MA50196: Sheet 7

1. Suppose a stock price $S(t)$ is a random quantity having a log-normal distribution with $\log S(t) \sim N(0,1)$.

   (a) Find $P[S(t) \leq 2]$.

   (b) Find the expected payoff for a binary call option with strike price 3 and maturity date t.

   (c) Find $E[S(t)]$

   (d) Find $E[S(t)^3]$

   (e) A (very) exotic option has payoff

   $$\exp\left(\frac{1}{2}(\log(S(t)))^2 - |\log S(t)|\right).$$

   Find the expected payoff for this option.

2. Suppose a stock price $S(t)$ is a random quantity having log-normal distribution where $\log S(t) \sim N(3, 16)$.

   (a) Find $P[S(t) > 20]$.

   (b) Find the expected payoff for a binary put option with strike price 20 and maturity date t.

   (c) Find $E[S(t)]$

   (d) Find $E[(S(t))^2]$.

3. Find a formula for $\mathrm{Var}(X)$ if $\log(X) \sim N(\mu, \sigma^2)$.

4. A share has a current price of 30 and its value evolves as a geometric Brownian motion with expected rate of return $\mu = 0.08$ and volatility $\sigma = 0.4$.

   (a) Find the expected payoff of a 9-month binary put option (on this share) with strike price 30.

   (b) Find the expected payoff of a 6-month binary call with strike price 40

   (c) Use a put-call parity argument to find the expected payoff of a 6-month binary put with strike price 40.

   (d) Find the expected payoff of a 9-month exotic option with payoff equal to $\log S(T)$, the logarithm of the share price at maturity.

   (e) Find the expected payoff of a 9-month exotic option with payoff equal to the $(\log S(T))^2$, the squared logarithm of the share price at maturity.