

MA50196: Sheet 5

1. Suppose a stock is currently worth 64, and each month its value either goes up by 25% (with probability 2/3) or goes down by 25% (with probability 1/3), with the stock's behaviour in successive months assumed independent.
 - (a) Find the probability distribution of the stock price in 3 months' time.
 - (b) Find the expected value of the payoff for a European call option with strike price 70 and maturity in 3 months' time.
 - (c) Find the expected value of the payoff for a straddle comprising a European call option and a European put option, both with strike price 70 and maturity in 3 months' time.
 - (d) Find the expected value of the payoff for a bull spread consisting of a long position in a European call option with strike price 70 together with the short position in a European call option with strike price 85.
 - (e) Find the expected value of the payoff for a discrete-time lookback call option with maturity in 3 months' time with payoff equal to $S(3) - \min(S(0), S(1), S(2), S(3))$ (here $S(n)$ denotes the stock price after n months).
 - (f) Find $E[S(8)]$, the expected value of the stock price after 8 months.
2. Suppose a stock price is currently 81. Suppose each year the stock price either goes up by one-third, or remains constant, each with probability 1/2, independent of other years. Find the expected value of the payoff for:
 - (a) a binary put option with strike price 150 and maturity in 5 years' time;
 - (b) a binary call option with strike price 150 and maturity in 5 years' time
3. Suppose a stock price is currently $S(0) = 100$. It is known that in 1 year's time the stock price $S(1)$ will be either 120 or 80. Also, the interest rate is zero so that the value at time 1 of a risk-free investment now of 1 is $\alpha = 1$.
 - (a) Find the value of $p = P[S(1) = 120]$ which ensures that $E[S(1)] = 100\alpha$.
 - (b) Find the no-arbitrage price C of a European call option on this stock with maturity in 1 year and strike price 90.
 - (c) Find a hedging strategy for this option, i.e. a way to invest total amount C in stocks and in a risk-free bank account so as to have wealth at time 1 that replicates the option's payoff.
4. Repeat the previous question with the risk-free interest rate (compounded annually) changed to 0.1 so that α is changed to 1.1
5. A stock is currently worth 50. Next year it will be worth either 70 or 40. A risk-free investment of 100 now will be worth 110 in one year's time.
 - (a) Find the no-arbitrage price of a European call option with strike 60 and maturity one year from now.
 - (b) Describe the hedging strategy (replicating portfolio) for the above call option.