MA50196: Sheet1

1. An investor enters into a short cotton futures contract where the futures price is 50 pence per kilo. The contract is for the delivery of 50 000 kilos. How much does the investor gain or lose if the cotton price at the end of the contract is (a) 48.20 pence per kilo (b) 51.30 pence per kilo?

2. Suppose you write a put option on 100 shares with a strike price of £115 and maturity 6 months. The current stock price is £118. What have you committed yourself to? How much could you gain or lose?

3. Suppose a European call option to buy a share for £70 costs £2.80 and is held until maturity. Under what circumstances will the holder of the option make a profit?

4. Suppose the interest rate r is 7 per cent per annum, compounded continuously. To give the same return after 1 year, what would be

   (a) The interest rate per annum compounded annually;
   (b) The interest rate per annum compounded semi-annually?

5. As mentioned in class, the value $A(t)$ of an investment A_0 at time zero satisfies the differential equation
$$\frac{dA(t)}{dt} = rA(t)$$
   with initial value $A(0) = A_0$. Find the corresponding differential equation and inital value for $\log A(t)$ and hence find the solution to this differential equation, and to the original differential equation without assuming the answer given in lectures.

6. Suppose the interest rate (compounded continuously) varies with time and is given by a function $r(t)$, i.e. suppose
$$\frac{d}{dt}A(t) = r(t)A(t).$$
   Assuming an initial investment A_0, find a formula for $A(t)$ in terms of $r(s), 0 \le s \le t$ (by a similar argument to the previous question).

7. The price of silver is currently 2.50 pounds per ounce. The futures price for delivery in 1 year is 3.00 pounds. An arbitrageur can borrow money at 10% per annum compounded annually. What should the arbitrageur do?

8. Suppose you enter into a short futures contract to sell gold for £520 per ounce. The size of the contract is 50 ounces. The initial margin is £4000 and the maintenance margin is £3000. What change in the futures price will lead to a margin call?

9. Suppose you enter a six-month forward contract on a non-dividend paying stock when the price is 30 pounds and the risk-free interest rate with continuous compounding is 12% per annum. What is the forward price?

10. A 1-year long forward contract on a non-dividend paying stock is entered into when the stock price is £40 and the risk-free interest rate is 10% per annum with continuous compounding.

   (a) What are the forward price and initial value of the forward contract? Assume the forward price is the delivery price in the contract.
   (b) Six months later, the price of the stock is £45 and the risk-free interest rate is still 10%. What are the forward price and the value of the original forward contract?

1