Equality

We now return to the question of equality: what does it mean when we say two things are equal?

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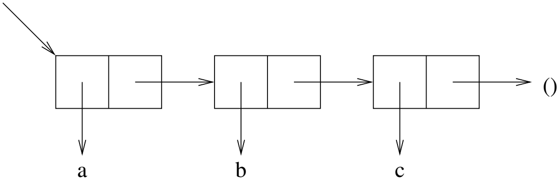
We will have to approach this carefully, starting with the way datastructures are stored in memory

Equality

Lists in Memory

We often draw pairs (also called *cons cells*) as blocks:

`(a b c)` is



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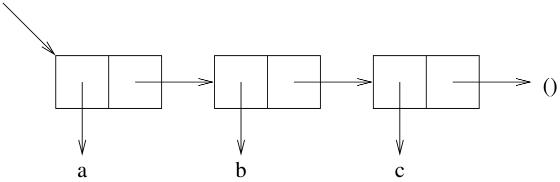

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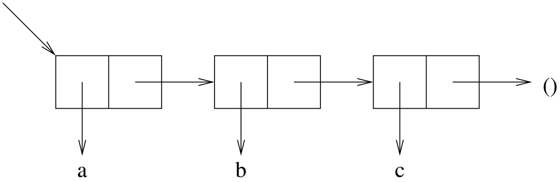

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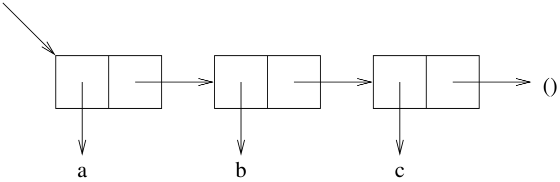

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A pair really is a pair of pointers in memory

Equality

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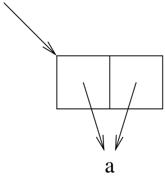
The pair made by `(cons 'a 'a)`, namely `(a . a)` is



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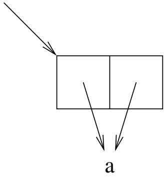

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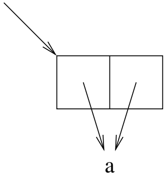

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In Lisp, a symbol of a given name is unique within the system, unlike strings: there may be several copies of `"cat"` in memory

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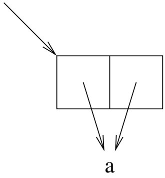

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Uniqueness of symbols is pretty much a defining property of symbols in Lisp

Equality

Aside

Another subtle point:

```
(let ((x 1))
  (let ((x 2))
    ... x ...)
  ... x ...)
```

Regarded as variables (code), the two `x`s are different, and they refer to different memory locations

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It's a matter of which properties you are thinking about

Equality

Lists in Memory

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Some implementations may even have the `car` and `cdr` parts in entirely separate areas of memory

It doesn't really matter and the Lisp system deals with it: you never see memory locations in Lisp (unless. . .)

Equality

Lists in Memory

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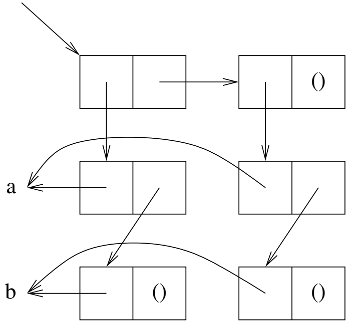
And, as a consequence, `list` guarantees to create an all-new list

Equality

Lists in Memory

The list made by `(list (list 'a 'b) (list 'a 'b))` is `((a b) (a b))`

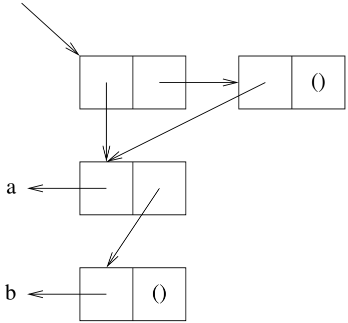


For convenience, I have drawn pointers to `()` as ()

