Answer 3 questions

1. (a) Briefly explain the terms "hedging" and "arbitrage". [4]

(b) Briefly describe in general terms the circumstances under which a company might wish to take a long position in a futures contract in a certain commodity, for hedging purposes, and the nature of risk which could thereby be hedged. [3]

(c) Suppose that certain stock index has value denoted $S(t)$ at time t, and pays dividends continuously at a rate of $qS(t)$, where q is a constant. Suppose the risk-free rate of interest is a constant r, compounded continuously, and suppose a forward contract on this stock index is available at time 0 with a maturity date $T > 0$ and a delivery price K which satisfies the inequality $K > S(0)e^{(r-q)T}$. Describe the strategy that an arbitrageur should adopt with regard to this index. [4]

(d) Suppose the term structure of interest rates both in Pounds Sterling and in Swiss Francs is flat, with the risk-free interest rate per annum at 6% for Sterling and at 4% for Swiss Francs (both compounded continuousuly). Suppose the exchange rate is £0.45 per Swiss Franc. What is the futures price (in Sterling) for a futures contract to buy 1000 Swiss Francs in 3 years' time? [4]

(e) Suppose at time $t = 0$ you took a long position in the contract described in part (d). Two years have passed, and now at time $t = 2$ the exchange rate is £0.50 per Swiss Franc, while the risk-free interest rate in Sterling has changed to 7% but the risk-free interest rate in Swiss Francs is still 4% (both compounded continuously). What is the value now (i.e., at time $t = 2$) of your long position in this contract? [5]

2. (a) Suppose a 2-year bond with nominal value £100 pounds, redeemable at par with coupon rate 4.5% payable annually, is on sale for £102. What is the yield on this bond, expressed as an annual interest rate compounded annually? [4]

(b) Suppose $P(t)$ denotes the price in pounds of a zero-coupon bond with nominal value £100, redeemable at par t years from now, and

$$P(0.25) = 99, \quad P(0.5) = 97.5, \quad P(0.75) = 96, \quad P(1) = 94, \quad P(1.5) = 89.$$

(i) Find the 1-year zero-coupon rate $Z(0,1)$ and the 1.5-year zero-coupon rate $Z(0,1.5)$. Also find the forward rate $f(0,0.75,1.5)$ for a 9-month loan agreed upon now to take place in 9 months' time. All rates should be expressed as annual interest rates compounded continuously. [6]

(ii) Find the swap rate for a 1.5 year interest rate swap versus LIBOR, with semi-annual payments. Express the swap rate as an annual rate compounded semiannually. [5]

(iii) A 1.5 year interest rate swap versus LIBOR with nominal principal of £100 and semiannual payments is entered into. Nine months later, assuming the values of $P(t)$ are still as given above, what is the value of the swap to the party paying floating? Assume the 6-month LIBOR 3 months ago was 5.2% per annum compounded semiannually. [5]

3. (a) What is (i) a European call option (ii) an American put option (iii) an up-and-out European call option? [6]

(b) Let C denote the price at time 0 of a European call option, and P the price at time 0 of a European put option with the same strike price K and the same expiry date T, for an asset which does not bring in any dividends but which incurs storage costs with total present value U for ownership of the asset between time 0 and time T. The risk-free rate of interest is r compounded continuously. Describe a strategy by which you could achieve arbitrage if, in violation of put-call parity, the inequality

$$C + Ke^{-rT} > P + S(0) + U$$

holds. Here $S(t)$ denotes the price of the asset at time t. [5]

(c) Suppose a share with no storage costs or dividends is currently on sale at a price of £42. Suppose the risk-free rate of interest is 7% per annum compounded continuously, and suppose the volatility of the share is 30% (with time measured in years).

(i) Using the Black-Scholes formula

$$C = S\Phi(d_0 + \sigma\sqrt{T}) - Ke^{-rT}\Phi(d_0), \quad d_0 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}},$$

find the price of a European call option on this share with strike price £40 and maturity 1 year from now. [5]

(ii) Find the price of a European put option on the same share with the same strike price and maturity date. [4]

4. (a) Suppose a share with no storage costs or dividends is currently on sale at a price of £35. Suppose the risk-free rate of interest is 9% per annum compounded continuously, and suppose the share price $S(t)$ follows a geometric Brownian motion with infinitesimal drift $\mu = 0.15$ and volatility $\sigma = 0.20$ (here time is measured in years), so that

$$\frac{S(t)}{S(0)} = \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \left(\sigma\sqrt{t}\right)Z\right)$$

where Z is a standard normal random variable.

Using the principle of risk-neutrality, find the price of a binary put option with payoff 1 after 2 years if the stock price is at least £45 at that time, and payoff zero otherwise. [7]

(b) Suppose a share price is currently £80, and it is known that in 1 year's time the share price will be either £100 or £70. Suppose the annual risk-free rate of interest is zero.

(i) Using the principle of risk-neutrality, or otherwise, find the price P of a put option with strike price £90. [5]

(ii) Describe a hedging strategy using initial funds P to replicate the payoff of this option.

Check that this strategy does indeed replicate the payoff from the option under both possible outcomes. [8]