Chebychev's inequality

Let $h: \mathbb{R} \to [0, \infty)$ be a non-negative function, then

$$\mathbb{P}(h(X) \geq a) \leq \frac{\mathbb{E}(h(X))}{a} \quad \forall\, a > 0$$

Proof Let $A = \{h(X) \geq a\}$

then $h(X) \geq a I_{[A]}$

so $\mathbb{E}(h(X)) \geq \mathbb{E}(a I_{[A]})$

$= a\,\mathbb{P}(A)$

as a property of indicator functions

□

Using $h(X) = X^2$ gives Chebychev's inequality

$$\mathbb{P}(X^2 \geq a) \leq \frac{\mathbb{E}(X^2)}{a} \quad \text{for } a > 0$$

In our §2.1 setting, $X = (T - \theta)$