Previously on MA40189:

- representation theorem for 0-1 exchangeable RVs

$$f(x_1, \ldots, x_n) = \int_0^1 \left\{ \prod_{i=1}^{n} \theta^{x_i}(1-\theta)^{1-x_i} \right\} f(\theta)\, d\theta$$

  where $y_n = \Sigma_{i=1}^n x_i$ and $\theta = \lim_{n\to\infty} \frac{y_n}{n}$

- general representation theorem (simplified form)

$$f(x_1, \ldots, x_n) = \int_\theta \left\{ \prod_{i=1}^{n} f(x_i \,|\, \theta) \right\} f(\theta)\, d\theta$$

Today on MA40189:

- infinitely exchangeable RVs may be viewed as being conditionally independent given a parameter θ
- typically use the shorthand exchangeable for this scenario
- example of exchangeable X_i:

$$X_i \,|\, \theta \sim N(\theta, \sigma^2), \;\; \theta \sim N(\mu_0, \sigma_0^2), \;\; \theta \,|\, x \sim N(\mu_n, \sigma_n^2)$$

  where

$$\frac{1}{\sigma_n^2} = \frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}, \;\; \mu_1 = \left(\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}\right)^{-1} \left(\frac{\mu_0}{\sigma_0^2} + \frac{n\overline{x}}{\sigma^2}\right)$$

- $t(X)$ sufficient for X for learning about θ if

$$f(x \,|\, \theta) = g(t, \theta)h(x)$$

- k-parameter exponential family:

$$f(x \,|\, \theta) = \exp\left\{ \sum_{j=1}^{k} \phi_j(\theta)u_j(x) + g(\theta) + h(x) \right\}$$