Contrast with the list made in

```
(let ((lab (list 'a 'b)))
   (list lab lab))
```

$\rightarrow$ `((a b) (a b))`



Equality

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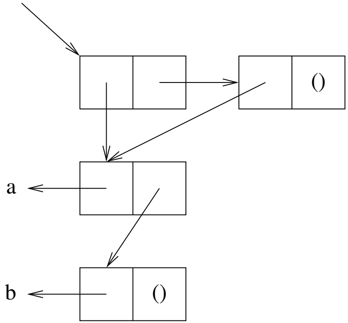

Very different from the previous picture!

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Lists in Memory

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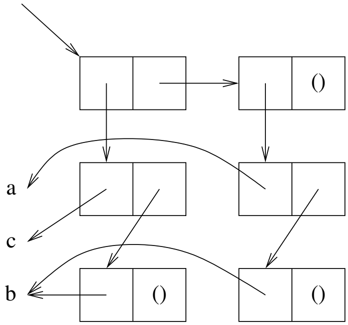
In the second, the sublists are *shared*: the second sublist is the *same* memory as the first sublist

In the first, the sublists are separate: the second sublist occupies *different* memory from the first sublist

If we take the first example and somehow update the first sublist to have a `c` instead of the `a` we get



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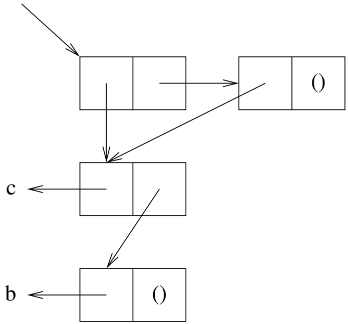

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But it's not "both" as there's only one

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And this applies to all such structures in all languages, not just Lisp

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Lists in Memory

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So the first example guarantees separate sublists

The second, with `(list lab lab)` we are being explicit about sharing the list `lab`

Exercise. Draw boxes and arrows to explain the differences between

- `(list '(a b) '(c d))`
- `(cons '(a b) '(c d))`
- `(append '(a b) '(c d))`

Each function here makes new cons cells: they *do not modify existing cons cells*

Also: the results from `append` shares the second argument, but makes a new copy of the first argument (Exercise: why?). This makes `append` a very expensive operation if the first argument is a long list

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It is rare that other languages are even aware of this issue, leading to all kinds of bugs from programmers using them

Equality

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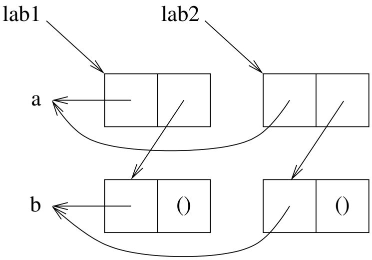
Suppose `lab1` and `lab2` have the values created by separate calls `(list 'a 'b)`

```
(let ((lab1 (list 'a 'b))
      (lab2 (list 'a 'b)))
   ...)
```

Equality

So the two lists occupy different chunks of memory



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`(eq lab1 lab2)` $\rightarrow$ `()`
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All objects are `eq` to themselves (except in Common Lisp…)

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7. Else return `()`

Equality

In brief, two pairs are `equal` if they are the same pair (`eq`), or both

- their `cars` are `equal`
- and their `cdrs` are `equal`

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Though `strcmp` is provided for strings

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Some implementations had `(eq 1023 1023)` true but `(eq 1024 1024)` false

Equality

Note: in the examples above we used

```
(let ((lab1 (list 'a 'b)
      (lab2 (list 'a 'b))) ...)
```

rather than

```
(let ((lab1 '(a b))
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So `(eq '(a b) '(a b))` could be either `t` or `()`

Equality

Exercise. What might you get from

