```
> restart;march('open',"\\\\myfiles\\masjhd\\Maple\\RegularChains.mla");libname:="H:\\Maple",libname;
```

libname := "C:\Program Files\Maple 2016\lib", "\\myfiles\masjhd\Maple\RegularChains.mla"

libname := "H:\Maple", "C:\Program Files\Maple 2016\lib", "\\myfiles\masjhd\Maple\RegularChains.mla" **(1)**

> *with*(*RegularChains*);

[*AlgebraicGeometryTools*, *ChainTools*, *ConstructibleSetTools*, *Display*, *DisplayPolynomialRing*, *Equations*, *ExtendedRegularGcd*, *FastArithmeticTools*, *Inequations*, *Info*, *Initial*, *Intersect*, *Inverse*, *IsRegular*, *LazyRealTriangularize*, *MainDegree*, *MainVariable*, *MatrixCombine*, *MatrixTools*, *NormalForm*, *ParametricSystemTools*, *PolynomialRing*, *Rank*, *RealTriangularize*, *RegularGcd*, *RegularizeInitial*, *SamplePoints*, *SemiAlgebraicSetTools*, *Separant*, *SparsePseudoRemainder*, *SuggestVariableOrder*, *Tail*, *Triangularize*] **(2)**

> *with*(*SemiAlgebraicSetTools*);

[*BoxValues*, *Complement*, *CylindricalAlgebraicDecompose*, *Difference*, *DisplayParametricBox*, *DisplayQuantifierFreeFormula*, *EmptySemiAlgebraicSet*, *Intersection*, *IsContained*, *IsEmpty*, *IsParametricBox*, *LinearSolve*, *PartialCylindricalAlgebraicDecomposition*, *PositiveInequalities*, *Projection*, *QuantifierElimination*, *RealRootCounting*, *RealRootIsolate*, *RefineBox*, *RefineListBox*, *RemoveRedundantComponents*, *RepresentingBox*, *RepresentingChain*, *RepresentingQuantifierFreeFormula*, *RepresentingRootIndex*, *SignAtBox*, *VariableOrdering*] **(3)**

> $G4 := [a+b+c+d, a\cdot b+b\cdot c+c\cdot d+d\cdot a, a\cdot b\cdot c+b\cdot c\cdot d+c\cdot d\cdot a+d\cdot a\cdot b, a\cdot b\cdot c\cdot d-1]$;
 # same examples as Groebner bases

$$G4 := [a+b+c+d, a\,b+a\,d+b\,c+c\,d, a\,b\,c+a\,b\,d+a\,c\,d+b\,c\,d, a\,b\,c\,d-1] \quad \textbf{(4)}$$

> $R := PolynomialRing([a, b, c, d])$;

$$R := polynomial\_ring \quad \textbf{(5)}$$

> $G5 := [a+b+c+d+e, a\cdot b+b\cdot c+c\cdot d+d\cdot e+e\cdot a, a\cdot b\cdot c+b\cdot c\cdot d+c\cdot d\cdot e+d\cdot e\cdot a+e\cdot a\cdot b, a\cdot b\cdot c\cdot d+b\cdot c\cdot d\cdot e+c\cdot d\cdot e\cdot a+d\cdot e\cdot a\cdot b+e\cdot a\cdot b\cdot c, a\cdot b\cdot c\cdot d\cdot e-1]$;

$$G5 := [a+b+c+d+e, a\,b+a\,e+b\,c+c\,d+d\,e, a\,b\,c+a\,b\,e+a\,d\,e+b\,c\,d+c\,d\,e, a\,b\,c\,d+a\,b\,c\,e+a\,b\,d\,e+a\,c\,d\,e+b\,c\,d\,e, a\,b\,c\,d\,e-1] \quad \textbf{(6)}$$

> $R5 := PolynomialRing([a, b, c, d, e])$;

$$R5 := polynomial\_ring \quad \textbf{(7)}$$

> $T := Triangularize(G4, R)$;

$$T := [regular\_chain, regular\_chain] \quad \textbf{(8)}$$

> $Display(T[1], R)$;

$$\begin{cases} a+c=0 \\ b+d=0 \\ d\,c+1=0 \\ d\neq 0 \end{cases} \quad \textbf{(9)}$$

> $Display(T[2], R)$;
 # These two give us the same structure as last time: note that they've done the 'square-free' for us

$$\begin{cases} a + c = 0 \\ b + d = 0 \\ d\,c - 1 = 0 \\ d \neq 0 \end{cases} \quad \textbf{(10)}$$

> $T5 := Triangularize(G5, R5);$

$T5 :=$ [regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain, regular_chain] **(11)**

> *# lots of special cases, so example*

> $Display(T5[7], R5);$

$$\begin{cases} a + b + c + 2\,e = 0 \\ (4\,c + e)\,b + e\,c - e^2 = 0 \\ c^2 + 3\,e\,c + e^2 = 0 \\ d - e = 0 \\ e^4 + e^3 + e^2 + e + 1 = 0 \\ 4\,c + e \neq 0 \end{cases} \quad \textbf{(12)}$$

> *# A side-condition (which is in fact always true) and then 8 solutions (4 for e, and 2 for each from c).*