

Block Cylindrical Decompositions (Possible PhD Project)

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1 Problem

The general theory of Cylindrical Algebraic Decomposition [Col75] requires that the cells be arranged cylindrically (i.e. if C_1 and C_2 are two cells in \mathbf{R}^n , and, for any k , π_k is the projection onto the first k coordinates, then $\pi_k(C_1)$ and $\pi_k(C_2)$ are either equal or disjoint). This is what allows Collins to use CAD for Quantifier Elimination: a key logical process.

The treatment in [Dav15, §3.5.4] suggests a weaker concept: *block cylindricity*, where we only enforce this for the k that correspond to quantifier alternations, e.g. for $\forall x_1 \forall x_2 \exists x_3 \exists x_4 \forall x_5$ we would only be concerned about π_2 and π_4 . But we have no algorithm to compute this.

2 Idea

Recent discussions with Chris Brown suggest that his process [Bro14] for constructing *Non-uniform Cylindrical Algebraic Decompositions*, where the projection property is not necessarily guaranteed, might prove a way to construct Block-Cylindrical Decompositions. JHD is prettys ure of collaboration with Chris Brown on this project.

References

- [Bro14] C.W. Brown. Model-based construction of Open Non-uniform Cylindrical Algebraic Decompositions. <http://arxiv.org/abs/1403.6487>, 2014.
- [Col75] G.E. Collins. Quantifier Elimination for Real Closed Fields by Cylindrical Algebraic Decomposition. In *Proceedings 2nd. GI Conference Automata Theory & Formal Languages*, pages 134–183, 1975.

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<http://staff.bath.ac.uk/masjhd/JHD-CA.pdf>, 2015.