MA50196: Sheet 8

1. A share has a current price of 30 and its value evolves as a geometric Brownian motion with volatility 40%. The risk-free rate of interest is 8% per annum compounded continuously.

   (a) Find the no-arbitrage price of a 9-month binary put option with strike price 30.

   (b) Find the no-arbitrage price of a 6-month binary call with strike price 40

   (c) Use the Black-Scholes formula to find the no-arbitrage price of a 3-month European call option with strike price 35.

   (d) Use put-call parity to find the no-arbitrage price of a 3-month European put option with strike price 35.

2. Suppose a stock price $S(t)$ follows a geometric Brownian motion with infinitesimal drift μ and volatility σ, and the risk-free rate of interest compounded continuously is r. Assuming $S(0)$ is known, use the principle of risk-neutrality to find the no-arbitrage price of the long position in a futures contract in the stock with maturity T and delivery price K.

   By considering the case where K is the futures price, show that this is consistent with the theory of futures prices discussed earlier on in the course.

3. Suppose a stock price process follows a geometric Brownian motion with infinitesimal drift $\mu = 0.15$ and with volatility $\sigma = 0.25$. Suppose the current stock price $S(0)$ is 12 and that the risk-free rate of interest is 0.10 compounded continuously.

   (a) Find the no-arbitrage price of a bull spread consisting of a long position in a 2-year call with strike 12 and a short position in a 2-year call with strike 14.

   (b) Find the price of a straddle with strike price 13 and maturity in 9 months' time.

   (c) Find the no-arbitrage price of an exotic option with payoff $(S(1.5))^2$.

4. Derive the Black-Scholes formula for the price of a call option directly from the formula $e^{-rT}\tilde{E}[f(S(T))]$ (instead of from the Black-Scholes formula for a put option and put-call parity, as in the lectures).