```
(eq 'cat 'cat)
```

```
(eq "cat" "cat")
```

Exercise. Try

```
(eq '(a b) '(a b))
```

on a few Lisps

Equality

Aside: Equality in Mathematics

In various branches of Mathematics you have to define your own equality

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We generally want:

- X equals X; reflexivity
- if X equals Y then Y equals X; symmetry
- if X equals Y and Y equals Z then X equals Z; transitivity

Equality

Aside: Equality in Mathematics

We also want

- if M equals N and $X[M/N]$ is what we get when we replace all occurrences of M by N in X, then X equals $X[M/N]$

Equality

Aside: Equality in Mathematics

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- if M equals N and $X[M/N]$ is what we get when we replace all occurrences of M by N in X, then X equals $X[M/N]$

This is substitionality, or substitution of equals by equals

In arithmetic, we can replace $1 + 1$ by 2 wherever we see it in an expression, and not affect the value of the expression

Equality

Aside: Equality in Mathematics

In other areas, e.g., Lambda calculus, a model of computation, $1 + 1$ and 2 are different, as it takes a step of computation to get from one to the other

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In summary: “equal” is a tricky and subtle concept

Exercise. Convince yourself that Lisp `equal` is an equality is the above sense (reflexive, symmetric, transitive, substitutional)

Exercise. And what about

`(eq l (cons (car l) (cdr l)))`

and

`(equal l (cons (car l) (cdr l)))`

for a list `l`?

Exercise. A related concept is *shallow copy* vs. *deep copy*. Read about this

Equality

Skip to the end…

The following was not covered in lectures

It is not examinable, but is worth reading nevertheless!

Equality

Recursion

Lisp is at its most powerful when we think recursively

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```
(defun factorial (n)
  (if (< n 2)
      n
      (* n (factorial (- n 1)))))
```

What happens when we try

```
(defun loop (n)
   (print n)
   (loop (+ n 1)))
```

Equality

Recursion

What happens when we try

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   (print n)
   (loop (+ n 1)))
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This is because each function call takes some stack space and the machine eventually runs out of memory

Equality

Tail Recursion

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So it replaces the function call by a simple jump back to the start of the function `loop`

```
(defun loop (n)
   (print n)
   increment n
   goto top
```

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Good compilers can spot tail calls and do this optimisation

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```
...
(foo a)
...
```

```
(defun foo (n)
  ...
  (bar n))
```

```
(defun bar (m)
  ...)
```

Equality

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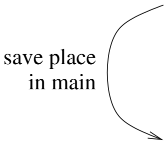


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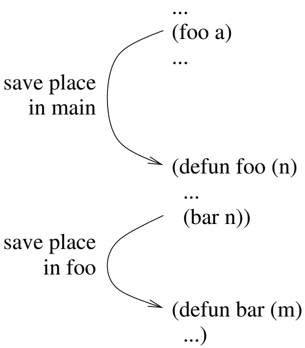

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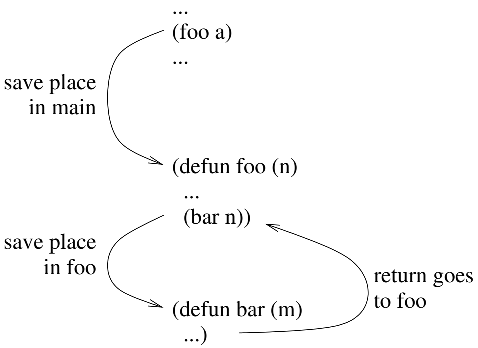

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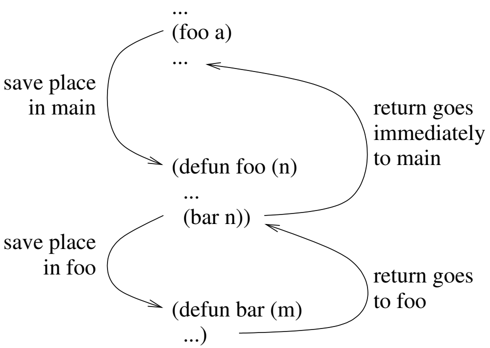

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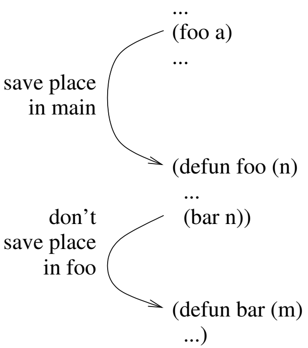

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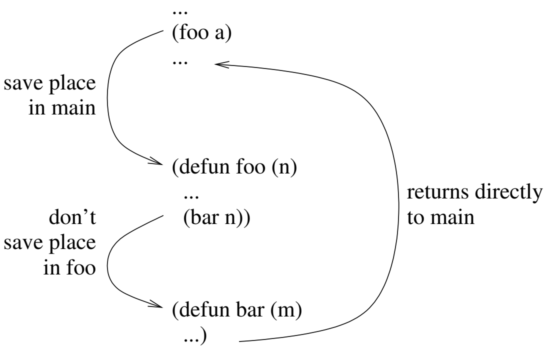

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This simple observation allows us to have arbitrary loops, but to write them naturally recursively

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```
(defun foo (n)
  ...
  (foo (+ n 1)))
```

