

## MA50196: Sheet 5

1. Suppose a stock is currently worth 64, and each month its value either goes up by 25% (with probability  $2/3$ ) or goes down by 25% (with probability  $1/3$ ), with the stock's behaviour in successive months assumed independent.
  - (a) Find the probability distribution of the stock price in 3 months' time.
  - (b) Find the expected value of the payoff for a European call option with strike price 70 and maturity in 3 months' time.
  - (c) Find the expected value of the payoff for a straddle comprising a European call option and a European put option, both with strike price 70 and maturity in 3 months' time.
  - (d) Find the expected value of the payoff for a bull spread consisting of a long position in a European call option with strike price 70 together with the short position in a European call option with strike price 85.
  - (e) Find the expected value of the payoff for a discrete-time lookback call option with maturity in 3 months' time with payoff equal to  $S(3) - \min(S(0), S(1), S(2), S(3))$  (here  $S(n)$  denotes the stock price after  $n$  months).
  - (f) Find  $E[S(8)]$ , the expected value of the stock price after 8 months.
2. Suppose a stock price is currently 81. Suppose each year the stock price either goes up by one-third, or remains constant, each with probability  $1/2$ , independent of other years. Find the expected value of the payoff for:
  - (a) a binary put option with strike price 150 and maturity in 5 years' time;
  - (b) a binary call option with strike price 150 and maturity in 5 years' time
3. Suppose a stock price is currently  $S(0) = 100$ . It is known that in 1 year's time the stock price  $S(1)$  will be either 120 or 80. Also, the interest rate is zero so that the value at time 1 of a risk-free investment now of 1 is  $\alpha = 1$ .
  - (a) Find the value of  $p = P[S(1) = 120]$  which ensures that  $E[S(1)] = 100\alpha$ .
  - (b) Find the no-arbitrage price  $C$  of a European call option on this stock with maturity in 1 year and strike price 90.
  - (c) Find a hedging strategy for this option, i.e. a way to invest total amount  $C$  in stocks and in a risk-free bank account so as to have wealth at time 1 that replicates the option's payoff.
4. Repeat the previous question with the risk-free interest rate (compounded annually) changed to 0.1 so that  $\alpha$  is changed to 1.1
5. A stock is currently worth 50. Next year it will be worth either 70 or 40. A risk-free investment of 100 now will be worth 110 in one year's time.
  - (a) Find the no-arbitrage price of a European call option with strike 60 and maturity one year from now.
  - (b) Describe the hedging strategy (replicating portfolio) for the above call option.