MA40092 PROBLEM SHEET 1

Example 1: Exponential families (§1.1, §1.2)
Which of the following families of distributions are exponential families?
i) $Geo(p)$
ii) $Bin(n, p)$ (Don't forget that we generally regard n as known.)
iii) $Beta(a, b)$
iv) $Cauchy(\theta, \eta)$ where the probability density function of the Cauchy random variable is given by

$$f_{\theta,\eta}(x) = \frac{1}{\theta\pi(1 + [(x - \eta)/\theta]^2)}, \ \theta > 0, \ \eta \in (-\infty, \infty), \ x \in (-\infty, \infty)$$

Example 2: Sufficient statistics (§1.3)
Find a single sufficient statistic for independent, identically distributed random variables $X_1, \ldots X_n$ if their distribution is
i) $Geo(p)$
ii) $Beta(a, 1)$
iii) Find a pair of jointly sufficient statistics for the case where $X_1, \ldots X_n$ are an independent sample from a $Beta(a, b)$ distribution.

Example 3: Exponential families, sufficient statistics (§1.2, §1.3)
Let $X_1, \ldots, X_n$ be independent identically distributed random variables with density

$$f_{\alpha,l}(x) = \frac{2l^\alpha x^{-(2\alpha+1)} e^{-l/x^2}}{\Gamma(\alpha)} \qquad\qquad x > 0,$$

for unknown parameters $\alpha > 0$, $l > 0$. Show that this density comes from a distribution which is a member of an exponential family. Find a pair of statistics which are jointly sufficient for α and l.

Example 4: Sufficient statistics (§1.3)
Suppose $X_1, \ldots, X_n$ are independent random variables with a Negative Binomial distribution with parameters r and p. Write down the joint distribution of $X_1, \ldots, X_n$.
Find the smallest possible set of sufficient statistics when
(i) r is fixed and known, p is unknown
(ii) p is fixed and known, r is unknown