x_i \,|\, \theta) \right\} f(\theta)\, d\theta$$

- can view the X_i as conditionally independent given θ
- $t(X)$ sufficient for X for learning about θ if

$$f(x \,|\, \theta) = g(t, \theta) h(x)$$

- k-parameter exponential family:

$$f(x \,|\, \theta) \;=\; \exp \left\{ \sum_{j=1}^{k} \phi_j(\theta) u_j(x) + g(\theta) + h(x) \right\}$$

Today on MA40189:

- exchangeable sequences with exponential family likelihood have sufficient statistics:

$$t_n \;=\; t_n(X_1, \ldots, X_n) \;=\; \left[n, \sum_{i=1}^{n} u_1(X_i), \ldots, \sum_{i=1}^{n} u_k(X_i) \right]$$

- conjugacy of exponential family distributions:
  - regard $f(x \,|\, \theta)$ as a function of θ,

$$f(x \,|\, \theta) \;=\; \exp \left\{ \sum_{j=1}^{k} u_j(x) \phi_j(\theta) + 1 g(\theta) + h(x) \right\}$$

  - posterior is

$$f(\theta \,|\, x) \;\propto\; \exp \left\{ \sum_{j=1}^{k} (a_j + u_j(x)) \phi_j(\theta) + (d+1) g(\theta) \right\}$$