MA40092 PROBLEM SHEET 2

Bookwork question (§1.2, §1.3)
Define the terms *exponential family* and *sufficient statistic.*
State the Generalised Factorisation Theorem and prove it in the special case of a single discrete statistic.

Example 2: Maximum likelihood, sufficient statistics (§1.3, §1.4)
In the following cases, find both the smallest set of sufficient statistics and the maximum likelihood estimators of the parameters:
i) $X_1, \ldots, X_n$ iid $Pois(\lambda)$
ii) $X_1, \ldots, X_n$ iid $U(0, \theta)$
iii) $X_1, \ldots, X_n$ iid $N(\mu, \sigma^2)$ when both parameters are unknown

Example 3: Invariance of maximum likelihood estimation (§1.4)
Suppose $X_1, \ldots, X_n$ are independent exponential random variables with rate parameter λ. Find the maximum likelihood estimator of λ.
Now suppose $X_1, \ldots, X_n$ have density

$$f_\theta(x) = \theta^{-1/2} e^{-x/\sqrt{\theta}},\ \theta > 0$$

What is the maximum likelihood estimator for θ?

Example 4: Bias (§2.1)
Suppose $X_1, \ldots, X_n$ are independent exponential random variables with rate parameter λ.
Show that X_1 and $\overline{X} = (\sum_{i=1}^n X_i)/n$ are unbiased for $(1/\lambda)$.
Find the expected values of $(1/X_1)$ and $(n/\sum_{i=1}^n X_i)$, if they exist. Hence find an unbiased estimator of λ.