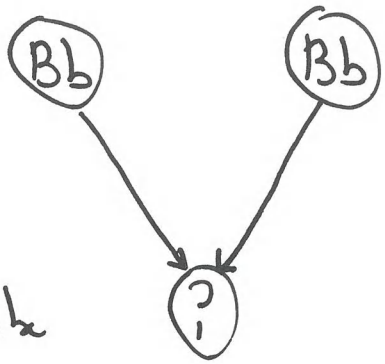
10/2/2022 PROBLEMS CLASS 0 – NOTES MADE ON THE VISUALISER

- Heterozygous (Bb)
- Homozygous (BB or bb)
- brown (bb)
- black (Bb or BB)

Generation 0 (Parents)



G0: both parents Bb

Inherited gene equally likely to be B or b.

a) Generation 1: offspring

Possibilities: G1 – BB offspring is BB } black mouse
G1 – Bb offspring is Bb
G1 – bb offspring is bb } brown mouse

Interested in how these probabilities change with data.

$$\mathbb{P}(G1\text{-}BB \mid G0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\mathbb{P}(G1\text{-}Bb \mid G0) = 2\left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{2}$$

$$\mathbb{P}(G1\text{-}bb \mid G0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Hence, $\mathbb{P}(G1\text{-black} \mid G0) = \mathbb{P}(G1\text{-}BB \mid G0) + \mathbb{P}(G1\text{-}Bb \mid G0)$
$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}.$

$$\mathbb{P}(G1\text{-brown} \mid G0) = \mathbb{P}(G1\text{-}bb \mid G0)$$
$$= \frac{1}{4}.$$

Learn that "G1-black" occurs.

Current knowledge: G0, $\overbrace{\text{G1-black}}^{\text{new data}}$

Update probabilities for G1-BB, G1-Bb, G1-bb given this new information.

$$\overbrace{\mathbb{P}(\text{G1-BB} \mid \text{G1-black}, \text{G0})}^{\text{"POSTERIOR"}} = \frac{\mathbb{P}(\text{G1-BB}, \text{G1-black} \mid \text{G0})}{\mathbb{P}(\text{G1-black} \mid \text{G0})}$$

(probability of data given the model) "LIKELIHOOD"

$$= \frac{\overbrace{\mathbb{P}(\text{G1-black} \mid \text{G1-BB}, \text{G0})}^{\text{"LIKELIHOOD"}}\ \overbrace{\mathbb{P}(\text{G1-BB} \mid \text{G0})}^{\text{"PRIOR"}}}{\mathbb{P}(\text{G1-black} \mid \text{G0})}$$

$$= \frac{1 \times 1/4}{3/4} = \frac{1}{3}.$$

$$\mathbb{P}(\text{G1-Bb} \mid \text{G1-black}, \text{G0})$$

$$= \frac{\mathbb{P}(\text{G1-black} \mid \text{G1-Bb}, \text{G0})\ \mathbb{P}(\text{G1-Bb} \mid \text{G0})}{\mathbb{P}(\text{G1-black} \mid \text{G0})}$$

$$= \frac{1 \times 1/2}{3/4} = \frac{2}{3}.$$

Similarly, $\mathbb{P}(\text{G1-bb} \mid \text{G1-black}, \text{G0}) = 0.$

Hence, $\mathbb{P}(\text{G1-homozygous} \mid \text{G1-black}, \text{G0}) = \mathbb{P}(\text{G1-BB} \mid \text{G1-black}, \text{G0})$
$= 1/3$

$\mathbb{P}(\text{G1-heterozygous} \mid \text{G1-black}, \text{G0}) = \mathbb{P}(\text{G1-Bb} \mid \text{G1-black}, \text{G0})$
$= 2/3$

"Parameter"	G1 - BB	G1 - Bb	G1 - bb
"Prior"	$1/4$	$1/2$	$1/4$
"Likelihood" (data G1-black)	1	1	0
Prior × Likelihood	$1/4$	$1/2$	0
Posterior	$\frac{1/4}{1/4+1/2+0} = \frac{1}{3}$	$\frac{1/2}{1/4+1/2+0} = \frac{2}{3}$	$\frac{0}{1/4+1/2+0} = 0$

b). Generation 2: "G1-black" mated with bb, "G1".

$$\mathbb{P}(G1\text{-}BB \mid G1\text{-black}, G0) = \mathbb{P}(G1\text{-}BB \mid G1, G0)$$
$$\mathbb{P}(G1\text{-}Bb \mid G1\text{-black}, G0) = \mathbb{P}(G1\text{-}Bb \mid G1, G0)$$

New data: 7B0, seven black offspring result from G1 mating.

New posteriors:

$$\underbrace{\mathbb{P}(G1\text{-}BB \mid 7B0, G1, G0)}_{\text{POSTERIOR}} \propto \underbrace{\mathbb{P}(7B0 \mid G1\text{-}BB, G1, G0)}_{\text{LIKELIHOOD}} \underbrace{\mathbb{P}(G1\text{-}BB \mid G0, G1)}_{\text{PRIOR}}$$
$$= 1 \times 1/3 = 1/3.$$

$$\mathbb{P}(G1\text{-}Bb \mid 7B0, G1, G0) \propto \mathbb{P}(7B0 \mid G1\text{-}Bb, G1, G0)\,\mathbb{P}(G1\text{-}Bb \mid G0, G1)$$
$$= \left(\frac{1}{2}\right)^7 \times 2/3 = \left(\frac{1}{2}\right)^6 \frac{1}{3}$$

Thus,

$$\underset{\text{(homozygous)}}{\mathbb{P}(G1\text{-}BB \mid 7B0, G1, G0)} = \frac{1/3}{1/3 + (1/2)^6\, 1/3} = \frac{2^6}{2^6+1} = \frac{64}{65}$$

$$\underset{\text{(heterozygous)}}{\mathbb{P}(G1\text{-}Bb \mid 7B0, G1, G0)} = \frac{(1/2)^6\, 1/3}{1/3 + (1/2)^6\, 1/3} = \frac{1}{2^6+1} = \frac{1}{65}$$