MA50196: Sheet 4

1. Suppose there are two kinds of stocks (Stock 1 and Stock 2) with price at time t denoted $S_1(t)$ and $S_2(t)$ respectively. Suppose $t = 0$ currently and T is a specified future time, and K is a specified cash amount.

   (a) Consider a 'barter option', i.e. a contract giving the option holder the right but not the obligation at time T to hand over Stock 1 and receive Stock 2 in exchange. Give arbitrage arguments to show that $B \geq S_2 - S_1$, and $B \leq C_2 + P_1$, where C_2 denotes the price of a call option in Stock 2 and P_1 denotes the price of a put option in Stock 1 (both with maturity at T and strike price K).

   (b) Consider a 'basket call option' giving the contract holder the right but not the obligation at time T to buy stocks 1 and 2 together for price K. Let C_{1+2} denote the price of this option. Suppose $K_1 > 0$ and $K_2 > 0$ with $K_1 + K_2 = K$. For $i = 1, 2$ let C_{i,K_i} denote the price of a call option in Stock i with strike price K_i. Give an inequality between C_{1+2} and $C_{1,K_1} + C_{2,K_2}$ and justify this with an arbitrage argument.

2. Suppose the current price of a stock is £100. Suppose as a mathematical model, that each year the stock price either goes up by £10 (with probability 0.7) or goes down by £10 (with probability 0.3), and that successive years are independent.

   (a) Find the probability that the stock goes up in exactly two of the next four years.

   (b) Find the probability that the stock goes up in at least two of the next five years.

   (c) Suppose you hold a call option in the stock with maturity in 2 years and strike price £105. What is the probability that it will be worth your while to exercise the option?

   (d) Suppose you hold a put option in the stock with maturity in 2 years and strike price £105. What is the probability that it will be worth your while to exercise the option?

   (e) Suppose you hold a binary call option with maturity in 6 years and strike 125. What is the probability that you will exercise the option?

3. Suppose the current interest rate in Dollars at 6% while those in Pounds and in Yen are at 5%. Suppose that it is thought that in the next 6 months the interest rates for Dollars will stay the same with probability 0.7, go up by 1% with probability 0.1 and go down by 1% with probability 0.2; likewise for Pounds and likewise for Yen. Also, it is thought that changes in these three interest rates in the next six months are independent of each other.

   Find the probability that in six months' time the Dollar interest rate is still greater than either the Pound or Yen interest rate.

4. Suppose a stock price $X = S(T)$ is modelled as a random quantity with possible values 100, 110, 120, 130 and with probabilities

$$P[X = 100] = 0.2, \quad P[X = 110] = 0.3, \quad P[X = 120] = 0.4, \quad P[X = 130] = 0.1$$

   (a) Find $P[X > 110]$ and $P[X \geq 110]$.

   (b) Let Y be the payoff for a European call option on this stock with maturity T and strike price 112. Find the probability distribution of the random quantity Y (i.e. the possible values of Y could take and the associated probabilities). What is the probability that this option is exercised?

   (c) Let Z be the payoff for a European put option on this stock with maturity T and strike price 112. Find the probability distribution of the random quantity Z.

   (d) Let W be the payoff for a binary call option with maturity T and strike 105. Describe the probability distribution of W.

5. Suppose the current stock price is £100 and each month the stock price either goes up by 20% or down by 10%, each with probability 1/2. Assume different months are mutually independent.

   (a) Let X denote the payoff (in £) from a European call option on this stock with strike price 100 and maturity in two months' time. Find the probability distribution of X.

   (b) Let Y denote the payoff (in £) from a European put option on this stock with strike price £100 and maturity in three months' time. Find the probability distribution of Y.

   (c) Let Z denote the payoff (in £) from a lookback call option of duration $T = 2$ months, with payoff equal to $(S(T) - m_T)^+$, where m_T is the minimum stock price over the times $t = 0, 1, 2$. Find the probability distribution of Z.