$G4 := [a+b+c+d, a \cdot b + b \cdot c + c \cdot d + d \cdot a, a \cdot b \cdot c + b \cdot c \cdot d + c \cdot d \cdot a + d \cdot a \cdot b, a \cdot b \cdot c \cdot d - 1];$

$$[a+b+c+d, a\,b + a\,d + b\,c + c\,d, a\,b\,c + a\,b\,d + a\,c\,d + b\,c\,d, a\,b\,c\,d - 1] \quad \textbf{(1)}$$

with(*Groebner*);

[*Basis*, *FGLM*, *HilbertDimension*, *HilbertPolynomial*, *HilbertSeries*, *Homogenize*, *InitialForm*, *InterReduce*, *IsBasis*, *IsProper*, *IsZeroDimensional*, *LeadingCoefficient*, *LeadingMonomial*, *LeadingTerm*, *MatrixOrder*, *MaximalIndependentSet*, *MonomialOrder*, *MultiplicationMatrix*, *MultivariateCyclicVector*, *NormalForm*, *NormalSet*, *RationalUnivariateRepresentation*, *Reduce*, *RememberBasis*, *SPolynomial*, *Solve*, *SuggestVariableOrder*, *Support*, *TestOrder*, *ToricIdealBasis*, *TrailingTerm*, *UnivariatePolynomial*, *Walk*, *WeightedDegree*] **(2)**

Basis(*G4*, *tdeg*(*a*, *b*, *c*, *d*)); # *a Groebner basis with total degree ordering*

$$[a+b+c+d, b^2 + 2\,b\,d + d^2, b\,c^2 - b\,d^2 + c^2\,d - d^3, b\,c\,d^2 - b\,d^3 + c^2\,d^2 + c\,d^3 - d^4 - 1, b\,d^4 + d^5 - b - d, c^3\,d^2 + c^2\,d^3 - c - d, c^2\,d^4 + b\,c - b\,d + c\,d - 2\,d^2] \quad \textbf{(3)}$$

Basis(*G4*, *plex*(*a*, *b*, *c*, *d*));
A groebner basis with pluely lexicographic ordering. There is no polynomial in d alone, so d is undetermined

$$[c^2\,d^6 - c^2\,d^2 - d^4 + 1, c^3\,d^2 + c^2\,d^3 - c - d, b\,d^4 + d^5 - b - d, c^2\,d^4 + b\,c - b\,d + c\,d - 2\,d^2, b^2 + 2\,b\,d + d^2, a+b+c+d] \quad \textbf{(4)}$$

G4b := [$b+d$, *op*(*G4*)]; # *If we just want the roots,* $(b+d)^2$ *is* **in** *the Groebner basis, so we can just add* $b+d$ (*losing multiplicity information, of course*)

$$[b+d, a+b+c+d, a\,b + a\,d + b\,c + c\,d, a\,b\,c + a\,b\,d + a\,c\,d + b\,c\,d, a\,b\,c\,d - 1] \quad \textbf{(5)}$$

Basis(*G4b*, *plex*(*a*, *b*, *c*, *d*));

$$[c^2\,d^2 - 1, b+d, a+c] \quad \textbf{(6)}$$

Now the solutions are obvious: d is free and c is either the reciprocal of d or its inverse, and b and a are determined