don't save
place in foo

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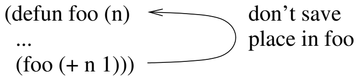

The compiler does clever stuff behind our backs, but compilers are always doing that

```
(defun loop2 (n)
  (loop2 (+ n 1))
  (print n))
```

is not tail recursive, as we need to save where we are before the recursive call to `loop2` to come back and do the `print`

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In reality, we don’t actually come back ever

This one would run until it ran out of stack space unless we have a really clever compiler

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Tail Recursion

```
(defun foo (n)
  (print n)
  (bar (+ n 1)))

(defun bar (m)
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are mutually tail recursive: the compiler can replace the function call to `bar` by a jump to `bar`; similarly the other way round

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This, too, will run forever (if the compiler spots it)

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EuLisp (Euscheme): yes. Clisp: interpreted, no; compiled, yes. Scheme: always, since defined in the language specification, Clojure: no, allegedly because Java doesn't but this is not a valid implication

Loops in other languages are replaced by tail recursive calls in the functional style

```
for (i = 0; i < 10; i++) {
  do something
}
```

becomes

```
(defun loopy (i)
  (when (< i 10)
        do something
        (loopy (+ i 1))))

(loopy 0)
```

Or even

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(labels ((loopy (i)
            (when (< i 10)
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There are better ways to do something to a sequence of objects

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But note within the body of the function `loopy` the variable `i` is never updated; the variable `i` does not vary

It would be quite easy to add a "`for`" form to Lisp (and some Lisps do) that implements

```
(for init test inc body1 body2 ...)
```

as

```
(labels ((loopy ()
             (when test
               body1 body2 ...
               inc
               (loopy))))
  init
  (loopy))
```

But that's not a route we shall follow

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Tail Recursion

Many functions can, with some effort, be converted to tail recursive style

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```
(defun factorial (n)
  (if (< n 2) 1 (* n (factorial (- n 1)))))
```

is not tail recursive

```
(defun fact (n)
  (factaux n 1))

(defun factaux (n sofar)
  (if (< n 2)
      sofar
      (factaux (- n 1) (* n sofar))))
```

is tail recursive

Equality

Tail Recursion

```
(defun fact (n)
  (factaux n 1))

(defun factaux (n sofar)
  (if (< n 2)
      sofar
      (factaux (- n 1) (* n sofar))))
```

is tail recursive

Whether it is worth it is a question you must address in each circumstance

Note: tail recursion is something done by the compiler, but the programmer should be aware it exists to make good use of it

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The loss is worth the gain, though