MA50196: Sheet 6

1. A stock is currently worth 80. Each year it goes up by 50% or down by 25%. The interest rate compounded annually is zero.

   (a) Find the no-arbitrage price of a 2-year call option with strike price 80.

   (b) Find Δ (delta) for this option. Find the investments in stock and in the risk-free bank account for the first year of a hedging strategy for this option.

   (c) Find the no-arbitrage price of a 4-year European put option with strike price 80.

   (d) Find the no-arbitrage price of a 5-year binary put option with strike price 100.

   (e) For the binary put option in the preceding part, find the value of the option after 1 year (i) if the stock goes up in the first year and (ii) if the stock goes down in the first year. Hence, find Δ for this option. Find the investments in stock and in the risk-free bank account for the first year of a replicating portfolio for this option.

2. Repeat the previous question when the interest rate compounded annually is 25%.

3. A stock is currently worth 100, and each year it goes up or down by 20%. The risk-free interest rate is 10% per annum compounded annually.

   (a) Find the no-arbitrage price for a 3-year Asian call option with strike price 100, i.e. with payoff $(A_3 - 100)^+$, where A_3 is the average of $S(1)$, $S(2)$ and $S(3)$.

   (b) Find the no-arbitrage price of a 3-year lookback option with payoff given by

   $$\max(S(0), S(1), S(2), S(3)) - \min(S(0), S(1), S(2), S(3)).$$

   (c) Find the no-arbitrage price of a 3-year down-and-out put option with strike 105 and barrier 70.

   (d) Find the no-arbitrage price of a two-year American put with strike price 88 (to decide whether to exercise the put early, at $t = 1$ compare the value of the put if you exercise it now with its value if you wait, and then likewise at $t = 0$).

4. Consider the one-period binary stock-price model, with notation as in lectures. Suppose the payoff at time 1 for a certain derivative is z_u if $S(1)/S(0) = u$ and is z_d if $S(1)/S(0) = d$.

   (a) Show directly that there is a replicating portfolio with initial value $P = \alpha^{-1}(pz_u + qz_d)$, and with amount $\Delta S(0)$ invested in the stock at time 0, where we set $p = (\alpha - d)/(u - d)$ and $q = 1 - p$, and $\Delta = (z_u - z_d)/(uS(0) - dS(0))$.

   (b) Suppose this derivative is being sold for price P' with $P' > P$. Describe an explicit arbitrage strategy.

   (c) Suppose this derivative is being sold for price P'' with $P'' < P$. Describe an explicit arbitrage strategy.

5. In the two-period binary stock-price model, suppose a certain derivative has payoff at time 2 of z_2 if $S(2)/S(0) = uu$, z_1 if $S(2)/S(0) = ud$, and z_0 if $S(2)/S(0) = dd$.

   By using the result from part (a) of the previous question, show that there is a replicating strategy for this derivative with initial value $\alpha^{-2}(p^2 z_2 + 2pqz_1 + q^2 z_0)$.

   Hint. You may find it useful to set $z_u = \alpha^{-1}(pz_2 + qz_1)$.